THE ELEMENTS
OF
PHYSICS

BY

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PREFACE

In this revision, as in former editions, a limited number of physical principles have been presented, in a way to encourage students to form clear and distinct physical concepts and to discourage the habit of being content with verbal knowledge. The central aim has been to introduce students to physics as a way of thinking that is largely characteristic of our daily lives and to develop in them a capacity for seeing relations between physical phenomena and for applying universal physical principles. Throughout the book the attempt has been to lay a basis for correct scientific thinking rather than to stress the accumulation of facts or to trace the historical development of the subject.

The training and habits of thought of students interested in a survey of physics and its applications have been kept clearly in mind. Topics that closely touch our daily lives have been chosen and the use of mathematical methods has been reduced to a minimum. A large number of applications of physics to engineering, agriculture, biology, and everyday life have been included in order to stimulate students to recognize the universality of physical principles and to find in them an explanation of daily experiences and observations. To encourage accuracy and concreteness of thinking, many solved examples have been introduced.

The practice of printing illustrative material and minor topics in fine print to differentiate them from fundamental concepts and principles has been continued. It is still hoped that this illustrative material makes the fundamental ideas, laws, and principles more concrete and intimate. Ordinarily it should not be necessary to discuss these illustrations either in the lecture or in the classroom. Students should be able to get their significance without the aid of the instructor.

This revision offers an opportunity to make a number of changes and additions that have been suggested as desirable by users of the book. Increased emphasis has been placed on the

importance of physics in the other sciences and in the industries. Many new figures and illustrations have been added in order to make the subject matter as vivid and as interesting as possible. The lists of problems have been thoroughly revised and rearranged. Many new problems have been added to replace others which seemed less desirable. Some important changes in the order of presentation have been made where classroom experience has suggested that such changes would facilitate the use of the book. Definitions of some important physical quantities and derivations requiring the methods of the calculus have been included in two additional appendices. Where students have a knowledge of the calculus, these definitions and derivations are to be preferred over those in the body of the text. Recent advances in physics have made it necessary to rewrite some of the paragraphs relating to modern physics and to introduce a separate chapter on nuclear physics. A chapter on astrophysics has also been added to give students an impression of the applications of physics in modern astronomy. This chapter tends to extend and unify the subject of physics. Wherever possible the methods and points of view of modern physics have been followed.

Again the author gratefully acknowledges his indebtedness to those who have made helpful suggestions and criticisms for the improvement of this book. Especial reference must be made to Professor G. E. Grantham of Cornell University, to Professor L. R. Ingersoll of the University of Wisconsin, and to two of the author's colleagues, Dr. Alva W. Smith and Dr. Harold P. Knauss. The author is also greatly indebted to those who have generously supplied material for illustrations.

As in the past, corrections and criticisms from those who use the book will be welcomed.

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THE ELEMENTS OF PHYSICS

INTRODUCTION

1. Subject Matter of Physics.—Physics is a broad science having wide applications to most aspects of modern life. It is a study about familiar things and an attempt to find satisfactory explanations for them. Its object is to determine exact relations between physical phenomena so that the sequence of physical phenomena can be clearly understood and definitely predicted. Physics deals primarily with phenomena which can be accurately described in terms of matter and energy. Hence, the basic concepts in all physical phenomena are the concepts of **matter and energy.** It therefore becomes of first importance in physics to determine accurately the characteristics of both matter and energy, the laws which govern th[illegible] transformations, and the fundamental relations which exist [illegible]een them. In order then to proceed with a discussion of [illegible]er and energy, it is first necessary to formulate a working [illegible]finition of each of these concepts. Later it will be possible [illegible] arrive at a more adequate understanding of each of these entities, but for the present preliminary definitions are necessary.

2. Matter.—Many definitions of matter have been given from time to time, but the following will serve our present purposes. *Matter is that which can occupy space.* This definition does not undertake to state what matter really is. It only indicates the most obvious characteristic of matter—the fact that it occupies space. Whatever the ultimate nature and structure of matter, it is obvious to the most casual observer that it has a certain constancy and permanence which makes it a useful entity in terms of which to analyze physical phenomena. Its more important characteristics do not change from time to time. It thus affords a basic unit in terms of which to make descriptions of physical phenomena.

3. Conservation of Matter.—Many careful experiments have shown that the quantity of matter existing at the end of a

chemical or physical change is exactly the same as the quantity of matter at the beginning of the experiment. Matter may be changed from one state to another or it may combine with other forms of matter, but in the end the total amount of matter is unchanged. Such experiments have led to one of the very important laws in physics—**the law of conservation of matter. The principle of the conservation of matter states that matter may be altered in form but it can never be created or destroyed.** The amount of matter in the universe remains completely unchanged.

4. Energy.—When matter is in motion, it has a new set of properties differing from those which it manifests when it is at rest. For example, water in falling over a waterfall acquires new properties. It can now be made to drive a water wheel and do useful work. We now say that the water has acquired *energy of motion* instead of its former energy of position. A stretched spring differs greatly in its behavior from an unstretched one. When the restraining forces are released, it is capable of doing work. We say that the stretched spring has energy. From this point of view **energy may be defined as the capacity for doing work.** It manifests itself in many forms. Its fundamental characteristics are of supreme importance in understanding physical phenomena.

5. Systems of Units.—In order to proceed with the study of the characteristics of matter and energy and to describe physical phenomena accurately in terms of them, it is necessary to have an agreement with respect to the units in terms of which measurements are to be made. Many different kinds of magnitudes are involved in the different branches of physics. Some of these magnitudes, like *length*, *time*, *mass*, and *speed*, are familiar. Others, like *power*, *electric current*, and *magnetic pole*, are unfamiliar. In all cases, however, it is necessary to set up conventional definitions for the units which are employed in measuring every quantity used in describing physical phenomena.

Some of these measurements are fundamental and seem to refer to concepts which cannot be further analyzed. There are three of these basic concepts: **time, space, and mass.** Because these quantities are regarded as fundamental magnitudes, the units used to measure them are called **fundamental units.** Hence, all units of time, space, and mass are known as

fundamental units. There are other magnitudes which can not be thought of without connecting them with two or more of these fundamental units, or with one of these fundamental units more than once. For example, the speed of an automobile cannot be described without considering both the space and the time, that is, the space per unit of time. As another example of such a unit, consider an area or a volume. In any unit of area a unit of length is used twice, and in a unit of volume the unit of length is used three times. In this way, an area is expressed in square feet and a volume in cubic feet. Units of this kind which involve more than one fundamental unit, or one fundamental unit more than once, are called **derived units.** Units of speed, units of volume, units of area, units of acceleration, etc., are derived units.

The two systems of units commonly employed are the **decimal metric system** devised by the French and the more familiar **English system.** (See Appendix B, p. 749.)

6. Units of Length.—The unit of length in the metric system is taken as the centimeter, which is 1/100 part of the standard meter. The standard meter is defined as the distance at the temperature of melting ice between two marks on a certain platinum-iridium bar which is kept at the International Bureau of Weights and Measures, near Paris. Two copies of this meter are kept at the U. S. Bureau of Standards at Washington. It is a familiar fact that a bar of metal changes in length when heated or cooled. In order then to have this unit of length accurate, it is necessary to keep the bar at a fixed temperature.

The unit of length employed by the English-speaking people for ordinary purposes is the yard. By an act of Congress the yard is defined as 3,600/3,937 m. Hence,

$$1 \text{ m.} = 39.37 \text{ in.}$$
$$1 \text{ in.} = 2.54 \text{ cm.}$$

7. Units of Mass.—The unit of mass in the metric system is called the gram. It is the 1/1,000 part of a kilogram, which is the mass of a metal cylinder kept at the International Bureau of Weights and Measures, near Paris. Two copies of this standard are kept at the U. S. Bureau of Standards at Washington.

It was the original intention of those who chose this standard that 1 g. should be the mass of 1 c.c. of water at 4°C. More

exact determinations have shown that this relation is not strictly true. The error, however, is so small that it may be neglected for all practical purposes.

Among the English-speaking peoples the pound is ordinarily used as the unit of mass. The pound is defined as the mass of a certain piece of platinum in the possession of the British Government. By act of Congress, the kilogram was declared to be equivalent to 2.2 lb. Hence, the relation between these units of mass is as follows:

$$1 \text{ kg.} = 2.2 \text{ lb.}$$
$$1 \text{ lb.} = 453.6 \text{ g.}$$

8. Units of Time.—For most scientific purposes the second is chosen as the unit of time. It is defined as the 1/86,400 part of the mean solar day. The mean solar day, as used here, means the average interval throughout the year between successive passages of the sun across the meridian. The minute, which is equivalent to 60 sec., and the hour, which is equal to 60 min., are also used as units of time.

9. Units of Volume.—In the English system of units, different units of volume are used, for example, the gallon, the cubic foot, and the cubic yard. In the metric system, the familiar units of volume are the cubic centimeter, the cubic meter, and the liter.

The liter is defined as the volume of 1 kg. of pure water at its maximum density (4°C.).

10. Density.—The mass per unit volume is called the density of a body. In the English system of units, densities are usually expressed in pounds per cubic foot. In the metric system, densities are measured in grams per cubic centimeter (see Table of Densities, page 766).

Let V = the volume of a body.
M = the mass of the body.
d = the density.

Then

$$d = \frac{M}{V}.$$

Example.—A quantity of mercury has a mass of 136.5 g. It has a volume of 10 c.c. What is its density?

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{136.5 \text{ g.}}{10 \text{ c.c.}} \quad 13.65 \text{ g. per cubic centimeter.}$$

Problems

1. The altitude of a certain town is given as 340 m. above sea level; what is the altitude expressed in feet?

2. Calculate the number of square centimeters in 1 sq. ft.; given that 2.54 cm. is the length of 1 in.

3. A machine part is made with an error no greater than 4/10,000 in. in its largest dimension. Express the allowed error in centimeters.

4. The standard meter was selected originally as 1/10,000,000 of the distance along the earth's surface from the pole to the equator. On this basis, find the distance from the surface of the earth to the center.

5. A flow of oil in a pipe line is maintained at the rate of 7.5 ft. per second; how far would the oil travel in 1 day at that rate?

6. Assuming an average density of 1 g. per cubic centimeter for the human body, find the volume occupied by a person weighing 70 kg.

7. A liter of air under ordinary conditions weighs about 1.3 g. Find the approximate weight of 1 cu. ft. of air.

8. The cubic contents of 1 gal. is 231 cu. in.; find how many cubic centimeters are the equivalent of 1 gal.

9. Solid carbon dioxide has a density of 1.53 g. per cubic centimeter. How much volume would be occupied by 5 c.c., after evaporation to gas with a density of 0.00019 g. per cubic centimeter?

10. A piece of brass with a density of 8.7 g. per cubic centimeter is heated until its volume has increased 0.5 per cent. What is the density after heating?

11. A tank of water is cubical in shape and measures 4 ft. 7½ in. each way. Calculate the volume in cubic feet and in gallons, and express the weight of the water in pounds. In metric units the above tank measures 1.41 m.; find the volume in cubic meters and in liters, and the weight of water in kilograms.

12. Bronze is an alloy consisting of 10 parts copper to 1 part of tin by weight. The densities of copper and tin are 8.9 and 7.3 g. per cubic centimeter respectively. What is the density of the bronze? Assume volumes are additive.

PART I.—MECHANICS

CHAPTER I

MOTIONS OF TRANSLATION

11. Uniform Motions.—There are many examples of bodies moving from one place to another. A stone lifted by a crane moves directly upward; an automobile in a race follows the track; an airplane flies from one city to another. Such motions may be studied by observing the time and the distance the body has moved. If the body moves over equal distances in the same time, the motion is said to be **uniform.** Hence when an automobile covers, without variation, a distance of 40 ft. per second its motion is **uniform,** or when a flywheel continuously makes 10 revolutions per minute its motion is **uniform.**

12. Types of Motion.—The motions of bodies may be divided into three classes: (1) **translation,** (2) **rotation,** (3) **vibration or oscillation.** A body is said to have a motion of translation when it moves on continuously in the same direction. A ball thrown from the hand and an automobile running on a straight road are illustrations of **motions of translation.** If a body instead of traveling forward turns on a fixed axis, it has a **motion of rotation.** Thus the flywheel of a stationary engine turns continuously around its axis without ever moving forward. Any point on the wheel returns again and again to its original position. This is a motion of rotation. The drive wheels of a locomotive are moving forward and are at the same time rotating. They, therefore, have two motions, one of rotation and the other of translation. Some bodies reverse their motions from time to time and return at regular intervals to their original positions. Such bodies are said to have a **motion of vibration or oscillation.** The pendulum of an ordinary clock swings back and forth at regular intervals, so that the same motion is repeated again and again. The bob of the pendulum has a motion of vibration.

13. Speed.—The speed of a body is defined to be the rate at which the body is passing through space or the space passed over in unit time. It is determined by dividing the space over which a body has passed by the time required to pass over that space.

$$\text{Speed} = \text{space per unit time} = \frac{\text{distance}}{\text{time}}.$$

Example.—A train which is moving at a uniform rate passes over 75 miles in 3 hr. What is its speed?

$$\text{Speed} \quad \frac{\text{distance}}{\text{time}} \quad \frac{75 \text{ miles}}{3 \text{ hr.}} = 25 \text{ miles per hour.}$$

14. Constant and Variable Speeds.—The speed of a body is constant or uniform when the body passes over equal distances in equal intervals of time. A body has a **variable speed** when it passes over unequal distances in equal intervals of time. Where the speed of a body changes from time to time, it is convenient to consider what is known as the **average speed.** The average speed is that constant speed which would cause the body to move from one point to another in the same time which is required when the speed is variable. Thus, the average speed of a train which runs 60 miles in 2 hr. is $60 \div 2 = 30$ miles per hour. This means that the train would travel this same distance in 2 hr. if it had a constant speed of 30 miles per hour instead of the variable speed with which it actually travels. In case the speed changes uniformly, the average speed over any interval of time can be found by taking the speed at the beginning and at the end of the time, adding them together and dividing the sum by 2. This gives the speed with which the body on the average has been moving. Part of the time it has moved with a speed greater than the average speed, and the remainder of the time it has moved with a speed less than the average speed. The distance covered is the same as if the body had moved with a speed equal to the average speed.

Example.—At the beginning of a certain time the speed of a body is 30 ft. per second. The speed changes uniformly for 5 min. and is then 80 ft. per second. What is the average speed over this time?

$$\text{Average speed} = \frac{\text{initial speed} + \text{final speed}}{2}$$

$$= \frac{30 \text{ ft. per second} + 80 \text{ ft. per second}}{2} = 55 \text{ ft. per second.}$$

15. Speed as a Scalar Quantity.—The speed of a body is completely known when its magnitude is given. There are many other quantities, for example, mass, time, etc., which have magnitude only. Such quantities are called **scalar quantities. A scalar quantity is a quantity which has magnitude only.** Such quantities obey the ordinary laws of addition and subtraction. A block weighs 8 lb. A piece weighing 3 lb. is sawed off. The remainder weighs 5 lb. A body has a speed of 4 miles per hour. Later it has a speed of 6 miles per hour. The change in speed has been 2 miles per hour.

16. Velocity.—The distinction between speed and velocity arises out of the fact that in defining the speed of a body no reference is made to the direction in which the body is moving. The term **velocity** differs from the term **speed** for the reason that in stating the velocity of a body, the direction of motion must also be specified. The magnitude of the velocity is the same as the numerical value of the speed. The velocity of a body is changed either by changing the numerical value of the speed or by changing the direction of motion. A body which is moving in a circular path with uniform speed, continually changes its direction. Its velocity is, therefore, variable. The magnitude of the velocity is

$$\text{Space per unit time} \qquad \frac{\text{distance}}{\text{time}}. \qquad \text{(Appendix D-1)}$$

17. Velocity as a Vector Quantity.—Velocity which has a directional quality in addition to its magnitude is known as a **vector quantity** in contrast to speed which is a scalar quantity. In order to describe a vector quantity completely, it is necessary to give its direction as well as its magnitude. **Displacement is a vector quantity.** If an object is moved 10 m. from its original position, it may be anywhere on a circle with a radius of 10 m. whose center is at the original position of the object. When the object is moved 10 m. **east,** its new position is clearly specified.

If it is stated that an automobile is running 30 miles per hour, the information is not sufficient to locate the machine. In addition to stating the speed of the machine, it is necessary to give the direction in which the machine is moving and the point from which it starts. Directed speed called velocity, is, there-

fore, a **vector quantity. Any quantity which possesses both magnitude and direction is a vector quantity.**

18. Addition of Velocities.—Suppose a railroad train is running east at 10 miles per hour and that a man walks forward on the train at the rate of 4 miles per hour. The man has a forward velocity due to the motion of the train and also a forward motion due to his walking. His forward velocity with respect to the earth is the sum of these two velocities or 14 miles per hour. Now suppose that he walks backward on the train at the rate of 4 miles per hour. Again he has two velocities, 10 miles per hour forward and 4 miles per hour backward. His net velocity with respect to the earth is the difference between his forward and his backward velocity or 6 miles per hour forward. In this case the two velocities lie along the same line and the resultant velocity is equal to the algebraic sum of the separate velocities. When the separate velocities lie along different lines, they must be added with proper regard for the directions of motion.

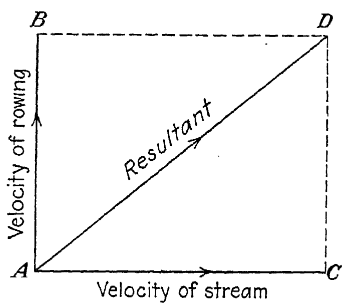


FIG. 1.—Addition of velocities.

To illustrate the addition of velocities which have different directions, consider a man rowing a boat across a stream. The man has a velocity down the stream and at the same time a velocity across the stream due to his rowing. If the man rows at right angles to the direction in which the water flows, conditions are as represented in Fig. 1. The effect of the combined velocities is that the boat is carried across the stream and at the same time is carried down the stream. The speed at which the boat actually moves and the direction of its motion are found by constructing a rectangle so that

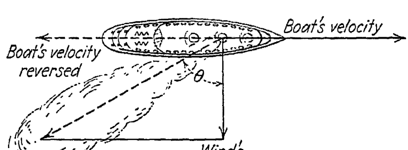


FIG. 2.—The composition of the velocity due to the boat and that due to the wind give the velocity of the smoke.

one side represents the speed and direction of motion of the boat due to the rowing, and the other side represents the speed and direction of motion of the boat due to the stream alone. The actual direction of motion and speed of the boat is given by the diagonal of this rectangle. The diagonal of the rectangle represents the resultant of the two velocities.

This is a simple illustration of a more general principle dealing with the addition of vectors. A single vector which acting alone produces the same

result as two or more vectors acting together, is known as the **resultant of these vectors.**

Vectors do not obey the ordinary laws of arithmetic but must be treated according to the laws of geometry. When vectors lie in the same direction, they are added algebraically.

Figure 2 shows another illustration of addition of velocities.

19. Acceleration.—The rate at which the velocity of a body changes is called the acceleration. It is found by dividing the change in velocity by the time in which the change takes place. It is, therefore, the change in velocity per unit of time. An acceleration like a velocity is a vector, that is, a quantity which has both magnitude and direction.

Example.—A ball which has a velocity of 30 ft. per second toward the north moves for 5 sec. and at the end of that time is found to have a velocity of 70 ft. per second. Find the acceleration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}.$$

$$\text{Change in velocity} = 70 - 30 = 40 \text{ ft. per second.}$$

$$\text{Acceleration} = \frac{70 - 30}{5} = \frac{40 \text{ ft. per second}}{5 \text{ sec.}} = 8 \text{ ft. per second per second.}$$

Hence, the velocity of the ball is increasing 8 ft. per second for each second during which it moves. It is important to observe that two units of time must be stated in order to express an acceleration. One of these units expresses the unit of time in which the original velocity is measured. The other of these units expresses the unit of time used to measure the interval of time over which the velocity is allowed to change.

An acceleration may be either **uniform** or **variable.** A uniform or constant acceleration is one in which equal changes of velocity take place in equal intervals of time. To calculate a constant acceleration, it is only necessary to divide the total change in velocity by the time in which it took place. Where the acceleration is variable, the total change in velocity divided by the time gives the average acceleration. (Appendix D-2.)

20. Illustrations of Uniform Acceleration.—When a train is stopping at a station, it is losing velocity. When it is leaving the station, it is gaining velocity. In the former case the acceleration is *negative*, in the latter it is *positive*. If the train gains velocity at the same rate, for example, 2 miles per hour per minute, the acceleration is constant. A falling body is a good illustration of uniformly accelerated motion. The body gains in

velocity 32 ft. per second each second that it falls. A loaded sled on the side of a hill will slide to the bottom, and its velocity will increase uniformly as it goes. When it reaches the bottom of the hill and coasts on the level, the velocity decreases and the acceleration is negative. In Fig. 3 the velocity of a train at first increases and then becomes constant.

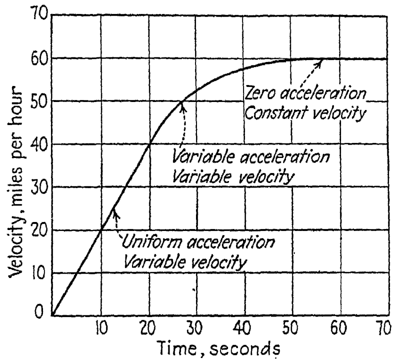


FIG. 3.—The acceleration of the train is at first constant, then variable, and finally zero.

21. Motion of Bodies with Constant Velocity.—The magnitude of the velocity of a body which is moving uniformly has already been defined as the space passed over in unit time. This relation may be written as

$$v = \frac{s}{t},$$

in which v represents the velocity, s the distance, and t the time during which the body was moving. This equation may be written in the form,

$$s = vt.$$

In this form the equation gives the space passed over in the time t by a body moving with constant velocity v. If it happens that the body is moving with variable velocity, the space passed over may be found by multiplying the average velocity by the time.

Example.—The velocity of an automobile is 30 miles per hour. How far will it move in 3 hr.?

Space passed over = velocity × time
= 30 miles per hour × 3 hr. = 90 miles.

Example.—The average velocity of a falling body for 5 sec. is 80 ft. per second. How far does the body fall in that time?

$$\text{Space passed over} = \text{average velocity} \times \text{time}$$
$$= 80 \text{ ft. per second} \times 5 \text{ sec.} = 400 \text{ ft.}$$

22. Motion of Bodies with Constant Acceleration Starting from Rest.—Suppose that a body starts from rest with an acceleration a. At the end of the first second its velocity is a (Fig. 4), at the end of the second second it is $2a$, at the end of the third second it is $3a$, and a velocity equal to a will be added during each second the body moves. At the end of t sec. its velocity will be at.

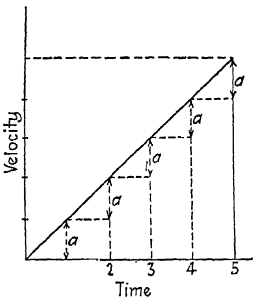


FIG. 4.—Relation between time, velocity, and space passed over for uniform acceleration when the initial velocity is zero. The area under the curve re.
over.

$$\text{Final velocity} = \text{rate of change of velocity} \times \text{time} = at.$$

The average velocity =

$$\frac{\text{initial velocity} + \text{final velocity}}{2} = \frac{0 + at}{}$$

The space passed over = average velocity $\times$ time

$$= \frac{1}{2}at \times t = \frac{1}{2}at^2.$$

In the case of bodies falling freely under the action of gravity the acceleration is approximately 980 cm. per second per second, or 32.2 ft. per second per second. For this special case these relations then become

$$v = 32.2t \text{ ft. per second.}$$
$$= 980t \text{ cm. per second.}$$
$$s = \frac{1}{2} \cdot 32.2t^2 \text{ ft.}$$
$$= \frac{1}{2} \cdot 980t^2 \text{ cm.}$$

Example.—A ball which is thrown upward leaves the hand of the thrower with a velocity of 80 ft. per second. How long before it comes to rest?

$$\text{Time to come to rest} = \frac{\text{initial velocity}}{\text{rate of losing velocity}} = \frac{\text{initial velocity}}{\text{acceleration}}$$
$$= \frac{80 \text{ ft. per second}}{32.2 \text{ ft. per second per second}} = 2.5 \text{ sec.}$$

Example.—A body starts from rest and falls freely for 10 sec. Find the space passed over in this time.

Final velocity = acceleration × time = 980 cm. per sec. per sec. × 10 sec.
= 9,800 cm. per second.

$$\text{Average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2}$$

$$= \frac{0 + 9{,}800}{2} = 4{,}900 \text{ cm. per second.}$$

Space passed over = average velocity × time = 4,900 × 10 = 49,000 cm.

In the two equations,

$$v = at \tag{1}$$

and

$$s = \tfrac{1}{2}at^2. \tag{2}$$

There are in all four unknown quantities: the space passed over, the velocity, the time, and the acceleration. If any two of these quantities are given, the other two can be calculated from these equations. It is convenient to eliminate the time from these equations and have a single equation involving three unknown quantities. If two of these three unknown quantities are given, the other one can be found.

From Eq. (1),

$$t = \frac{v}{a},$$

$$t^2 = \frac{v^2}{a^2}.$$

Substituting in Eq. (2),

$$s = \frac{1}{2}\frac{av^2}{a^2},$$

$$2as = v^2.$$

Example.—A body having an acceleration of 4 ft. per second per second starts from rest and moves a distance of 200 ft. in a given time. What is its final velocity?

$$v^2 = 2as$$
$$= 2 \times 4 \text{ ft. per second per second} \times 200 \text{ ft.} \quad 1{,}600.$$
$$v = 40 \text{ ft. per second.}$$

23. Motion of Bodies with Constant Acceleration and Initial Velocity.—In the preceding discussion it was assumed that the body started from rest and moved with a constant acceleration. Let us now consider bodies that are moving at the instant we begin to count time. In such cases the velocity of the body at the end of a given time is the velocity with which it started plus or minus its change in velocity during this time. To find the final velocity in these cases (Fig. 5), it is only necessary to take the initial velocity and add to it or subtract from it the change in velocity. For example, a body is thrown directly downward from the top of a tower with a velocity of 80 ft. per second. What is its velocity at the end of 10 sec.? During each second the velocity increases 32.2 ft. per second. In 10 sec. the increase will be 322 ft. per second. Hence, the final velocity will be $80 + 322 = 402$ ft. per second.

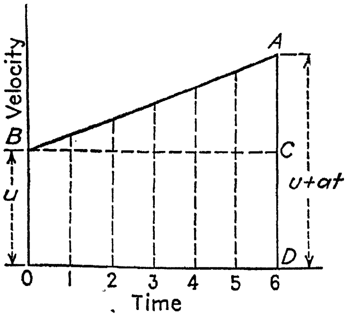


FIG. 5.—Relation between time, velocity, and space passed over for uniform acceleration when the body has an initial velocity. The area under the curve represents the space passed over.

Let u = the initial velocity.
v = the final velocity.
t = the time.

$$a = \text{the acceleration} = \frac{v - u}{t}, \text{ by definition.}$$

$$v = u + at, \text{ when the velocity is increasing.}$$

$$v = u - at, \text{ when the velocity is decreasing.}$$

The average velocity during the time t is

$$\frac{\text{Initial velocity} + \text{final velocity}}{2} = \frac{u + v}{2}.$$

Space passed over = average velocity $\times$ time

$$u + v,$$

Replacing v by $u + at$,

$$= \left(\frac{u + u + at}{2}\right)t = ut + \frac{1}{2}at^2.$$

In the case of freely falling bodies, the acceleration is usually represented by g instead of a, where g is 32.2 ft. per second per second in the English system and 980 cm. per second per second in the c.g.s. system (c.g.s. is an abbreviation for centimeter-gram-second, appropriately descriptive of the metric system). The equations for freely falling bodies are then obtained by substituting g for a in the preceding equations.

Since,

$$v - u \tag{1}$$

and

$$v + u \tag{2}$$

it is possible by multiplying these equations to eliminate t and have a single equation involving the space passed over, the acceleration, the initial velocity, and the final velocity. It is,

$$as = \frac{v^2 - u^2}{2}$$

Whence,

$$v^2 = u^2 + 2as.$$

Example.—A body is moving on ice with a velocity of 60 ft. per second. It has a negative acceleration of 6 ft. per second per second. Find the velocity at the end of 5 sec. and the space passed over in that time.

$$\begin{aligned}\text{Final velocity} &= \text{initial velocity} - \text{loss in velocity}\\ &= \text{initial velocity} - \text{acceleration} \times \text{time}\\ &= 60 \text{ ft. per second} - 6 \text{ ft. per second per second} \times 5 \text{ sec.}\\ &= 30 \text{ ft. per second.}\end{aligned}$$

$$\begin{aligned}\text{Space passed over} &= \text{average velocity} \times \text{time}\\ &= \frac{\text{initial velocity} + \text{final velocity}}{2} \times \text{time}\\ &= \frac{60 \text{ ft. per second} + 30 \text{ ft. per second}}{2} \times 5 \text{ sec.}\\ &= 45 \times 5 = 225 \text{ ft.}\end{aligned}$$

Example.—A mass has an initial velocity of 40 ft. per second. Its acceleration is 3 ft. per second per second. What will be its velocity when it has moved a distance of 800 ft.?

$$v^2 = 2as + u^2.$$
$$u = 40 \text{ ft. per second.}$$

$$a = 3 \text{ ft. per second per second.}$$
$$s = 800 \text{ ft.}$$
$$v^2 = 2 \times 3 \times 800 + (40)^2.$$
$$= 6{,}400.$$
$$v = 80 \text{ ft. per second.}$$

24. Path of a Projectile Fired Horizontally.—If a body is projected horizontally from the top of a tower of height h (Fig. 6) with a velocity V, it continues to move forward with the same horizontal velocity it had at the beginning of its path, except for whatever decrease is caused by the resistance of the air. For the present purposes the effect of the air may be neglected. At the same time the body falls because of the attraction of the earth on it. Hence, at any instant, the projectile has two velocities—a forward velocity which remains constant, and a downward velocity which increases with the time. The effect of the force of gravitation on a body which is falling and, at the same time, moving forward with a horizontal velocity is exactly the same as it is on a body falling freely from rest without any horizontal motion. Hence, the downward velocity of the body at the end of any time t is

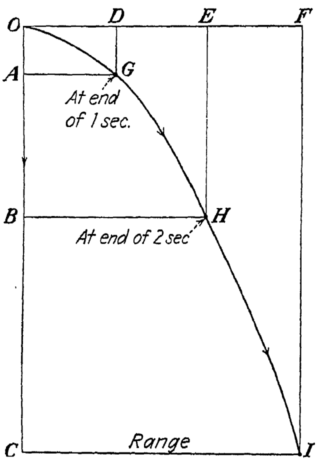


FIG. 6.—Path of a projectile fired in a horizontal direction. The forward velocity remains constant. The downward velocity increases uniformly.

$$v = gt.$$

Since the resultant velocity at any instant is made up of a horizontal component V and a vertical component gt, the resultant velocity tangent to the path of the body is

$$V' = \sqrt{V^2 + (gt)^2}.$$

The distance which the body moves forward in the time t is

$$x = Vt. \tag{1}$$

The distance the body falls in the time t is

$$y = \frac{1}{2}gt^2. \tag{2}$$

Since both of these equations involve the time t, we can eliminate t from them and have a single equation connecting the distance along the x-axis and the distance along the y-axis. This equation describes the path along which the particle moves.

From Eq. (1),

$$t = \frac{x}{V}$$

$$t^2 = \left(\frac{x}{V}\right)^2$$

Substituting in Eq. (2),

$$x^2 = \frac{2V^2}{g}y.$$

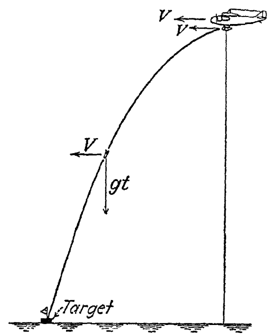


FIG. 7.—Path of bomb released from an airplane flying horizontally. Bomb moves forward as well as downward.

The body moves in a path which is a parabola. When $x = 0$ and $y = 0$, the body is at the origin just starting on its motion. When $y = h$, $x = R$, where R = the distance the body goes before it strikes the ground. This distance is called the range of the projectile. When the body strikes the ground,

$$h = \frac{1}{2}gT^2$$

where T is the time of flight.

$$\text{Time of flight} = T = \sqrt{\frac{2h}{g}}.$$

$$\text{Range } R = VT = V\sqrt{\frac{2h}{g}}$$

When a bombing plane flying in a horizontal direction (Fig. 7) at a height h above the ground releases a bomb, the bomb is moving forward with the same velocity as the plane and travels forward with a constant horizontal velocity until it strikes the target. When the bomb is released, it has no downward velocity but its downward velocity increases with the time of fall owing to the acceleration of gravity. The bomb does not strike the ground

at a point directly below the point at which it was released. In order to hit the target the bomb must be released before the plane is directly above the target. The time of release is determined by the speed of the plane and its height above the ground.

Example.—A ball is projected horizontally from the top of a tower which is 64 ft. high with a velocity of 20 ft. per second. Find the time of flight and the range.

$$\text{Time of flight} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 64}{32}} = 2 \text{ sec.}$$

$$\text{Range} = V\sqrt{\frac{2h}{g}} = 20\sqrt{\frac{2 \times 64}{32}} = 40 \text{ ft.}$$

25. Projectile Fired at an Angle with Horizontal.—If a projectile is fired at an angle θ with the horizontal, it traces out a parabolic path as indicated in Fig. 8. The horizontal velocity remains constant. The upward velocity

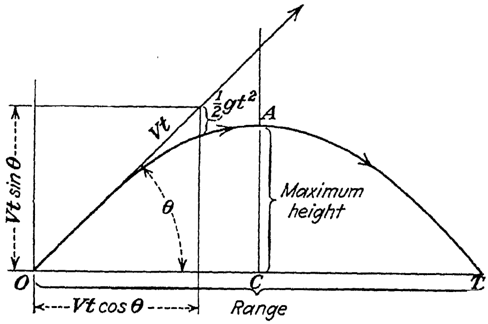


Fig. 8.—Path of a projectile fired at an angle to the horizontal. The maximum height and the range depend on the angle of projection.

decreases to zero and then increases to the same value it had at the moment the projectile was fired. The first step in analyzing the motion of such a projectile is to resolve the velocity of projection into vertical and horizontal components. If the velocity of projection V makes an angle θ with the horizontal, the horizontal component is $V \cos \theta$ and the vertical component is $V \sin \theta$. The horizontal velocity remains constant during the motion but the vertical component changes due to the acceleration of gravity. After a time t the vertical component is $v = V \sin \theta - gt$.

The horizontal distance passed over in the time t is,

$$x = Vt \cos \theta$$

and the vertical distance is,

$$y = Vt \sin \theta - \frac{1}{2}gt^2.$$

Eliminating t from these equations, we have for the equation of the path of the particle,

$$y = x \tan \theta - \frac{1}{2} \frac{x^2 g}{V^2 \cos^2 \theta}$$

When the vertical velocity becomes zero, the projectile has attained its greatest height h. At that time t_1,

$$0 = V \sin \theta - g t_1$$

and

$$t_1 \quad \frac{V \sin \theta}{g}.$$

The time for the projectile to return to the ground is the same as the time to rise to its maximum height. Hence the time of flight is,

$$2t_1 = T \quad 2V \sin \theta$$

and the range,

$$X = \left(\frac{2V \sin \theta}{g}\right)(V \cos \theta) \quad 2V^2 \sin \theta \cos \theta$$

The maximum height is

$$y = h = V \sin \theta \left(\frac{V \sin \theta}{g}\right) - \frac{1}{2} \quad V^2 \sin^2 \theta$$

$$h = \frac{V^2 \sin^2 \theta}{2g}.$$

Problems

1. A mail clerk threw the mail bag from a moving train so that it moved at right angles to the track with a speed of 15 ft. per second. Find the direction in which he threw it, if the train was moving 20 ft. per second.

2. An airplane can fly at a velocity of 120 miles per hour in still air. It is headed north when the wind is blowing with a velocity of 60 miles per hour due east. Find the actual velocity and the direction of motion of the airplane.

3. How long will it take for a steamer capable of doing 18 miles per hour in still water to reach a point 40 miles upstream against a current of 2 miles per hour?

4. During an interval of 40 sec., a train on a straight track changes its velocity from 15 to 25 miles per hour. Determine the average velocity during that period, assuming that the change occurred uniformly.

5. Find the time required for a locomotive traveling at the rate of 40 ft. per second to come to rest if the brakes are applied to produce a negative acceleration of 8 ft. per second per second.

6. A passenger train starts from a station just as a freight train is passing on a parallel track in the same direction with a speed of 45 ft. per second. The passenger train has an acceleration of 1.5 ft. per second per second.

What time will elapse before the trains are again side by side and how far from the station will they then be?

7. A sled is started with an initial velocity of 10 ft. per second down a hill on which it accelerates 8 ft. per second per second. How far will it go in 6 sec.?

8. A freight train starts from rest and travels 300 m. in 30 sec.; find the acceleration, the average speed, and the final speed. (Assume constant acceleration.)

9. An elevator starting from rest has a constant upward acceleration of 8 ft. per second per second. It reaches a maximum speed of 24 ft. per second and is then brought to rest with a negative acceleration of 6 ft. per second per second. Find the distance the elevator moved during the time which elapsed between starting and stopping.

10. With what speed and in what direction must a baseball leave the hand of the pitcher to rise to a height of 30 ft. and travel horizontally a distance of 150 ft.?

11. A ball is thrown at an angle of 45 deg. with the horizontal. Its initial speed is 120 ft. per second. Find the time which will elapse before the ball returns to the ground, if it is assumed that the ground is level.

12. Two trains leave stations at different times traveling toward each other. One of them travels at a speed of 40 miles per hour, the other at a speed of 50 miles per hour. The stations are 120 miles apart. How much later should the faster train start in order that the two trains meet halfway between the stations?

13. A ball is projected upward from the bottom of a tower which is 375 ft. high, and, at the same instant, another ball is dropped from the top of the same tower. If the balls meet at a point halfway between the top and the bottom of the tower, with what initial velocity was the ball projected upward?

14. A body is thrown in a horizontal direction from the top of a tower which is 300 ft. high, with a velocity of 60 ft. per second. Find its range.

15. A balloon ascending with a velocity of 3.2 ft. per second releases a bag of sand at a height of 300 ft. from the surface of the ground. Find the time for the bag of sand to reach the earth.

16. Find the range of a shell fired from a cannon with a muzzle velocity of 1,200 ft. per second, when the shell is fired at an angle of 30 deg. with the horizontal.

CHAPTER II

FORCES AND MOTIONS

26. Force.—Our first ideas of force come from muscular effort exerted to produce changes in the motion of a body. If a ball is thrown into the air or a heavy stone lifted into a wagon, a certain muscular effort is required. This muscular effort is greater for large bodies than for small ones. When bodies are once set in motion, they require muscular effort or some other force to stop them; and the more rapid their motion, the greater is the force required to bring them to rest in a given time.

A force is an action exerted by one body on another tending to change the state of motion of the body acted upon. Thus, a loaded wagon is drawn by a team of horses or an automobile is driven forward by its engine. When a man lifts on a bucket of water, his hand exerts a force on the bucket which tends to change the state of motion of the bucket. It may happen that the force exerted on the bucket is not sufficient to lift it. In that case the force only tends to change the state of motion of the body. The primary effects of a force are two: a force may cause a change in the size or shape of a body; or, if the body is free to move, a force can cause a change in velocity. This change in velocity may be a change of the direction of the motion, or a change in the magnitude of the velocity, or both.

27. Newton's First Law of Motion.—A person stepping from a moving car observes a tendency to keep on moving after leaving the car. On the other hand, a person finds it easier to jump on a moving car if he is running forward at the same rate as the car. In like manner it is found that if a heavy flywheel is standing still, it requires a considerable effort to set it in rotation about its axis. These are illustrations of an important law known as Newton's first law of motion. It states that **every body continues in a state of rest or uniform motion in a straight line, unless it is compelled to change that state by the application of some external force.** This form of statement applies to motions of translation. For motions of rotation the law states **that every**

body continues in a state of uniform motion of rotation about a fixed axis unless acted upon by some impressed force applied at some point not on the axis of rotation. This law states in effect that a stone lying on the ground will remain there until some outside force compels it to move. When a ball is thrown into the air, it will move on indefinitely unless some outside force offers a resistance to its motion. The earth will continue to rotate about its axis until some external force is applied to stop the rotation. Even after such forces are applied, time is required to bring the body to rest. The changes in motion are not instantaneous.

28. Inertia.—In Newton's first law of motion an important property of matter appears. It is known as **inertia** and is defined as **that property of matter by virtue of which it tends to remain at rest or in uniform motion unless external forces are operating on it.** There are two kinds of inertia as there are two general types of motion—translation and rotation. Bodies show opposition to being translated and also opposition to being rotated. The former opposition is called **linear inertia**; the latter is called **rotary inertia.** The linear inertia of a body is proportional to the mass of the body, but rotary inertia depends also on the distribution of the mass about the axis of rotation.

29. Momentum.—Certain properties of moving bodies depend jointly on the mass and the velocity. Thus, the time for a locomotive to start a train depends on the mass of the train and the velocity which is given to it. It is more difficult to give a freight train a speed of 10 miles per hour than it is to give a passenger train that speed. This property is called the **momentum** of the body, and **it is defined as the product of the mass and the velocity.** Since the velocity has direction as well as magnitude and the mass has only magnitude, the momentum will be a directed quantity and due regard must be paid to its vector properties.

Example.—A tractor having a mass of 7,500 lb. has a velocity of 8 ft. per second. What is its momentum?

$$\begin{aligned}\text{Momentum} &= \text{mass} \times \text{velocity}\\ &= 7{,}500 \text{ lb.} \times 8 \text{ ft. per second} \quad 60{,}000 \text{ lb.-ft. per second.}\end{aligned}$$

30. Newton's Second Law of Motion.—When an unbalanced force acts on a body, it produces an acceleration which is in the

direction of the force and proportional to it. For example, let a force of 32 lb. act on a mass of 64 lb. The mass will have an acceleration in the direction of the force, and, if the force is increased to 64 lb., the acceleration will be doubled. The force effective in producing this acceleration is not the entire force acting on the body but the unbalanced force. By an unbalanced force is meant the amount by which the pull or push in one direction exceeds the pull or push in the opposite direction. When the engine in a tractor exerts a force on the tractor, there are, besides this force on the tractor, the forces due to the resistance which must be overcome in pulling the plows through the earth. This resistance acts in the opposite direction to the pull of the engine on the plows. The unbalanced or net force is the difference between the force pulling the tractor forward and the force tending to stop it. When this net force is zero, the tractor moves forward with constant velocity. If the forward force exceeds the backward force, the tractor moves forward with constant acceleration. If the resistance exceeds the forward force, the motion of the tractor slows down at a constant rate.

Experiments also have shown that if equal accelerations are imparted to bodies of unequal masses, the forces required to produce these accelerations are proportional to the masses of the bodies. Thus, if an acceleration of 2 ft. per second per second is imparted to a mass of 5 lb. and to another of 15 lb., it requires three times as much force to produce this acceleration in the 15-lb. mass as it does in the 5-lb. mass. The larger the mass the more difficult it is to produce a given acceleration.

Combining these statements, we get Newton's second law of motion. **The net or effective force acting on a body is proportional jointly to the mass and the acceleration produced by the force.** The product of the mass and the acceleration is therefore a measure of the force acting on the body. This law can be more conveniently expressed as follows:

$$F = kMa,$$

where F is the net force acting on the mass, M is the mass of the body, a is the acceleration, and k is a constant factor of proportionality whose numerical value is determined by the units in which the force, mass, and acceleration are measured. In this form the law states that the acceleration of a body is

proportional to the force acting on it and inversely proportional to its mass. **More generally, Newton's second law states that the time rate of change of momentum is proportional to the impressed force.** (Appendix D-3.)

31. Absolute Units of Force.—Newton's second law of motion gives a satisfactory way of defining an absolute unit of force. By suitably choosing the unit of mass, the unit of force, and the unit of acceleration, the factor of proportionality k becomes equal to unity and Newton's second law of motion takes the form,

$$F = Ma.$$

This equation implicitly defines the absolute unit of force as the force which will produce unit acceleration when acting on unit mass. In the c.g.s. system of units this unit of force is called a **dyne. A dyne is defined to be that force which will produce an acceleration of 1 cm. per second per second when acting on a mass of 1 g. A poundal is defined to be that force that will produce an acceleration of 1 ft. per second per second in a mass of 1 lb.**

Example.—A force of 30 dynes acts on a mass of 10 g. Find the acceleration.

Force in dynes = mass in grams × acceleration in centimeters per second per second.

$$\begin{aligned} F &= Ma. \\ 30 \text{ dynes} &= 10 \text{ g.} \times a. \\ a &= 3 \text{ cm. per second per second.} \end{aligned}$$

Example.—What force is necessary to give a mass of 10 lb. an acceleration of 3 ft. per second per second?

Force in poundals = mass in pounds × acceleration in feet per second per second.

$$\begin{aligned} F &= Ma \\ &= 10 \times 3 = 30 \text{ poundals} \end{aligned}$$

32. Gravitational Units of Force.—The force arising from the pull of the earth, on bodies which are on or near its surface, is so easily observed that it is convenient to make it the basis for a system of units in which to measure other forces. For this reason the weight of a gram or a pound is often used as a unit of force. Although the attraction of the earth for a given mass varies from place to place on the surface of the earth, this attrac-

tion at a given place is constant and can be used as a standard. Such units of force are known as **gravitational units of force.**

A force of a pound is defined as a force equal to the force with which the earth attracts a mass of 1 lb. Since the attraction of the earth for a mass of 1 lb. is not the same at all points, it is necessary to specify the place at which this standard pound is used.

Similarly, a **kilogram of force is defined as a force equal to the attraction of the earth for a mass of 1 kg., and a force of 1 g. as a force equal to the attraction of the earth for a mass of 1 g.**

33. Mass and Weight.—It is important to distinguish between the mass and the weight of a body. The **mass** is in a sense the body itself or the amount of matter which it contains. Hence, the **mass** is independent of the position of the body on the earth. It is the same whether the body is on this planet or on some other planet. The **weight** is a measure of the attraction of the earth for the mass which the body contains. The weight of a body at a particular place on the surface of the earth is proportional to its mass, but the weight depends on where the body happens to be located with respect to the earth.

One of the most convenient methods of determining the mass of a body is to measure the attraction of the earth for it. This is such a familiar method that it has become a general practice to use the same word to denote the unit of mass and the corresponding unit of force. Thus, the word pound or gram is used to denote a certain amount of matter and also the attraction of the earth for this matter; that is, its weight. In order to avoid confusion, it is necessary to specify whether we mean a pound of mass or a pound of force, a gram of mass or a gram of force. When pound is used as a unit of force, and there is any danger of ambiguity, it will be called a **pound weight**; and when it is used as a unit of mass, it will be referred to simply as a **pound.** The corresponding distinction will be recognized in the case of the **gram** and the **kilogram.**

34. Relation between Absolute and Gravitational Units.—The force acting on a freely falling body is sufficient to give it an acceleration of 980 cm. per second per second, or an acceleration of 32.2 ft. per second per second. The relation between the force and the mass on which it acts is given by the formula

$$F = Ma.$$

If the force is expressed in dynes and the mass in grams, the force acting on 1 g. is

$$F = 1 \times 980 = 980 \text{ dynes.}$$

The force with which a mass of 1 g. is attracted to the earth is known as the weight of 1 g. Hence a force equal to the weight of 1 g. is equivalent to a force of 980 dynes.

Example.—Find the force in grams weight to which 2,940 dynes are equivalent.

$$\text{Force in grams weight} = \frac{\text{force in dynes}}{980} = \frac{2{,}940 \text{ dynes}}{980} = 3 \text{ g. weight.}$$

In the same way the force in poundals with which the earth attracts a mass of one pound is,

$$F = 1 \times 32.2 \text{ poundals.}$$

Hence the force with which 1 lb. is attracted to the earth is equal to 32.2 poundals or the force of 1 lb. weight is equal to 32.2 poundals or

$$\text{Force in pounds weight} = \frac{\text{force in poundals}}{32.2}.$$

Example.—What pull in pounds weight must be applied by a locomotive to give a train of 250 tons an acceleration of 2.5 ft. per second per second?

Force in poundals = mass in pounds $\times$ acceleration in feet per second per second.

Force in poundals = $500{,}000 \times 2.5 = 1{,}250{,}000$ poundals.

Force in pounds weight = $\dfrac{1{,}250{,}000}{32.2} = 38{,}800$ lb.

Example.—A force equal to the weight of 30 g. acts on a body which has a mass of 98 g. Find the acceleration in centimeters per second per second.

Force in dynes = mass in grams $\times$ acceleration in centimeters per second per second.

Force in dynes = force in grams weight $\times$ 980
= $30 \times 980 = 29{,}400$ dynes

29,400 dynes = 98 g. $\times$ acceleration in centimeters per second per second.

Acceleration = $\dfrac{29{,}400}{98}$ 300 cm. per second per second.

35. Newton's Third Law of Motion.—The third law of motion like the first and second laws is the result of experience. The hand of a person holding up a given mass is subjected to the

downward force of gravity acting on the mass. The hand must apply an equal upward force in order to keep the mass in position. The wheels of an automobile push backward on the road, but the road pushes forward on the wheels with an equal force. When the road is icy so that the wheels cannot grip it, the road cannot push forward on the wheels, and the car does not move forward although the motor is at full speed. If a horse pulls a load along the road, the horse pushes back on the road with a certain force and the road pushes forward on the horse with an equal force. This fact is made use of in a treadmill where the track on which the horse stands is movable. The horse attempts to walk forward, but in reality the track moves back under his feet. The horse remains in one position while the track on which he stands moves and thus develops the power which turns the mill.

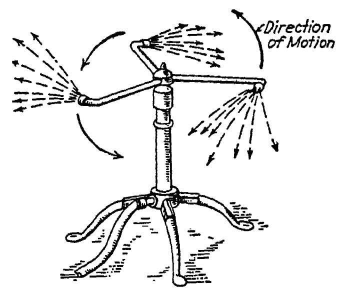


FIG. 9.—Reaction due to the emission of water from a nozzle. The greater the velocity of the water, the greater the reaction.

Wherever one force is found in nature, it is possible to find another force which has the same magnitude but is always in the opposite direction. If, for example, a man pulls on a rope with a force of 40 lb., the rope pulls on the man's hands with a force of 40 lb. The train pulls back on the locomotive with a force which is just as great as the force with which the locomotive pulls forward on the train. The sun pulls on the earth and the earth pulls on the sun with an equal and opposite force. A stream of water (Fig. 9) issuing from a spray causes rotation.

These facts may be stated as the third law of motion in the following form: **To every force or action there is always an equal and opposite reaction.**

33. Universal Gravitation.—There is a tendency for every body in the universe to move toward every other body. This tendency arises out of the fact that every particle of matter attracts every other particle of matter with a force whose direction is that of the line joining the particles. When the masses of these bodies are small, this attractive force is small; but the attraction between two bodies like the sun and the earth is very large. It is this

force of attraction between the sun and the earth which causes the earth to describe its orbit about the sun. The weight of a body is the attractive force of the earth on the body. This force pulls the body toward the center of the earth. The greater the mass of the body, the greater is this pull and the greater the weight. The farther the body from the center of the earth the less is this pull. Hence, the weight of a body varies with the altitude and also the latitude at which the observations are made.

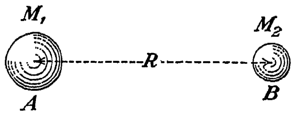


FIG. 10.—The force of gravitation varies directly as the product of the masses and inversely as the square of the distance between them.

Newton found that **the force of attraction between two small bodies or between two spherical bodies of any size (Fig. 10) is proportional to the product of their masses and inversely proportional to the square of the distance between their centers.** This law may be written as

$$F = k\frac{M_1M_2}{R^2}$$

where F is the force of attraction, M_1 and M_2 the masses of the bodies, R the distance between their centers, and k a constant known as the constant of gravitation.

The best evidence for the correctness of this law is obtained from astronomical observations. By an application of this law, it is possible to calculate and predict the motion of the planets and their satellites with the greatest accuracy years in advance. Such accurate predictions make a severe test of the correctness of the law.

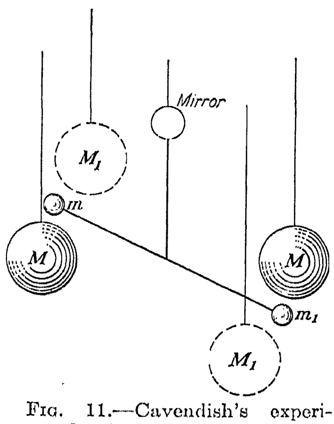


FIG. 11.—Cavendish's experiment showing attraction between fixed and movable spheres. Direction of rotation is reversed when position of spheres is reversed.

This law has also been verified by direct experiment. Cavendish by means of a torsion balance (Fig. 11) measured with great accuracy the attraction of one body for another. Two spheres m and m_1 were fixed to the end of a light rod which was suspended by a long, fine wire so that it could turn easily. In its normal position the wire is untwisted. When two heavy lead balls are placed at M and M, the rod carrying the two small balls is turned into a new position. When the lead balls are placed at M_1 and M_1, the light rod with the small balls is turned in the opposite direction.

By measuring the elastic constant of the suspending wire, it becomes possible to determine the attractive force between the movable and fixed spheres. By means of such an experiment Cavendish carefully verified the law of universal gravitation. It is one of the most important laws in nature.

Example.—Calculate the attraction between two lead spheres which weigh 10 and 20 kg., respectively, when the distance between their centers is 40 cm.

$$F \quad k\frac{M_1M_2}{R^2}.$$

$$k \quad 6.67 \times 10^{-8}.$$

$$F = 6.67 \times 10^{-8} \times \frac{10{,}000 \times 20{,}000}{40^2}$$

$$= 6.67 \times 10^{-8} \times \frac{2 \times 10^8}{1{,}600}$$

$$= 0.0083 \text{ dyne.}$$

37. The Mass and Density of the Earth.—From Newton's law of gravitation it is possible to calculate the mass of the earth. It is first necessary to know the numerical value of the gravitational constant k with proper regard for the unit of force, the unit of mass, and the unit of distance used in the law of gravitation. Suppose the mass is measured in grams, the force in dynes, and the distance in centimeters, then the observed gravitational constant $k = 6.67 \times 10^{-8}$.

Consider the earth a perfect sphere of radius R. The force which it exerts on a mass of 1 g. at its surface is 980 dynes. Newton's law of gravitation states that the force of attraction between this gram and the earth is

$$k\frac{mM}{R^2},$$

where m = 1, the mass of the 1-g. weight.
M = the mass of the earth in grams.
R = the radius of the earth in centimeters.
F = the force of attraction in dynes.

Substituting the proper values in this equation,

$$980 \quad 1 \cdot M \times 6.67 \times 10^{-8}.$$

$$R = 6.4 \times 10^8 \text{ cm., approx.}$$

$$M = \frac{980R^2}{6.67 \times 10^{-8}} = \frac{980 \times (6.4 \times 10^8)^2}{6.67 \times 10^{-8}}$$

$$= 6.018 \times 10^{27} \text{ g., approx.}$$

B'lore

Since the mass of the earth is 6.1×10^{27} g. and the volume of the earth is $\frac{4\pi}{3}R^3 = 1.08 \times 10^{27}$ c.c., the mean density of the earth can be calculated.

$$\text{ity} = \frac{\text{ss}}{\text{volume}} = \frac{6.1 \times 10^{27}}{1.08 \times 10^{27}} \quad 5.6 \text{ g. per cubic centimeter.}$$

38. Variation of Gravity.—The decrease in the force of gravity as one goes upward from the surface of the earth is not rapid because the radius of the earth is large. The earth has its greatest radius at the equator and bodies weigh less at the equator for this reason. A mountain rising above a plain has a gravitational attraction for a plumb bob so that it no longer hangs exactly vertical. There are now available methods for measuring small variations of the force of gravity at different points on the earth's surface. These methods have proved valuable in the discovery of salt domes in certain oil fields. In such fields the distribution of oil is largely determined by these salt domes which cannot be located by ordinary methods. The location of other valuable mineral deposits and structural features of the crust of the earth have been determined by gravitational methods.

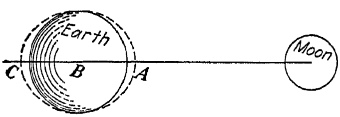


FIG. 12.—The difference between the attraction of the moon for water on opposite sides of the earth causes tides.

39. Formation of Tides.—Assuming for the sake of simplicity that the earth is not rotating and that it is covered uniformly with a deep ocean, Fig. 12 shows how the difference in the attractive forces of the moon for the water, nearest and farthest from it, furnishes tide-rising forces. A gram of water nearest the moon is attracted with a greater force than the force on a gram of matter situated at the center of the earth. On the other hand, a gram of matter at the center of the earth is attracted with a greater force than the force on a gram of water on the side of the earth farthest from the moon. The differences in these forces of attraction cause the water to pile up on the side of the earth nearest the moon and on the side farthest from it. The attraction of the sun causes a similar set of tide-rising forces. The lunar tides have their greatest height twice in a period of 1 day and 51 min. The solar tides have their greatest height twice in a period of 24 hr. The resulting tide is a combination of these two tides. In fact, this discussion is oversimplified. The inertia of the water, forces of friction and other causes produce lags in the motion of the water and otherwise complicate the picture.

The crust of the earth is not perfectly rigid. It also yields to tidal forces. The actual rise and fall of the earth's surface due to tidal forces have been measured. At the time of the spring tides it is as much as 9 in.

Problems

1. A force of 600 dynes acts continuously on a mass of 15 g. during a period of 2 min. What velocity does the body acquire if it was originally at rest?

2. What retarding force must be exerted on a mass of 100 lb., in order to permit it to fall with an acceleration of 12 ft. per second per second?

3. If the cable which supports an elevator exerts an upward force of 4.5 tons on an elevator which weighs 5 tons, what is the acceleration of the elevator? Neglect frictional forces.

4. A string which can sustain a tension of 2.5 kg. is fastened to a mass of 1.5 kg. lying on a smooth horizontal table. What is the largest acceleration which can be imparted to the mass without breaking the string?

5. A porter carries a bag weighing 16 lb. into an elevator. What force must he exert on the bag in order to hold it when the car is started upward with an acceleration of 5 ft. per second per second?

6. The car of an elevator and its contents exert a force of 2 tons on the cables when at rest. How great is the force when an upward acceleration of 4 ft. per second per second is being given to the elevator? when the acceleration is numerically the same but downward?

7. A man weighing 160 lb. slides down a rope which can sustain only 150 lb.; what must be the acceleration of the man in order that the rope will not break?

8. Taking the mass of the earth as 6×10^{27} g., the mass of the moon as 7×10^{25} g., and the distance between their centers as 3.8×10^{10} cm., calculate the attraction of the earth for the moon.

9. When a sphere of lead is placed 30 cm. from another sphere of lead with a mass of 2,500 g., the attraction of one for the other is found to be 1×10^{-5} g. What is the mass of the other sphere?

10. The diameter of the earth is 7,900 miles and that of the moon is 2,160 miles. The mass of the earth is 81 times that of the moon. Find the acceleration of gravity on the moon.

11. A body weighs 200 kg. on the surface of the earth. Find its weight 1,000 miles above the surface of the earth. Assume the radius of the earth to be 4,000 miles.

12. A mass of 20 lb. rests on a smooth table. It is fastened by means of a flexible cord passing over a frictionless pulley to a weight of 3 lb. which hangs freely. Find the acceleration of the system.

CHAPTER III

COMPOSITION AND RESOLUTION OF FORCES

40. Representation of Forces.—Since forces are vector quantities, in order to completely define a force, it is necessary to give its magnitude, its direction, and its point of application. Forces like other vector quantities can be represented by straight lines. The length of the line represents the magnitude of the force. The direction of the line represents the direction in which the force acts, and the head of an arrow on the line shows whether the force acts up or down, to the right or to the left, east or west, etc.

It may be that two forces act on a body at the same time. Suppose a force of 40 lb. is acting north and a force of 30 lb. is acting east. Two arrows are then drawn, one pointing north and the other pointing east. The two arrows begin at the same point, showing that the forces are applied at the same point. If the arrow pointing north is made 4 in. long, then each inch of its length represents a force of 10 lb. The arrow pointing east must now be made 3 in. long in order to represent the other force.

41. Resultant.—If a force acts on a body which is free to move, the body moves in the direction of the force. When two horses pull in opposite directions on a load, the load moves in the direction of the stronger pull. The force tending to displace the load in this case is the difference between the two pulls. When the forces act in the same direction, the effective force is the sum of the two pulls. This effective or equivalent force is known as the **resultant of the forces.** In the case of two oppositely directed forces acting at the same point this resultant is found by taking the difference between the applied forces, and its direction is the direction of the greater force. When the forces act in the same direction, the resultant is found by adding the applied forces.

42. Resultant of Forces Acting at Right Angles.—When two forces acting at right angles are applied to a body, the effect is the same as if some single force acted on the body. For example, if two ropes are fastened to a sled and two men pull on them at right angles to each other with a force of 50 and 75 lb., respec-

tively, the sled will move along the diagonal of a parallelogram of which the forces form the sides. Lay off, as in Fig. 13, a line OB, 3 in. long. Each inch represents a force of 25 lb. Now, at right angles to this line, lay off another line which is 2 in. long. This line represents the other force in magnitude and direction. Under the simultaneous action of these forces the body will move along the diagonal OC. This diagonal represents the direction and magnitude of the resultant or equivalent force.

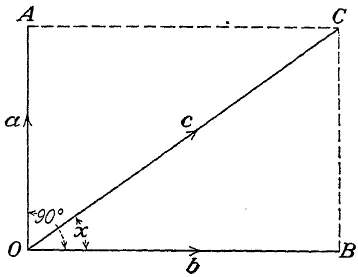


FIG. 13.—The resultant of forces which are at right angles is given by the diagonal of the rectangle.

To calculate the magnitude of the resultant force when the forces are acting at right angles, it is only necessary to make use of the familiar relation between the sides and the hypotenuse of a right triangle. Thus,

$$\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 \text{ or } R = \sqrt{X^2 + Y^2}.$$

The direction of the resultant is found from the equation,

$$\tan x \quad \frac{BC}{OB}.$$

Example. A body lying on a horizontal table is acted on by two forces which are at right angles to each other. One of these forces has a magnitude of 75 lb. and points in the direction OB. The other has a magnitude of 50 lb. and points in the direction BC. Find the magnitude and direction of the resultant force.

$$\begin{aligned} R &= \sqrt{X^2 + Y^2}. \\ \tan x &= \frac{Y}{X}. \\ R^2 &= (75 \text{ lb.})^2 + (50 \text{ lb.})^2. \\ R &= 90.1 \text{ lb.} \\ \tan x &\quad \frac{50}{75} = 0.67. \\ \therefore x &= 34.0 \text{ deg.} \end{aligned}$$

43. Resultant of Forces Not at Right Angles.—Suppose that two men on opposite sides of a canal are pulling on a boat by means of two ropes. When the boat is some distance behind the men, the angle between the forces will be less than a right angle.

In order to find the resultant of these two forces, arrows are drawn to represent the forces as in the preceding case, but in this case a parallelogram (Fig. 14) instead of a rectangle is constructed. The resultant is then represented by the diagonal of the parallelogram. By this graphical method the magnitude and direction of the resultant can be obtained. If it is desired to calculate either the direction or the magnitude of the resultant, it is necessary to use the following trigonometric relation (Appendix E-1):

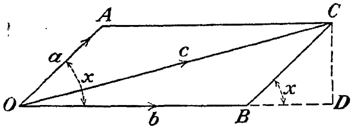


FIG. 14.—The resultant of forces not at right angles is represented by the diagonal of a parallelogram.

$$c^2 = a^2 + b^2 + 2ab \cos x.$$

The correctness of this equation can be tested by suspending three weights as in Fig. 15 and measuring the angle ACB for different magnitudes of the weights.

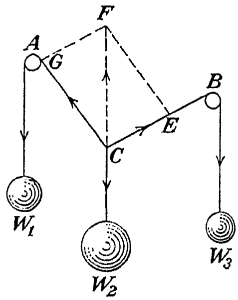


FIG. 15.—Experimental test of relation between two forces and their resultant.

44. Illustrations of Composition of Forces.—In Fig. 16 is shown a pendulum consisting of a heavy bob A suspended by a cord attached at O and having a spring balance at C. By means of a second spring balance at D, the bob is pulled aside from its vertical position. This pull is exerted in the horizontal direction and is read on the spring balance at D. The spring balance at C gives the tension in the string for any position of the bob. When the bob is in the vertical position, the reading of the spring balance C is the weight of the bob. When the bob is pulled aside from this position, there are three forces acting on the bob —the horizontal pull, the tension in the string, and the weight of the bob. The reading of the spring balance C is the resultant of the horizontal and vertical forces on the bob.

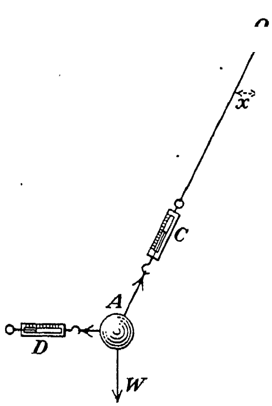


FIG. 16.—Equilibrium of forces acting on a pendulum bob.

A simple crane (Fig. 17) illustrates the composition of the two forces. Let the top of the beam BC be connected by means of a cord to the wall at A, in such a way that when the beam is in equilibrium the cord AB is at right angles to the wall. A spring balance inserted in this cord indicates the tension in it. From B is hung a weight W, and under the action of these two forces the beam exerts a thrust which is just enough to overcome their combined action. This thrust is equal in magnitude and opposite in direction to the resultant of these two forces. To get the force diagram, lay off on a vertical line a distance which represents the weight W, and at right angles to this line lay off a second line which represents the magnitude and direction of the tension T in the cord AB. Now, draw the rectangle of which T and W are the two sides. The diagonal of this rectangle represents the resultant of the horizontal and vertical forces and is equal to the force exerted on the beam. If a weight of 300 lb. is hung from B, and if the tension in the cord AB is 200 lb., then the resultant force on the beam is

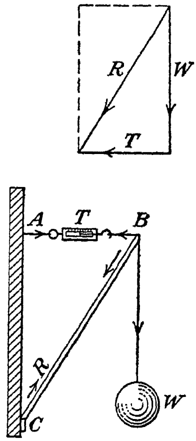


FIG. 17.—Forces acting on a simple crane.

$$R = \sqrt{(300 \text{ lb.})^2 + (200 \text{ lb.})^2} = 360.6 \text{ lb.}$$

45. Derrick Crane.—A derrick crane (Fig. 18) is a modification of a simple crane. It differs from the simple crane in the fact that the force in the rope AB is not always perpendicular to the post to which it is fastened. Instead of obtaining the resultant by drawing the rectangle of forces, it is now necessary to draw a parallelogram.

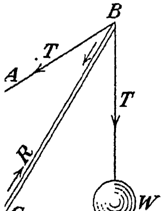


FIG. 18.—Forces acting on a derrick crane.

46. Resultant of More than Two Forces.—If more than two forces act simultaneously on a particle, the resultant can be obtained by constructing a polygon with its sides parallel respectively to these forces, the lengths of the sides representing the magnitudes of the forces. In case there are three forces A, B, and C (Fig. 19) acting on a particle P, let the length of a line ab, parallel to the force B, represent the magnitude of this force.

In like manner, let the line *ac* represent the force *A* in both magnitude and direction, and the line *bc* the force *C*. If the resultant force acting on the particle *P* is zero, a closed triangle *abc* will be formed. If the resultant force on the particle is not

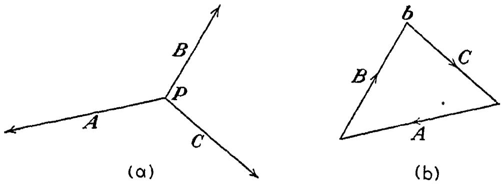


FIG. 19.—Equilibrium of three forces acting at a point.

zero, the triangle will not be closed, and the line necessary to make a closed figure will represent the magnitude of the resultant or effective force. The condition that the particle be in

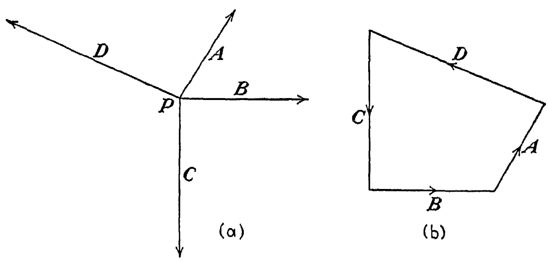


FIG. 20.—Equilibrium of four forces acting at a point.

equilibrium under the action of these three forces is that the three forces, when plotted in the way indicated, shall form a closed triangle.

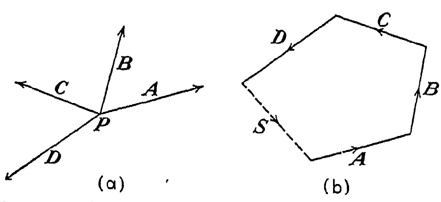


FIG. 21.—Four forces acting at a point do not produce equilibrium when the polygon does not close.

In like manner, let a particle *P* (Fig. 20) be acted on simultaneously by four forces and draw a polygon making its sides represent these forces in both magnitude and direction. If this polygon is closed, the resultant force on the particle is zero. If,

however, the figure is not closed (Fig. 21), there will be a resultant force acting on the particle. The length of the line necessary to close the polygon represents the magnitude of the force required to keep the particle in equilibrium. The forces on an airplane (Fig. 22) illustrate the way effective forces are balanced.

47. Resolution of Forces.—It has been seen that two forces can be combined into a single force. On the other hand, a single

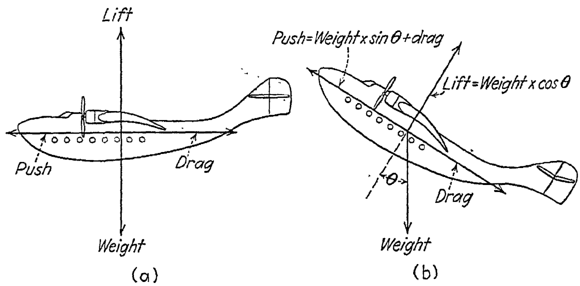


FIG. 22.—Forces on an airplane. (a) Flying horizontally; (b) flying at an angle with the horizontal.

force may be broken up into two or more forces to which it is equivalent. When one force is given, it is possible to find two other forces which when applied simultaneously will produce the same effect as the single force. This process of splitting up a single force into two or more parts is known as **the resolution of forces,** and the parts into which the force is split up are called the **components of the force.**

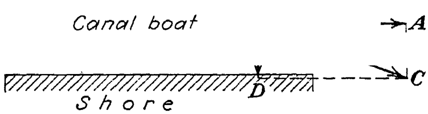


FIG. 23.—Components of the force on a canal boat.

Consider a canal boat as indicated in Fig. 23, and let a force be applied to it in the direction BC. This force produces two distinct effects on the boat. It moves the boat forward and also pulls it to the bank. Two separate forces might have been applied with the same result. The single force BC is, therefore, equivalent to two forces, one urging the boat forward and the other pulling it to the shore. These forces which produce the same effect as BC are the components of BC. The components

of this force are represented by the sides of a parallelogram of which the original force is the diagonal. In Fig. 24 the force due to the wind acting on a sailboat is resolved into components perpendicular and parallel to the sail, and the force perpendicular to the sail is then resolved into components perpendicular and parallel to the axis of the boat.

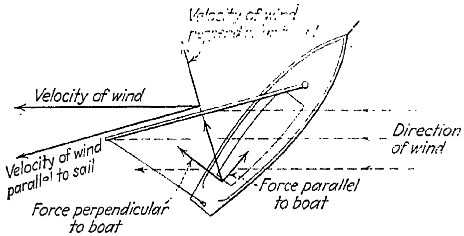


FIG. 24.—Resolution of the force due to the wind on a sailboat.

To find the components of a force, it is necessary to know the direction in which the components are desired, since there are any number of parallelograms which may be constructed on a given diagonal. When the directions of the components have been given, the magnitudes of the components are found by constructing a parallelogram whose diagonal represents the original force in magnitude and direction, and whose sides have the directions of the desired components.

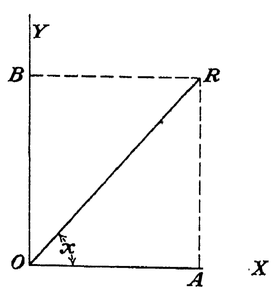


FIG. 25.—Rectangular components of a force.

48. Rectangular Components of a Force.—Most frequently the force is resolved in such a way that the components are at right angles to each other. In Fig. 25, OR represents a given force, and the components are desired along OX and OY, two directions which are at right angles to each other. By completing the rectangle $OARB$, the magnitudes of the components are found to be OA and OB. The relation between the components and the original force is given by the trigonometric formulae,

$$OA = OR \cos x.$$
$$OB = AR = OR \sin x.$$

Example.—A mass weighing 10 lb. is kept at rest on a smooth plane, which is inclined 30 deg. to the horizontal, by a force which acts parallel to the plane. Find the force parallel to the plane and the force at right angles to the plane.

Force parallel to plane = weight × sin 30 deg.
Force at right angles to plane = weight × cos 30 deg.
Force parallel to plane = $10 \times \frac{1}{2} = 5$ lb.
Force perpendicular to plane = $10 \times \frac{1}{2}\sqrt{3} = 8.6$ lb.

When a weight W hangs from a horizontal bar AC as indicated in Fig. 26, the tension T in the string BC can be resolved into two components, one horizontal and the other vertical. The horizontal component,

$$R = T \cos x,$$

causes a pressure against the vertical wall. This pressure is just balanced by the push of the wall against the bar. The vertical component,

$$Y = T \sin x,$$

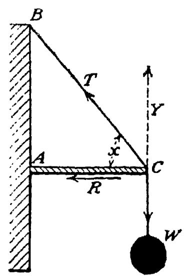


FIG. 26.—Resolution of tension into rectangular components.

tends to lift the end C of the bar AC. This tendency to lift the bar AC is just balanced by the pull of gravity on the weight W. Hence,

$$Y = T \sin x = W.$$

Example.—A mass of 100 lb. hangs from the bar at C (Fig. 26). Find the tension in the string BC which makes an angle of 45 deg. with the bar. Assume AC is without weight.

Weight = tension × sin 45 deg.

$$W = T \times \sin 45 \text{ deg.}$$
$$100 = T \times 0.707.$$
$$T = \frac{100}{0.707} \quad 141 \text{ lb.}$$

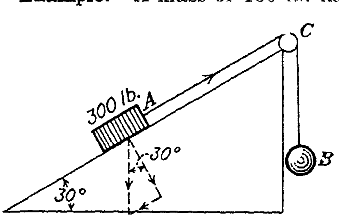


FIG. 27.—Resolution of force due to weight into rectangular components along and at right angles to the plane.

If a mass rests on an inclined plane (Fig. 27), the force of gravity acting on the mass can be resolved into two components, one parallel to the plane and the other perpendicular to the plane. The former is $W \sin x$ and the latter is $W \cos x$. The component parallel to the plane causes the mass to slide down the plane, and

the component perpendicular to the plane presses the mass against the plane.

Example.—A mass of 300 lb. rests on a plane making an angle of 30 deg. with the horizontal. What is the force down the plane and the force at right angles to the plane?

Force down the plane = W sin 30 deg. = 300 lb. × 0.5 = 150 lb.

Force perpendicular to plane = W cos 30 deg. = 300 lb. × 0.866 = 260 lb.

Problems

1. A body is subject to the action of a vertical force of 100 lb. upward and a horizontal force of 40 lb. What are the amount and the direction of the resultant of the two forces?

2. Two forces at right angles have a resultant of 320 lb. If one of the forces is 200 lb., what must the other one be?

3. A pendulum bob with a mass of 2 kg. is hung on a cord 1.6 m. long. A horizontal force is applied to the bob, sufficient to bring the cord to an angle of 30 deg. with the vertical. Find the horizontal force and the tensile force in the cord.

4. A picture is supported by two wires fastened to the ends of the upper edge of the picture frame which is horizontal. Each of the wires makes an angle of 60 deg. with the vertical. What is the tension in each wire if the picture weighs 15 lb.?

5. A plank which is 12 ft. long has one end resting on the ground and the other end at a point 4 ft. higher. On the plank is a trunk weighing 125 lb., which is kept from sliding by the friction. How great must the friction be?

6. A train of 450 tons is drawn up at 1.5 per cent grade. The frictional forces opposing the motion amount to 2 per cent of the weight of the train. What is the total force required to keep the train moving?

7. A cart standing on an inclined plane which makes an angle of 5 deg. with the horizontal is kept from rolling down hill by a force of 90 lb. applied in a direction parallel to the plane. What is the weight of the cart?

8. A piece of wire is 60 cm. long and strong enough to support 50 kg. without breaking. The ends of the wire are fastened to two points on the same horizontal line, 48 cm. apart. What is the greatest load which can be suspended from the middle of the wire?

9. The end of an electric transmission line in which there is a tension of 850 lb. is fastened to the top of a vertical pole and also to a guy wire which can sustain a tension of 1,600 lb. What is the smallest angle which the guy wire can make with the pole?

10. A cake of ice weighing 500 lb. slides down an inclined plane which is 50 ft. long and makes an angle of 30 deg. with the horizontal. Neglecting the force of friction, what is the speed of the block of ice when it reaches the bottom of the inclined plane?

CHAPTER IV

EQUILIBRIUM OF FORCES

The forces acting on a body may neutralize each other in such a way that there is no tendency for the body to change either its motion of translation or its motion of rotation. The body is then said to be in **equilibrium** under the action of the applied forces. If the body is at rest, it will remain at rest, and if it is in uniform motion,—either motion of translation or motion of rotation,—it will continue to move with uniform motion.

49. Torque.—The tendency of a force to produce rotation depends on the magnitude of the force and on the perpendicular distance between the line of action of the force and the axis about which the rotation takes place (Fig. 28). It is proportional to the magnitude of the force and also to the distance between the line of action of the force and the axis of rotation. **It is convenient to define torque or moment of force as the product of the force and the perpendicular distance between the line of action of the force and the axis of rotation.**

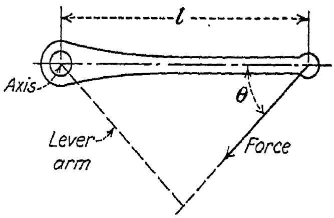


Fig. 28.—Torque = force times perpendicular distance from force to axis.

$$\text{Torque} = \text{force} \times \text{distance from axis.}$$

Example.—A man pulls at right angles to the spoke of a wheel with a force of 75 lb. If the distance from the axis to the hand is 3 ft., what moment of force does he apply to the wheel?

$$\begin{aligned}\text{Torque} &= \text{force} \times \text{distance to axis.}\\ &= 75 \text{ lb.} \times 3 \text{ ft.} = 225 \text{ lb.-ft.}\end{aligned}$$

50. Conditions of Equilibrium.—In order that a body be in equilibrium under the action of any number of forces, two conditions must be satisfied: (1) **The sum of the forces acting on the body in any direction must be equal to zero.** When this

condition is satisfied, the body will have no tendency to change its motion of translation, since there is no net force acting on it. (2) **In order that the body may have no tendency to change its motion of rotation, the sum of the moments of force tending to produce clockwise rotation about an axis must be equal to the sum of the moments of force tending to produce counterclockwise rotation about this same axis.** Where this second condition is fulfilled, there is no net torque acting on the body, and its motion of rotation will not change with time: that is, if the body is already at rest, it will not start into rotation; and, if it is already in rotation, its rate of rotation will not change.

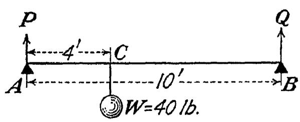


FIG. 29.—Reactions due to a weight supported by a weightless rod.

Example.—A weightless rod AB (Fig. 29) has its ends resting on supports. The rod is 10 ft. long and carries a weight W of 40 lb. at a point 4 ft. from A. Find the force exerted by each support.

$$\text{Forces downward} = \text{forces upward.}$$
$$40 = P + Q.$$

Moments of force clockwise = moments of force counterclockwise. Taking moments about C,

$$4 \times P = 6 \times Q,$$

whence

$$P = 24 \text{ lb., and } Q = 16 \text{ lb.}$$

51. Center of Gravity.—Every particle of a body possesses weight, so that the pull of the earth on the body is made up of a large number of forces directed toward the center of the earth. For bodies of ordinary size these forces are essentially parallel to each other. Now if two or more parallel forces act on a body, they may be replaced by a single force which will produce the same translation and rotation as were produced by all of the forces. The magnitude of this single force is obtained by adding together all the parallel forces, and the point at which this resultant force must be applied is found by requiring that the sum of the moments tending to produce clockwise rotation about this point is equal to the sum of the moments tending to produce counterclockwise rotation.

Suppose that there are two particles of mass, m and M (Fig. 30), at A and B, respectively, and that these particles are connected

with a light rod. These particles are attracted to the earth with forces which are nearly parallel to each other. If a point C in the rod is so chosen that

$$mg \times AC = Mg \times BC,$$

then the moments of force tending to turn the rod clockwise are just equal to the moments tending to turn it counterclockwise. If the rod is turned into some other position, the forces will be inclined at different angles to the rod, but the moments of force about C will still balance each other. Hence, it is possible to regard the two masses as concentrated at C, since the action of gravity on these two masses concentrated at C is the same as its action when the masses are at the ends of the rod. Such a point at which it is possible to assume the masses concentrated without changing the action of gravity on them is called **the center of gravity** of the masses.

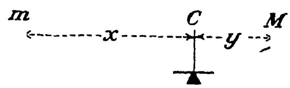


FIG. 30.—Center of gravity of two spheres connected by a light rod.

If instead of these two masses we take a long thin rod (Fig. 31), then every particle of matter in the rod is acted on by the force of gravity, and the directions of these forces are essentially parallel to one another. By an extension of the reasoning by which two masses were considered, it is possible to find a point O in the rod such that the moments of force tending to turn the rod clockwise about O are equal to the moments tending to turn it counterclockwise. This point is the center of gravity of the rod, and at it we may consider all the mass of the rod to be concentrated.

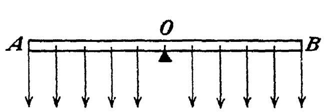


FIG. 31.—Center of gravity of a uniform bar.

Whatever the shape and size of the body, it is always possible to find one point at which a force equal and opposite to the weight of the body can be applied so that the body will remain at rest. About this point the body has no tendency to rotate under the action of gravity, and at this point we may consider all the mass of the body to be concentrated. If the body is balanced on a knife edge, this point will lie directly above the knife edge. The center of gravity need not necessarily lie in the substance of the

body. Thus the center of gravity of a uniform ring lies outside the material of the ring at its center.

Example.—Two weights, one of 5 lb. and the other of 15 lb., are connected by a light stiff rod 3 ft. long. Find the distance of the center of gravity from the 5-lb. weight.

Let x = the distance of the center of gravity from the 5-lb. weight.
$3 - x$ = the distance of the center of gravity from the 15-lb. weight.

Taking the center of gravity as the point about which to take moments

$$\begin{aligned} 5x &= 15(3 - x). \\ 5x - 45 &= -15x. \\ 20x &= 45. \\ x &= 2.25 \text{ ft. distance from center of gravity to 5-lb. weight.} \end{aligned}$$

52. Method of Finding the Center of Gravity.—If a sheet of metal of uniform thickness be freely suspended from a point near one of its extremities, it will come to rest in such a position that the center of gravity is in the vertical line passing through the support. If this line be indicated by the position of a plumb line and marked on the body, the center of gravity lies in this line. If another point of suspension be selected and a vertical line through it marked in the same way, it will also contain the center of gravity. Since the center of gravity lies in each of these lines, it must lie in their intersection.

53. Simple Cases of Center of Gravity.—The center of gravity of a rod of uniform density and of the same breadth and thickness throughout its length lies at the center of the rod. If such a rod is divided into particles, for every particle on one side of the middle point of the rod there is an equally heavy particle on the other side at an equal distance from the middle. In a rectangular plate of uniform thickness and density, the center of gravity lies at the intersection of the diagonals of the rectangle. This fact can be seen by dividing the plate up into narrow strips which may be considered as uniform rods in the preceding case. In the same way it can be shown that the center of gravity of a triangular lamina of uniform thickness and density lies at the point of intersection of the medians drawn from the angles of the triangles to the opposite sides. The centers of gravity of a sphere and a solid cube lie at their geometric centers.

54. Types of Equilibrium.—The equilibrium of a body may be **stable, unstable,** or **neutral.** When a body returns to its original position after being slightly disturbed, the equilibrium is said to be stable. A cone standing on its base is an illustration of this type of equilibrium. When a cone (Fig. 32) on its base is raised slightly from the table on which it rests, it returns to its original position on being released. It is in **stable** equilibrium. If, however, the cone rests on its vertex and is then slightly displaced,

it tends to fall into a new position rather than return to its original position. In this case the cone is in unstable equilibrium. Any body which tends to get as far as possible from its original position when disturbed is in **unstable** equilibrium. A sphere resting on a horizontal table when slightly displaced tends neither to return to its former position nor to go still farther away from it, but it remains in any position in which it finds itself. Such a body is in **neutral** equilibrium. Figure 33 shows how a cone may be in stable, unstable, or neutral equilibrium.

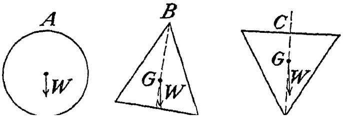


FIG. 32.—Neutral, stable, and unstable equilibrium.

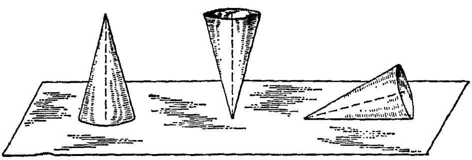


FIG. 33.—Stable, unstable, and neutral equilibrium of a cone.

55. Stability of a Body.—The position of the center of gravity is of much importance in determining the stability of a body. The lower the center of gravity, the greater the stability of the body and the more difficult it is to overturn. A body becomes unstable as soon as the vertical line through the center of gravity falls outside its base. The body which must be lifted the greater amount, in order to make the vertical through its center of gravity fall outside the base, is the more stable. In case of a wagon (Fig. 34), in which the load is some distance above the ground, the center of gravity of the load is high. If the vertical line through the center of gravity of the load fall inside the wheels of the wagon, the load will not overturn. If, however, the wagon is driven over ground which is inclined so that one wheel is raised above the other, the vertical line through the center of gravity approaches

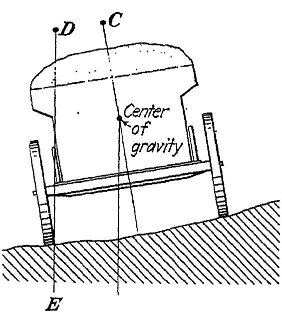


FIG. 34.—For equilibrium the perpendicular through the center of gravity must fall inside of the wheels.

the point at which the wheel is in contact with the ground. If a further elevation of one wheel above the other causes the line of action of gravity on the load to fall outside the wheels, the load will overturn. In order to avoid upsetting, it is, therefore, desirable to have the load as low as possible so that the center of gravity of the load may be as near as possible to the ground.

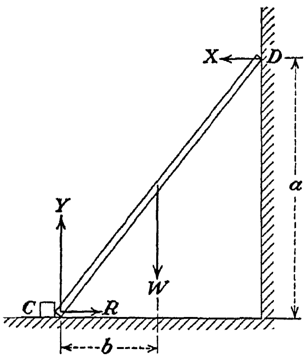


FIG. 35.—Forces up equal forces down, and forces to the right equal forces to the left.

56. Illustrations of Conditions for Equilibrium.—The forces acting on a ladder (Fig. 35) give an easy illustration of the two principles of equilibrium. If the wall is smooth, it exerts a force X on the top of the ladder in the horizontal direction. At the foot of the ladder, there is a vertical force Y and a horizontal force R. The only other force is the weight of the ladder, W, which may be considered to be concentrated at its center of gravity. Since the forces up must equal the forces down, and the forces to the right must equal forces to the left,

$$Y = W$$

and

$$X = R.$$

Take moments about C. Since the lever arm of R and the lever arm of Y are zero, neither of them produces a torque, but X produces a torque counterclockwise, and W a torque which is clockwise. Hence,

$$Xa = Wb,$$

where a is the perpendicular distance from C to X and b the perpendicular distance from C to W.

Problems

1. A beam 18 ft. long is supported at its ends by two walls. Find the reactions of the walls against the beam when a man weighing 160 lb. stands on the beam at a distance of 8 ft. from one end. Neglect the weight of the beam.

2. A pole 14 ft. long weighing 60 lb. can be balanced at a point 6 ft. from the thicker end. If it were to be supported at its ends, how much force would be needed at each end?

3. A uniform rod 15 ft. long weighing 70 lb. is supported at one end and at a point 3 ft. from the other end. Where can a weight of 30 lb. be attached to the rod so that the total force on each support will be the same?

4. A uniform bar a yard long has fastened to it at one end a mass of 2 lb., and at the other a mass of 3 lb. The bar itself weighs 1 lb. Where could a single force be applied to balance the system, and how great would the force have to be?

5. A very light rod 120 cm. long has attached to it a mass of 2 kg. at one end, 15 kg. at the other end, and 12 kg. in the middle. Find the position of the center of gravity.

6. A telegraph pole is placed on a two-wheeled truck located 8 ft. from the thicker end, and an upward force of 6 lb. at the thinner end is required to keep it horizontal. The pole is 18 ft. long and weighs 90 lb. Where is the center of gravity?

7. A uniform bar weighs 40 lb. and is 8 ft. in length. From one end is suspended a mass of 18 lb., and from the other end a mass of 24 lb. At what point must the rod be suspended in order that it be in equilibrium in a horizontal plane?

8. A square is acted upon by forces of 3, 6, 9, and 12 lb., along each side. The forces all act to produce rotation in the same direction. If the length of the side of the square is 12 in., what is the resultant torque tending to rotate the square about an axis through its center?

9. A ladder weighing 40 lb. makes an angle of 30 deg. with the vertical. The length of the ladder is 20 ft., and its center of gravity is 10 ft. from either end. A man weighing 180 lb. is at a point 15 ft. from the lower end of the ladder. Find the horizontal force against the foot of the ladder, necessary to keep it from slipping. Neglect frictional forces of wall on ladder.

10. Four weights of 2, 6, 8, and 12 lb. are placed along a straight bar at equal distances of 18 in. Neglecting the weight of the bar, find the position of the center of gravity.

11. From a circular disk whose radius is 20 cm., there is cut out a circle whose diameter is 10 cm. Find the center of gravity of the remainder of the disk if the circular hole is in contact with the circumference of the disk at one point.

12. A ladder resting against a smooth wall (Fig. 35) makes an angle of 30 deg. with the vertical wall. If the ladder weighs 100 lb. and is 20 ft. long, find the force it exerts on the vertical wall and the horizontal and vertical force on the floor at C.

CHAPTER V

WORK, POWER, ENERGY

57. Work.—When the point at which a force is applied moves, work is done. When a bucket of water is carried from the cellar to the attic, work is done which is equal to the weight of the bucket times the vertical distance through which it is lifted. **The work is found by taking the product of the force and the distance through which the object moves.** The force and the displacement must be measured in the same direction.

$$\text{Work} = \text{force} \times \text{distance}.$$
$$W = F \times S. \quad \text{(Appendix D-4.)}$$

If the force does not produce a displacement of the body, no work is done in the sense in which work is used in physics. A man holding a 10-lb. weight does no work in this sense because there is no displacement or movement of the mass. He exerts a force sufficient to overcome the pull of gravity on the mass. This, of course, requires an effort, but it is not work in the sense in which the word is used here.

58. Gravitational Units of Work.—Since work is measured by the product of the force times the distance through which it acts, in order to measure work it is necessary to measure two quantities—force and distance. In the English system, the force is measured in terms of a unit of force which is equal to the pull of gravity on a mass of 1 lb., and the distance is measured in feet. In this system the unit of work is called the **foot-pound.**

One foot-pound of work is defined as the work which is done when a force equal to the weight of 1 lb. acts through a distance of 1 ft.

For example, 1 ft.-lb. of work is done when a mass of 1 lb. is raised a distance of 1 ft. at constant speed against the action of gravity.

In the metric system the unit of work may be chosen as the gram-centimeter or the kilogram-meter.

One gram-centimeter of work is defined as the amount of work which is done when a force equal to the weight of 1 g. acts through

a distance of 1 cm., and the kilogram-meter as the work which is done when a force equal to the weight of 1 kg. acts through a distance of 1 m.

The gram-centimeter is the amount of work done when a mass of 1 g. is lifted a vertical distance of 1 cm. at constant speed against the action of gravity.

Example.—A ton of coal is lifted from the street into a building. The height through which the coal is lifted is 10 ft. Find the work done.

$$\text{Work} = \text{force} \times \text{distance}.$$
$$2{,}000 \text{ lb.} \times 10 \text{ ft.} = 20{,}000 \text{ ft.-lb.}$$

59. The Erg.—The gravitational units of work, like the gravitational units of force which enter into them, depend on the place on the surface of the earth at which they are used. For this reason an absolute unit of work, the **erg,** is frequently used. **An erg of work is the work done when a force of 1 dyne acts through a distance of 1 cm.** Since the weight of 1 g. is equivalent to 980 dynes, a gram-centimeter of work is equivalent to 980 ergs; that is, when a mass of 1 g. is lifted a distance of 1 cm. against the force of gravity, 980 ergs of work are done.

Example.—Find the work done by a force of 750 dynes acting through a distance of 20 cm.

$$\text{Work} = \text{force} \times \text{distance}.$$
$$\text{Work in ergs} = \text{force in dynes} \times \text{distance in centimeters}.$$
$$\text{Work} = 750 \text{ dynes} \times 20 \text{ cm.} = 15{,}000 \text{ ergs}.$$

The erg is such a small unit of work that it is convenient to define a much larger unit, the joule. **A joule is defined to be 10,000,000 ergs;** *i.e.*, **1 joule = 10^7 ergs.**

Example.—How many joules of work are done by a force equal to the weight of 2 kg. acting through a distance of 2 m.?

$$\text{Work in joules} = \frac{\text{force in dynes} \times \text{distance in centimeters}}{10^7}.$$
$$2 \text{ kg.} = 2{,}000 \times 980 \text{ dynes}.$$
$$\text{Work in joules} = \frac{2{,}000 \times 980 \text{ dynes} \times 200 \text{ cm}}{10^7}$$
$$= \frac{39.2 \times 10^7}{10^7} = 39.2 \text{ joules}.$$

60. Power.—In defining work as the force multiplied by the distance through which it acts, it is to be observed that the ele-

ment of time does not enter. The same work is done in lifting a mass of 300 lb. a distance of 100 ft. whether the work is done in a day or in a minute. The same work is done whether the mass is carried at a single load or in two or more loads. The amount of work done is measured by the end result, and it does not in any way depend upon the time to do the work. In the consideration of a machine it is necessary to know more than the total amount of work which the machine can do. It is desirable to know the rate at which the machine can work. **The time rate of doing work is called power.** (Appendix D-4.) Hence,

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}}$$

$$= \frac{F \times s}{t} = \text{work per unit of time.}$$

Since

$$\frac{s}{t} = v,$$

$$\text{Power} = \text{force} \times \text{velocity} = Fv$$

Example.—A horse hitched to a load of hay hauls it from the ground to the loft of the barn in 1 min. If the load weighs 500 lb. and if the height through which it is lifted is 20 ft., find the rate at which the horse is working.

$$\text{Work} = \text{force} \times \text{distance} = 500 \text{ lb.} \times 20 \text{ ft.} = 10{,}000 \text{ ft.-lb.}$$

$$\text{Power} = \frac{\text{force} \times \text{distance}}{\text{time}} = \frac{500 \text{ lb.} \times 20 \text{ ft.}}{60 \text{ sec.}} = 166.7 \text{ ft.-lb. per second.}$$

Horsepower and Watt.—The English unit of power is called the horsepower. A horsepower denotes the ability of a machine to do 33,000 ft.-lb. of work in 1 min. or 550 ft.-lb. in 1 sec.

Example.—The maximum walking draft of a horse is about one-half of his weight. If the horse weighs 1,600 lb. and walks at the rate of 0.5 mile per hour, he is working at the rate of

$$\text{Horsepower} = \frac{0.5 \times 5{,}280 \times 1{,}600}{60 \times 60 \times 550 \times 2} = 1.06 \text{ (hp.)}.$$

The absolute unit of power is called a watt. **A watt is a rate of work in which 10^7 ergs of work are done per second. A watt is a joule per second.**

Example.—At what rate is a motor working when it lifts a weight of 25 kg. through a distance of 4 m. in 5 min.?

$$\text{Power in watts} = \frac{\text{work in joules}}{\text{time in seconds}}$$
$$= \frac{25{,}000 \times 980 \times 400}{5 \times 60 \times 10^7} = \frac{19.6 \times 10^8}{6 \times 10^8} = 3.3 \text{ (watts).}$$

62. Definition of Energy.—The flywheel of an engine will keep the machinery running for some time after the power has been shut off. The flywheel, therefore, has energy and can do work. Inanimate bodies possess energy and can do work because work has been done on them at some previous time. Suppose that an elevator has lifted a 100-lb. mass from the ground to a height of 20 ft. The operation of lifting this mass has required the expenditure of work which is equal to the work done in lifting a weight of 100 lb. a distance of 20 ft., or 2,000 ft.-lb. of work. The mass in this new position possesses a capacity for doing work. If it is now allowed to slide down an incline in a suitable way, it may be made to return this work to some sort of a machine.

In order to set a wheel spinning about its axis, it is necessary to exert a push or a pull on the spokes or on the tire. During this effort, energy is stored up in the wheel. When the wheel is once in motion, it continues to spin unless a force is applied to stop it. The work done on the wheel is stored up as energy of rotary motion. The energy which is thus stored up may be made available for doing work on other bodies. In all cases energy is required to set bodies in motion, and this energy is returned when the bodies are stopped.

63. Potential and Kinetic Energy.—It is necessary to distinguish between two types of energy: (1) potential energy and (2) kinetic energy. The energy which a body has by virtue of its position or configuration is called **potential energy.** When a mass has been lifted above the surface of the earth, it has energy on account of its position. When a spring has been compressed or a bow has been bent, energy has been stored up. This energy can be again recovered and is potential energy.

Any body which is in motion is able to set other bodies in motion by colliding with them. A bullet leaving the muzzle of a gun with a velocity of 300 m. per second has energy stored up in it. This is seen by the fact that when it strikes a body, it can overcome a considerable resistance. Common experience teaches that it is difficult to stop or change the direction of motion of a body moving with considerable velocity. This is because the

body possesses energy. That energy which a body has on account of its motion is called **kinetic energy.** Every body in motion has kinetic energy, and the faster the body is moving, the greater is this kinetic energy. The heavier the body, the greater the kinetic energy.

64. Conservation of Energy.—The study of the various forms in which energy may occur and of the transformation of one kind of energy into another has led to the statement of a very important principle known as the **conservation of energy.** This principle may be stated as follows: **In any body or system of bodies which is not receiving or giving up energy, the total amount of energy is unchanged.** This principle states that energy can never be created or destroyed. It can be transformed from one form into another, but the total amount in the end is unchanged.

A bullet leaves the muzzle of the gun with kinetic energy which it received on account of the work done on it by the expanding gases. As it passes through the air, it loses some of this kinetic energy because of the heat developed by the friction in the air. When it strikes the target, sound waves are sent out which carry away some of the energy. There may also be a flash of light which uses up some energy. Heat will be developed in the target, and fragments of the bullet may carry away some of the energy. If all these energies are added together, they will be found just equal to the energy with which the bullet left the muzzle of the gun.

65. The Measure of Potential Energy.—The measure of the potential energy which a lifted body, such as a pile driver, has on account of its position is equal to the work which has been spent in lifting the body. If the height in feet through which the body has been lifted is h and its weight in pounds is W, then the potential energy of the lifted body is

$$\text{Potential energy} = Wh \text{ ft.-lb.}$$

Example.—A clock weight weighing 3 lb. is lifted 6 ft. against gravity. What is the potential energy thus stored up?

$$\begin{aligned}\text{Potential energy} &= Wh \\ &= 3 \text{ lb.} \times 6 \text{ ft.} = 18 \text{ ft.-lb.}\end{aligned}$$

Other examples of potential energy are found in the work done in winding a watch and in the work required to stretch an india-

rubber band. In these cases, the material of the spring or the rubber band is in a state of strain, and, on this account, the body possesses potential energy. It has been seen that when a body is at a height h and is then allowed to fall freely, all the potential energy is changed into kinetic energy. For example, if a pile driver weighing 2,000 lb. falls 15 ft., the potential energy is all changed into kinetic energy.

66. Measure of Kinetic Energy.—To find the kinetic energy which a body possesses by virtue of its motion, consider the work which must be done on it in order to give it a certain speed. When the body is stopped, it will give up an amount of energy

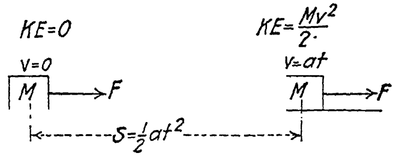


FIG. 36.—Kinetic energy equals the work to produce the velocity $= \frac{M}{2}v^2$.

which is equal to the work done in starting it. By definition, this latter is its kinetic energy.

From Newton's second law of motion, the force necessary to make a body move with an acceleration a is

$$F = Ma,$$

where force is in dynes or poundals, mass in grams or pounds, and acceleration in centimeters per second per second or feet per second per second.

Let s (Fig. 36) be the distance in centimeters or feet through which the body moves. Then the work done on the body in this distance is

$$\text{Work} = Fs = Mas \text{ (ergs or foot-poundals)}.$$

If the body starts from rest, its velocity in centimeters per second or feet per second at the end of t sec. is

$$v = at$$

where a is the acceleration in centimeters per second per second or in feet per second per second.

Since the space passed over in t sec. is $s = \frac{1}{2} at^2$ and since $t = \frac{v}{a}$, we have, by substitution,

$$v^2 = 2\ as$$

and

$$as = \frac{1}{2} v^2.$$

Substituting this value of as in the expression for the work done on the body,

$$\text{Work} = Fs = \frac{Mv^2}{2} \text{ ergs or foot-poundals.} \quad \text{(Appendix D-5)}$$

This expression gives the work necessary to cause a mass M to acquire a velocity v. This work does not depend on the distance covered or on the acceleration. It is determined solely by the mass of the body and its speed. If a retarding force is applied to this body so that it is brought to rest, the moving body will do work against this retarding force. When the body has come to rest, the amount of work which has been done will be just equal to the work done in starting the body. According to the law of conservation of energy, the energy spent in starting the body must be just equal to that derived from the body when it is stopped. Hence,

$$\text{Kinetic energy} = \frac{Mv^2}{2} \text{ ergs or foot-poundals.}$$

If the body has an initial velocity v_0 and an initial kinetic energy $\frac{1}{2}Mv_0^2$, the gain in kinetic energy is the work done on the body by the accelerating force.

$$\begin{aligned}\text{Gain in kinetic energy} &= \text{accelerating force} \times \text{distance} \\ &= Fs = Mas\end{aligned}$$

Since $as = v^2 - v_0^2$ (see page 15),

$$\text{Gain in kinetic energy} = \frac{M}{2}(v^2 - v_0^2) \text{ ergs or foot-poundals.}$$

If the body is decreasing in velocity, the loss of kinetic energy equals the work done by the body against the retarding force.

$$\text{Loss of kinetic energy} = \text{retarding force} \times \text{distance}$$
$$= Fs = Mas$$

Since in this case $as = \dfrac{v_0^2 - v^2}{2}$

$$\text{Loss of kinetic energy} = \frac{M}{2}(v_0^2 - v^2) \text{ ergs or foot-poundals}$$

When it is desired to express the kinetic energy in foot-pounds or in gram-centimeters, instead of expressing it in foot-poundals or ergs,

$$\text{Kinetic energy} = \frac{(\text{mass in pounds}) \times (\text{velocity in feet per second})^2}{2 \times 32.2}$$

where

$$32.2 = \text{the number of foot-poundals in one foot-pound,}$$

or

$$\text{Kinetic energy} = \frac{(\text{mass in grams}) \times (\text{velocity in centimeters per second})^2}{2 \times 980}$$

where

$$980 = \text{the number of ergs in one gram-centimeter.}$$

Example.—If an automobile weighing 3 tons is moving with a velocity of 30 ft. per second, what is its kinetic energy in foot-pounds?

$$\text{Kinetic energy} = \frac{6{,}000 \times (30)^2}{2 \times 32.2} \text{ ft.-lb.}$$
$$= \frac{3{,}000 \times 900}{32.2} = 83{,}800 \text{ (ft.-lb.)}$$

Example.—A boy weighing 75 lb. starts to slide on ice with a speed of 25 ft. per second. The retarding force due to the friction is 30 lb. How far will he go before coming to rest?

Loss of kinetic energy = retarding force × distance.

$$\text{Loss of kinetic energy} = \frac{75 \times (25)^2}{32.2 \times 2} = 30 \times d$$

whence,

$$d = \frac{75 \times 625}{32.2 \times 60} = 24.3 \text{ (ft.)}$$

67. Transformations of Potential and Kinetic Energy.—The energy of a body is capable of changing its form from potential to kinetic or from kinetic to potential. Thus, suppose that a stone of weight W is supported on the edge of a cliff at the height h above the base of the cliff. The potential energy is the work done in lifting the stone a height h. Since the force with which the earth attracts the stone is W lb., the potential energy is Wh ft.-lb. If the stone is allowed to fall freely, it loses its potential energy but gains in kinetic energy. When the stone has reached the base of the cliff, all added potential energy has been changed into kinetic energy. At any point in the path of the stone, the sum of the potential and the kinetic energy is the same as at every other point.

The swinging of a pendulum or the oscillation of a weight attached to a spring illustrates the way in which potential energy may be transformed back and forth. When the spring is elongated to the greatest extent, the mass has no velocity and no kinetic energy.. As the mass is released, it moves upward and acquires kinetic energy. Meanwhile, the potential energy stored in the spring is being transformed into kinetic energy. When the mass has risen to its highest point, the spring is compressed to the greatest extent and all of the energy is now stored in the spring as potential energy. The mass now begins to move downward and again acquires velocity and kinetic energy. This interchange of kinetic and potential energy continues as long as the oscillations of the spring persist.

68. Illustrations.—A windmill furnishes a good illustration of the transformation of kinetic into potential energy. The motion of the air imparts a certain motion of rotation to the wheel. The kinetic energy which the wheel possesses on this account is used to pump water into a tank. The work or energy required to lift this water comes from the energy of rotation in the wheel. The work done in lifting the water is stored up as potential energy. This energy may be again transformed into kinetic energy by allowing the water to flow out of the tank.

Another illustration of the transformation of kinetic into potential energy is seen in a sprayer. Water emerging from the nozzle of the sprayer possesses kinetic energy. If the sprayer is directed upward, the water rises to a certain height before it stops. When the water comes to rest, all the kinetic energy has been transformed into potential energy. As the water falls back to the earth, the velocity increases and the kinetic energy increases, while the potential energy decreases.

Problems

1. To what height can a piece of structural steel weighing ½ ton be lifted if work amounting to 500,000 ft.-lb. is done on it?

2. How many foot-pounds of work are required to lift stone from the ground to build a cylindrical wall, 40 ft. high and 3.5 ft. thick, with an outside diameter of 70 ft.? Density of stone, 140 lb. per cubic foot.

3. A standpipe 80 ft. high has an internal diameter of 14 ft. How much work would be required to fill the standpipe with water: (*a*) if the water were pumped in at the bottom; (*b*) if it were pumped in at the top?

4. Three men using a block and tackle are lifting a safe weighing 2,200 lb. to a height of 30 ft. If each man develops ⅛ hp., how long will it take to do the work?

5. A crank 15 in. long is turned by hand at the rate of 100 revolutions per minute, a force of 6 lb. applied at a tangent to the circle described by the handle being required. Calculate the horsepower applied.

6. A boat is moving through the water at the rate of 25 ft. per second. Its engines have a horsepower of 750. Find the resistance which must be overcome in moving the boat through the water.

7. The brakes of an automobile which weighs 4,500 lb. can exert a retarding force of 600 lb. Find the distance the car will move before stopping, if it is traveling at the rate of 40 miles per hour when the brakes are applied.

8. Find the average resisting force which is exerted in stopping the hammer of a pile driver weighing 500 lb. after it has fallen a distance of 20 ft., if the pile which is being driven moves a distance of 4 in. and no energy is lost at impact.

9. A machine gun fires 300 bullets each minute with a velocity of 1,800 ft. per second. If the mass of each bullet is 0.025 lb., what horsepower is developed by the gun?

CHAPTER VI

FRICTION

69. Nature of Friction.—When a heavy block of wood is pushed along the top of a table, a certain resistance is encountered. By making the surface of the table and the surface of the block very smooth, the amount of this resistance can be much decreased. No matter what the nature of the surfaces which are moving over each other, there is always some resistance or opposition to the motion. This resistance, the amount of which depends on the characteristics of the rubbing surfaces, is called **friction.**

70. Kinds of Friction.—Friction always opposes the motion, whatever its direction. It never tends to push the body either forward or backward. It merely tends to stop the motion and make it more difficult to move the body. It is more difficult to start the body than it is to keep it in uniform motion when once started. There are for this reason two kinds of friction, **kinetic friction** and **static friction.** The former is the force to keep the body moving with uniform speed. The latter is the force to start the body from rest. Kinetic friction is less than static friction.

71. Laws of Friction.—The laws which determine the amount of friction between the dry surfaces of solids are as follows:

1. **The friction between two sliding surfaces is nearly independent of the velocity.**

2. **If the force perpendicular to the surface remains the same, the friction does not depend on the area of the rubbing surfaces. This statement is only approximately true.**

3. **The force of friction is proportional to the total force pressing one surface against another.**

4. **The force of friction is greater at the start than after motion has begun.**

72. Coefficient of Friction.—In order to describe with some accuracy the characteristics of the rubbing surfaces, it is customary to define what is known as the coefficient of friction. If

the force at right angles to the rubbing surfaces (Fig. 37) is P, and the force required to just draw one surface over the other with uniform speed is F, **then F/P is called the coefficient of kinetic friction.** It states that the quantity obtained by dividing the force of kinetic friction by the force perpendicular to the rubbing surfaces is a constant which is characteristic of the rubbing surfaces.

$$\text{Coefficient of kinetic friction} = \frac{\text{force of friction}}{\text{force pressing bodies together}} = \frac{F}{P}.$$

$$\text{Coefficient of static friction} = \frac{\text{force necessary to produce motion}}{\text{force pressing bodies together}}.$$

An approximate idea of the magnitude of this coefficient of kinetic friction is found by observing by means of a spring balance the force necessary to draw the body along a horizontal surface with uniform velocity and then dividing this force which is equal to the force of friction by the weight of the body (see table, page 767).

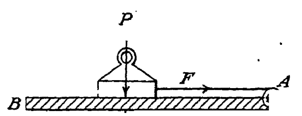


FIG. 37.—The coefficient of friction is equal to the force of friction divided by force perpendicular to the plane.

Example.—A mass of 25 lb. rests on a rough horizontal plane. The force required to pull this mass along with uniform velocity is 2.5 lb. What is the coefficient of kinetic friction?

$$\text{Coefficient of kinetic friction} \quad \frac{\text{force equal to friction}}{\text{force pressing bodies together}}$$

$$\therefore \frac{F}{P} = \frac{2.5 \text{ lb.}}{25 \text{ lb.}} = 0.10.$$

73. Rolling Friction.—The friction of a solid rolling on a surface is less than the friction of a solid sliding over a surface. For this reason, instead of hauling on a sled, a farmer uses a wagon. In this case, there is still some friction between the wheels of the wagon and the road bed and between the surface of the axle and the hub sliding over it. This latter source of friction may be reduced by using what is known as roller or ball bearings (Fig. 38). Then the sliding friction in the hub is replaced by rolling friction which is much smaller. When a car wheel rolls on a level

track (Fig. 39), it is flattened a little where it rests on the track and also makes a slight depression in the track. As the wheel rolls, it is forced continually to climb up out of this depression. This requires an additional force which is **rolling friction.** If

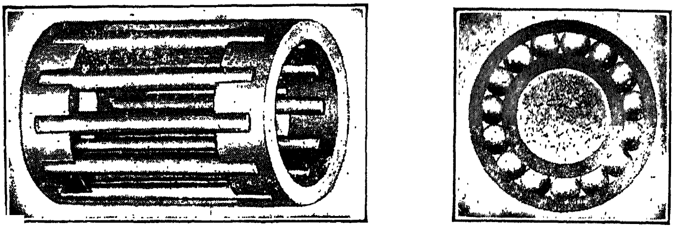


FIG. 38.—Roller bearings and ball bearings reduce frictional forces.

the road bed were made of steel, this depression would be very slight and difficult to detect. On soft dirt road it is considerable. It is less where the wheels of the wagon are provided with wide tires. On steep inclined railways running up the sides of mountains, it is necessary to provide the drive wheels of the engine with cogs which fit into a cogged rail. This is because sliding friction is not large enough to prevent the wheels from slipping. When the rails of an ordinary railroad are wet or covered with ice, the friction between the rails and the wheels is small, and the wheels slip on the track making it difficult for the engine to pull the load. For this reason the engine is provided with means of supplying sand to the track, by which the friction is increased and slipping is prevented.

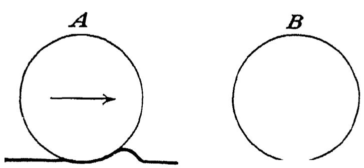


FIG. 39.—Deformation of surfaces by roller bearing.

74. Advantages of Friction.—Friction has many advantages as well as many disadvantages in machines. Except for friction between the shoes and the pavement, a person could not walk. When the pavement is covered with ice, friction is small and walking is difficult. On account of friction, belts cling to the pulleys and drive the machinery. The brakes on a wagon or on an automobile stop the wagon or automobile by means of friction. Screws and nails hold their places in the objects into which they are driven by means of friction.

75. Efficiency of Machines.—Every machine wastes energy on account of friction. Consequently, the work put into the

machine is always more than that obtained from it. This loss decreases the efficiency of the machine. **The efficiency of a machine is defined to be the ratio of the work done by the machine to the work done on the machine.** It is, therefore, the output of the machine divided by the input.

$$\text{Efficiency} = \frac{\text{work done}}{\text{energy supplied}} = \frac{\text{output of energy}}{\text{input of energy}}.$$

Example.—With a block and tackle a horse exerting a force of 500 lb. lifts 2,000 lb. a distance of 10 ft. If the horse moves a distance of 42 ft. in moving this load, what is the efficiency of the block and tackle?

$$\text{Efficiency} = \frac{\text{work done by the machine}}{\text{work done on the machine}} = \frac{2{,}000 \text{ lb.} \times 10 \text{ ft.}}{500 \text{ lb.} \times 42 \text{ ft.}} = 95 \text{ per cent.}$$

76. Action of Lubricants.—If a layer of liquid is introduced between two rubbing surfaces, the liquid flows over each of these surfaces and adheres to them and a layer of liquid is held rigidly to each of the surfaces. In such cases, instead of having friction between the liquid and the solid, there is friction between the layers of the liquid. When oil is poured into the bearings of a machine, it forms two layers over the metal surfaces. The sliding then takes place for the most part between these layers of oil. Since the friction between the layers of the oil is much less than it is between the surfaces of the metal, the energy lost by friction is much decreased.

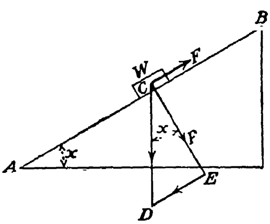


Fig. 40.—Coefficient of friction = tangent of angle of repose.

77. Friction on an Inclined Plane.—Let AB (Fig. 40) be a board on which rests a block W, and let the board be inclined at such an angle that the block will just slide down the plane with uniform velocity when once started. If CD represents the weight of the block, its components parallel to the plane and at right angles to the plane are DE and CE. The force of friction between the block and the plane is parallel to the plane and in the opposite direction to DE, *i.e.*, up the plane. Since the block experiences no acceleration, the force up the plane is equal to the force down the plane.

$$F = DE = W \sin x.$$

The force perpendicular to the plane $= CE$.

$$P = CE = DC \cos x = W \cos x.$$

$$\text{Coefficient of kinetic friction} = \frac{F}{P} = \frac{DE}{CE} = \frac{W \sin x}{W \cos x} = \tan x.$$

Hence, the tangent of the angle at which the block slides down the plane with uniform velocity when once started is numerically equal to the coefficient of kinetic friction. This angle is known as the **angle of repose.**

78. Power Transmitted by a Belt.—Where a pulley is being driven by a belt (Fig. 41), the tensions in the straight parts of the belt are not equal. They differ by the friction which is exerted between the belt and the pulley. Let T_2 be the tension in that part of the belt which is moving toward the pulley and T_1 the tension in that part which is moving away from the pulley, and let V be the velocity with which the belt is moving. The force of friction between the belt and the pulley is $T_1 - T_2$.

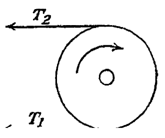


FIG. 41.—Friction between the belt and the pulley = $(T_1 - T_2)$.

The work done per minute by the belt = $(T_1 - T_2)V$ ft.-lb.

$$\text{The horsepower delivered to the pulley} = \frac{(T_1 - T_2)V}{33{,}000}$$

Example.—The tension on one side of a belt is 300 lb. and that on the other side is 100 lb. The belt is moving 300 ft. per minute. Find the horsepower delivered to the pulley.

$$\text{Horsepower} \quad \frac{(300 - 100)\ 300}{33{,}000} = 1.8 \text{ (hp.)}$$

Problems

1. A locomotive has 60 tons of its weight resting on its driving wheels. What is the greatest pull the engine can exert before slipping begins, if the coefficient of friction between wheels and rails is 0.15?

2. A weight of 6 lb. hanging over the edge of a table on a cord is just sufficient to drag a 48-lb. mass along the horizontal surface of the table with unchanging velocity. What is the coefficient of friction?

3. A brake shoe is pressed against the rim of a wheel with a force of 80 lb. If the coefficient of friction between the surfaces is 0.18, how much frictional force is developed?

4. A casting weighing 220 lb. is lifted 2 ft. from the floor by means of a chain hoist by the application of a force of 50 lb. through a total distance of 30 ft. What is the efficiency of the chain hoist?

5. Water is to be pumped to a height of 40 ft. at the rate of 165 gal. per minute. If the pump is 60 per cent efficient, what horsepower must be available to drive it?

6. The angle of repose for a 20-lb. block of metal on an incline is found to be 10 deg. How much force, parallel to the incline, is necessary to cause the body to begin to move up the incline?

7. A block slides down a plane which is inclined 40 deg. with the horizontal. If the coefficient of friction between the block and the surface of the incline is 0.2, find the acceleration of the block.

8. A block rests on a plane which is inclined at an angle of 30 deg. to the horizontal. The coefficient of friction between the block and the plane is 0.12. If the block weighs 30 kg., what is the force, parallel to the incline, necessary to cause it to slide up the plane?

9. A block of ice weighing 25 kg. is given a speed of 10 m. per second on the surface of a pond where the coefficient of friction is 0.015. How far will the ice go before coming to rest?

10. Find the greatest speed at which a locomotive and train can run if the engine and train weigh 150 tons and frictional resistance to motion on the level is 12 lb. per ton weight. The engine develops 180 horsepower.

11. What force acting at an angle of 30 deg. with the horizontal is necessary to move a mass of 10 lb. with uniform velocity along a horizontal surface, the coefficient of friction being 0.2?

CHAPTER VII

SIMPLE MACHINES

79. Levers.—A lever is a very simple form of machine. In Fig. 42 the three possible cases are shown. In one of these cases the fulcrum *F* is between the forces, and in the other cases it is at the end of the lever.

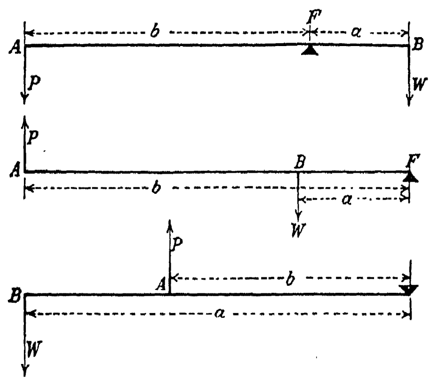


Fig. 42.—Three different types of lever classified according to the position of the fulcrum and forces.

The simplest kind of lever is one in which the arms are of equal length. The scale beam on a pair of ordinary balances (Fig. 43) is such a lever. In this case equal forces or weights at the ends of the lever just balance each other. Usually the distances of the forces from the point at which the lever is supported are not equal, and for equilibrium in such cases the forces must also be unequal. The larger force, at a smaller distance from the fulcrum, then has the same tendency to tip the lever as does the smaller force at a greater distance.

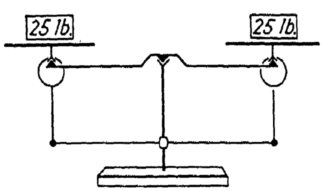


Fig. 43.—Platform balance—an equal-arm lever.

80. Law of the Lever.—Any lever is balanced when the sum of the moments of force tending to produce rotation clockwise is

equal to the sum of the moments of force tending to produce rotation counterclockwise. If only one force is applied and one force overcome, this law may be stated as follows: A lever is balanced when

$$\text{Weight} \times \text{weight arm} = \text{force} \times \text{force arm}.$$

This law states that when a lever is balanced under the action of two forces, the forces applied to the lever are inversely proportional to their distances from the fulcrum.

81. Levers in the Body.—In order to get food, and preserve life, the higher animals make use of a series of levers to move their bodies in whole or in part. These levers are generally made of bone and generally work against a bony fulcrum. Normally, the head acts as a lever in which the force is applied close to the fulcrum. Other illustrations of the lever are found in the movements of the forearm, the foot, and the lower jaw.

82. Mechanical Advantage.—In all simple machines like levers a certain advantage is obtained by the use of the machine. This advantage does not consist in an increase or in a decrease of the work performed by the machine. **Neglecting friction, the work done on the machine must always be the same as the work done by the machine.** This law follows from the law of conservation of energy. By means of a suitable lever or other machine it is possible, however, to exert a large force by the application of a small force. The large force will act through a small distance, and the small force through a large distance, so that the work done in the two cases will be the same.

The ratio of the resistance or the force overcome to the applied force is called the actual mechanical advantage. Because of friction the actual mechanical advantage is always less than the ideal or theoretical mechanical advantage.

Example.—By means of a crowbar which is 6 ft. long, a man lifts a stone which weighs 330 lb. The distance from the fulcrum to the stone is 0.5 ft. and the distance from the fulcrum to the point at which the man pushes is 5.5 ft. The man finds it necessary to exert a force of 30 lb. What is the mechanical advantage of the lever?

$$\text{Mechanical advantage} = \frac{\text{force overcome}}{\text{force applied}} \quad \frac{330\text{ lb.}}{30\text{ lb.}} \quad 11.$$

Also applying the principle of moments,

Force overcome $\times$ lever arm = force applied $\times$ lever arm.

$$R \times a = F \times b.$$
$$\frac{R}{F} = \frac{b}{a} = \frac{5.5}{0.5} = 11.$$

83. Levers of the First Class.— When the fulcrum is between the force applied and the force overcome, the lever is known as a lever of the first class. A pair of scissors is an illustration of this type of lever. If the distance from the screw which acts as fulcrum to the handles at which the force is applied is three times as great as the distance from the screw to the object on which the scissors are cutting, then a force of 10 lb. applied to the handles is capable of exerting a force of 30 lb. on the material which is being cut.

Example.—The length of a pump handle from the point at which the force is applied to the bolt which is acting as fulcrum is 30 in. The length of the weight arm is 3 in., and a force of 50 lb. is applied to the handle. Find the weight which can be lifted and the mechanical advantage.

Weight $\times$ weight arm = force $\times$ force arm.
Weight $\times$ 3 in. = 50 lb. $\times$ 30 in.

$$\text{Weight} = \frac{1{,}500}{3} = 500 \text{ lb.}$$

$$\text{Mechanical advantage} = \frac{\text{force overcome}}{\text{force applied}} = \frac{500}{50} = 10.$$

84. Levers of the Second Class.—Sometimes the fulcrum of the lever is located at one end, as shown in Fig. 44. The principle of moments applies in this case as in the preceding case. The

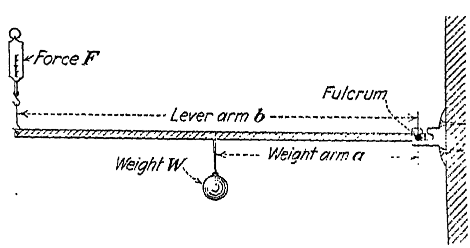


FIG. 44.—A lever of the second class. Weight $\times$ weight arm = force $\times$ force arm.

moment of force tending to tip the lever down must be equal to the moment of force tending to tip it upward when the lever is balanced. One of these forces tends to rotate the lever in the direc-

tion in which the hands of a clock moves; the other tends to rotate it in the opposite direction. For equilibrium,

$$F \times b = W \times a.$$

This type of lever is found in the ordinary wheelbarrow. The axle of the wheel is the fulcrum, the load is the weight, and the force applied to the handles corresponds to F in Fig. 45. When the force W multiplied by its distance from the axle is just equal to F multiplied by its distance from the axle, the legs of the wheelbarrow can just be lifted from the ground.

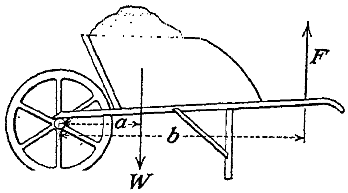


FIG. 45.—A wheelbarrow is a lever of the second class. The axle is the fulcrum.

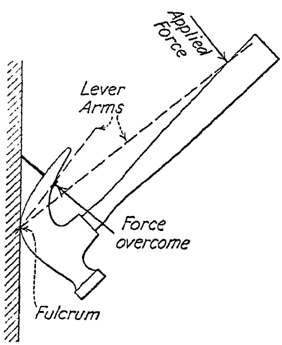


FIG. 46.—Principle of lever illustrated in a claw hammer.

A claw hammer (Fig. 46) used to draw nails illustrates this type of lever. The damper regulator of a hot-water boiler (Fig. 47) is also a lever of this kind. By moving the weight w outward, the force at g necessary to operate the damper is increased.

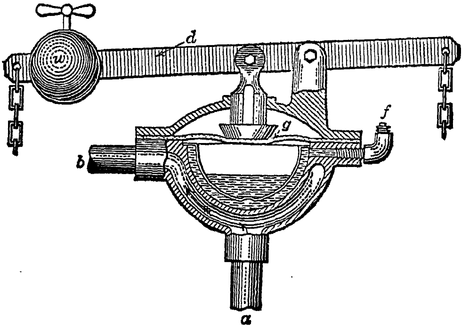


FIG. 47.—Damper regulator for hot-water boiler is a type of lever.

85. Levers of the Third Class.—A lever of the kind shown in Fig. 48 is known as a lever of the third class. The applied force is exerted between the fulcrum and the force which is overcome. A pair of fire tongs or a pair of shears for cutting grass belong

to this type of lever. The treadle of a sewing machine and the bones of the forearm together with the muscles operating them are levers of this class.

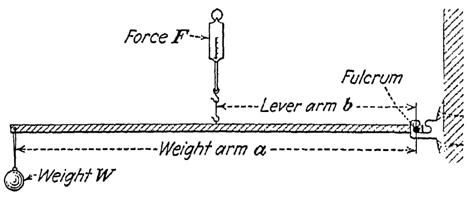


Fig. 48.—A lever of the third class. The force overcome is less than the force applied.

86. The Balance.—The balance (Fig. 49), which is an instrument for comparing masses, is a lever of the first class with equal arms. It consists of a light strong beam supported at its center

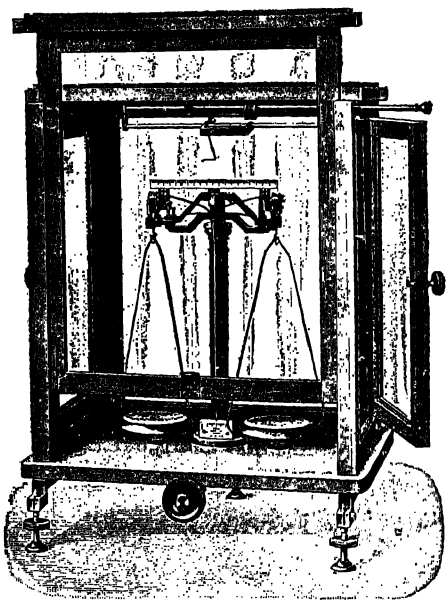


Fig. 49.—A balance. For accuracy the arms must be equal in length.

by a knife edge which rests on an agate plate. From either end of this beam is suspended a pan which holds the weights to be compared. These pans are also suspended from the end of the

beam by knife edges. When a mass is placed in one of the pans, there is a moment of force tending to rotate the beam about the central knife edge. When an equal mass is placed in the other pan, there is an equal moment tending to rotate the beam in the opposite direction. If the lengths of the arms of the balance are equal and the masses are equal to each other, the moment of force tending to rotate the beam in one direction is just equal to the moment of force tending to rotate it in the opposite direction. When the center of gravity of the balance lies below the knife edge, the beam of the balance will return to its original position of equilibrium. The pointer which projects downward from the center of the beam will oscillate back and forth about its equilibrium position.

87. Double Weighing.—In case the arms of the balance do not have the same length, the true weight of the body may be determined by weighing it first on one side and then on the other side of the balance. Let l_1 and l_2 denote the lengths of the arms, and W be the true weight of the body. Suppose the body when hanging from the arm l_1 is balanced by a weight W_2 in the other pan. Suppose also that when the body hangs from the arm l_2, it requires a weight W_1 in the pan hanging from the arm l_1 to balance it. Then for the first case, by the principle of moments,

$$W \times l_1 = W_2 \times l_2,$$

and for the second case

$$W_1 \times l_1 = W \times l_2.$$

By dividing the first of the equations by the second,

$$\frac{W}{W_1} = \frac{W_2}{W}.$$

$$W^2 = W_1 W_2.$$

$$W = \sqrt{W_1 W_2}.$$

Hence, in order to get the true weight of a body when the balance does not have equal arms, weigh the body first in one scale pan and then in the other. The square root of the product of the weights required to balance the body when placed first in one pan and then in the other is the true weight of the body.

Example.—An iron sphere when placed in the right-hand pan of a scale balance required 80.0 g. to balance it. When placed in the left-hand pan it required 80.5 g. to balance it. Find the true weight of the sphere.

$$\text{True weight} = W = \sqrt{W_1 W_2}$$
$$= \sqrt{80.0 \text{ g.} \times 80.5 \text{ g.}} = 80.249 \text{ g.}$$

88. Wheel and Axle.—A wheel and axle (Fig. 50) consists of a large and a small wheel fastened together on the same axle. The small wheel is usually itself the axle. It is in reality a lever of the first class with the center of the axle as fulcrum. To a rope wound around the axle is fastened the weight which is to be lifted. The force is applied to the rim of the wheel and will be called F. If r is the radius of the axle and R the radius of the wheel, then the moment of force tending to produce rotation counterclockwise is FR and that tending to produce rotation clockwise is Wr.

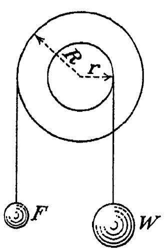


FIG. 50.—Wheel and axle. Equivalent to a simple lever with the fulcrum at the center of the wheel.

$$W \times r = F \times R.$$
$$\frac{W}{F} = \frac{R}{r} = \text{the mechanical advantage.}$$

From the above equation it is seen that the mechanical advantage of a wheel and axle, which is equal to the force exerted by the wheel and axle divided by the force applied to it, is equal to the radius of the wheel divided by the radius of the axle.

Example.—Let the radius of the wheel be 30 cm. and the radius of the axle be 7 cm. When a force of 10 kg. is applied to the wheel, find the force exerted on the rope wound around the axle.

Applying the law of moments,

$$W \times r = F \times R.$$
$$W \times 7 = 10 \times 30.$$
$$W = \frac{300}{7} = 42.8 \text{ kg.}$$
$$\text{The mechanical advantage} = \frac{W}{F} = \frac{R}{r}$$
$$= \frac{30}{7} = 4.28.$$

89. Pulleys.—A pulley consists of a wheel with a grooved rim, called a sheave, free to move about an axle which is mounted in a frame called a block. A flexible rope or cord passes over the groove in the rim of the wheel. To the ends of this rope are applied the weight and the force which overcomes this weight. In the case of a simple fixed pulley, as in Fig. 51, equal forces or

weights applied to the ends of the rope just balance each other. Neglecting friction, the tension in the rope is everywhere the same, and the mechanical advantage of the pulley is unity. There is, therefore, no advantage in such a pulley except that it is sometimes more convenient to pull down on the rope than it is to lift the weight directly.

When the pulley is movable as in Fig. 52, and the weight is attached to it, it is evident that the weight is supported by two parts of the cord. It is, therefore, necessary for each of the

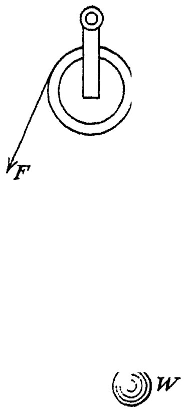


FIG. 51.—Single pulley. It only changes the direction of the applied force.

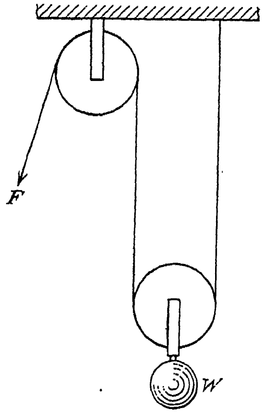


FIG. 52.—A fixed and movable pulley. The force overcome is double the applied force.

cords to exert a pull equal to only one-half of the weight. If the weight is lifted, it moves only one-half as far as the free end of the cord to which F is applied. Applying the principle of work to this simple machine, it is seen that if the weight W is lifted a ft. and the force F moves b ft., then

$$W \times a = F \times b.$$

$$\frac{W}{F} = \frac{b}{a} = 2 = \text{the mechanical advantage.}$$

90. Combination of Pulleys.—Fixed and movable pulleys are combined in a variety of ways according to the needs. It is quite common to use a fixed block with two or more sheaves and a movable block with two or more sheaves in it, as shown in

Figs. 53 and 54. The weight is fastened to the movable block, one end of the rope is attached to the movable or to the fixed block, and the force applied to the other end of the rope. If the weight is supported by five ropes (Fig. 53), the pull on each rope is one-fifth of the weight. Hence, the force required to move the weight is only one-fifth as great as the force required to lift it without the system of pulleys. If the weight is supported as in Fig. 54, the force overcome is four times the applied force. Figure 55 shows another possible way in which pulleys can be combined.

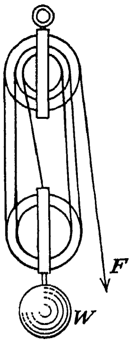


FIG. 53.—Block and tackle with rope fastened to movable pulley $(W/F) = 5$.

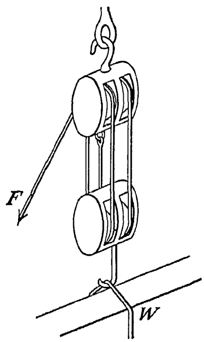


FIG. 54.- Another form of block and tackle with rope fastened to the fixed pulley (W/F) 4.

91. The Differential Pulley.—Where heavy weights are to be lifted, use is often made of a differential pulley (Fig. 56). This pulley consists of two sheaves of different diameters in the fixed block. These sheaves are rigidly fastened together. In the movable block there is one sheave. An endless chain passes over each of the sheaves. Teeth-like projections extend from the rim of the sheaves. Over these teeth the links of the chain fit so that the chain will not slip.

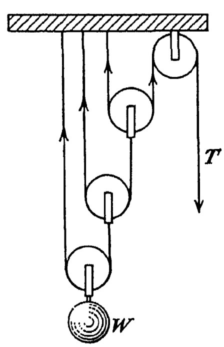


FIG. 55.—Combination of fixed and movable pulleys $(W/F) = 8$.

FIG. 56.—Differential pulley with endless chain.

Let the force P move down far enough to cause the fixed pulley to turn around once. If R is the radius of the larger sheave and r the radius of the smaller sheave, the work done by P will be $2\pi R \times P$. Since r is the radius of the smaller sheave in the fixed pulley, the length of chain unwound in one revolution is $2\pi r$. The weight W will be lifted $\frac{1}{2}(2\pi R - 2\pi r)$, and the work done on W is therefore $W\pi(R - r)$. Applying the principle of work to the pulley, since **input = output,**

$$W \times 2\pi\frac{(R - r)}{2} = P \times 2\pi R.$$

$$\frac{W}{P} \quad \frac{2R}{R - r} = \text{the mechanical advantage.}$$

The mechanical advantage will be determined by the radius of the larger sheave in the fixed pulley and by the difference between the radii of the sheaves in the fixed pulley. When the difference between the radii of these two sheaves is small, the mechanical advantage of the differential pulley is large.

Example.—A man wishes to lift a mass of 1 ton by means of a differential pulley. The radius of the larger pulley is 8 in. and that of the smaller 7 in. Find the force the man must exert.

$$\frac{\text{Weight lifted}}{\text{Force exerted}} = \frac{2R}{R - r}.$$

$$\frac{W}{F} - \frac{2R}{R - r} = \frac{2 \times 8}{8 - 7} = \frac{16}{}$$

$$W = 2{,}000 \text{ lb.}$$

Then

$$\frac{2{,}000 \text{ lb.}}{F} = \frac{16}{1}$$

and

$$F = \frac{2{,}000 \text{ lb.}}{16} \quad 125 \text{ lb.}$$

92. Inclined Plane.—Suppose a man must raise a block of ice from the ground into a wagon. Evidently the simplest way is to lift it vertically. But if the weight is too heavy for the man to lift, he can get a plank and put one end of the plank on the ground and the other on the rear of the wagon. Although the ice is too heavy to lift, the man can push it up the plank which acts as a simple machine known as an **inclined plane.**

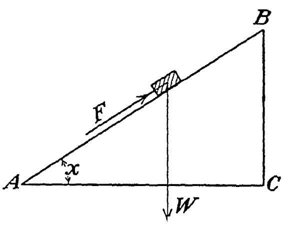


FIG. 57.—Mechanical advantage of an inclined plane—force parallel to the plane.

When a body rests on such an inclined plane (Fig. 57), the weight of the body acts vertically downward. The plane pushes on the body in a direction perpendicular to the surface of the plane. If there is no friction between the plane and the body, it is necessary to apply a third force to the body in order to keep it from slipping or to make it move up the plane. Every street or road which is

not level is such an inclined plane. Experience teaches that the steeper the plane, the greater the force required to haul the load up the incline.

Suppose a weight of W lb. is pushed from the bottom to the top of the plane a distance l ft. It has been lifted a height h, and the work done is

$$Wh \text{ ft.-lb.}$$

If F is the force parallel to the face of the plane, the work done by this force in moving the body the entire length of the plane is

$$Fl \text{ ft.-lb.}$$

Neglecting friction,

$$F \times l = W \times h.$$

$$\frac{W}{F} = \frac{l}{h} = \text{the mechanical advantage.}$$

Since the mechanical advantage is defined to be the ratio of the force overcome to the force applied, W/F is the mechanical advantage of the plane, and this is equal to the ratio of the length of the plane to its height. In order, therefore, to have the mechanical advantage as large as possible, the inclination of the plane should be as little as possible.

Example.—A mass rests on an inclined plane whose length is 8 ft. and whose height is 3 ft. Find the mechanical advantage.

$$\text{The mechanical advantage} = \frac{\text{force to lift mass}}{\text{force to push it up plane}} = \frac{\text{length}}{\text{height}} = \frac{l}{h}$$

93. Grade.—The ratio of the height of an inclined plane to the length of its base is of much importance in the construction of roads. This ratio is expressed by engineers as so many feet rise per 100 ft. along the horizontal distance. Suppose a road rises 3 ft. per 100 ft. measured horizontally, then the road has a 3 per cent grade. On such a grade the force required to move the load up the incline is only about 3 per cent of the force required to lift it directly. Consequently a team of horses can pull a much heavier load up such a grade than can be lifted directly. In the construction of roads it becomes important to have the slope as gentle as possible.

94. Wedge.—If, instead of pulling the load up the inclined plane, we push the inclined plane under the load, the inclined plane is known as a wedge (Fig. 58). Wedges are usually double

inclined planes with the force acting parallel to the base of the plane. The force is often applied by a blow from a hammer. To make the mechanical advantage of the wedge large, the angle of the wedge should be small. It then requires less force to drive the wedge. It is impossible to give a simple expression for the mechanical advantage in this case because friction cannot be neglected, and the resistance to be overcome is not constant all over the face of the plane. The principle of the inclined plane is used in many cutting tools such as the ax, the chisel, and the plane.

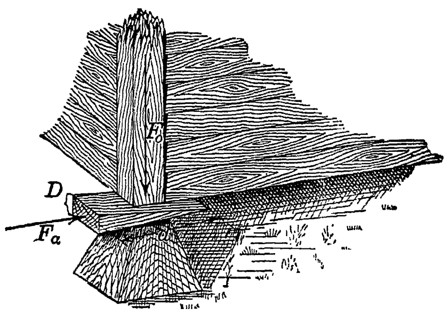


FIG. 58.—The wedge—two inclined planes.

95. The Screw.—The screw, which consists of a cylindrical rod of metal around which has been cut a groove or thread rising uniformly, is really another form of the inclined plane. A similar thread cut in the iron base or nut fits the thread of the screw. When the screw is turned around once, it moves upward a distance equal to the distance between the successive threads. This distance between the successive turns of the thread is called the **pitch of the screw.** If the head of the screw is supporting a weight of 1 ton, and the screw makes one complete turn, the weight is lifted the distance between two successive threads, *i.e.*, through a distance equal to the pitch of the screw. If the pitch of the screw is 0.01 ft., the work done by the screw when turned once around is $2{,}000 \times 0.01 = 20$ ft.-lb. The moving force is exerted through a distance equal to the circumference of the circle traced out by one end of the bar or handle used to turn the screw.

96. The Jackscrew.—When large forces must be exerted, a type of screw known as the jackscrew is often used (Fig. 59).

Let p be the pitch of the screw, W the weight to be lifted, F the force applied to the lever arm, and r the radius of the lever arm. In one complete turn of the screw, the output is the weight lifted times the distance through which it is lifted.

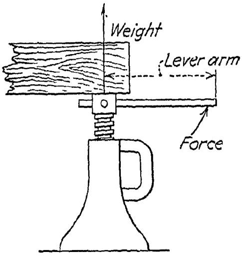


FIG. 59.—The jackscrew combines the principle of the lever and that of the inclined plane.

$$W \times p.$$

The input is the force applied times the distance through which it acts.

$$F \times 2\pi r.$$

By the principle of work,

$$W \times p = 2\pi r F.$$

$$\frac{W}{F} = \frac{2\pi r}{p} = \text{the mechanical advantage.}$$

Hence, the mechanical advantage is equal to the circumference traced out by the end of the lever in one complete revolution divided by the pitch of the screw. The mechanical advantage may be made large by making the pitch of the screw small or by making the lever arm long.

Example.—Suppose the length of the lever arm used with a jackscrew is 5 ft., the pitch of the screw is 1 in., and the weight to be lifted is 10 tons.

$$\text{Output} = \text{input}.$$

$$20{,}000 \times \frac{1}{12}\ (\text{ft.-lb.}) =$$

$$F \times 5 \times 2\pi\ (\text{ft.-lb.},$$

$$F = \frac{20{,}000}{120\pi} = 53\ \text{lb}.$$

The friction in the screw will make it necessary to apply a force somewhat greater than the theoretical value.

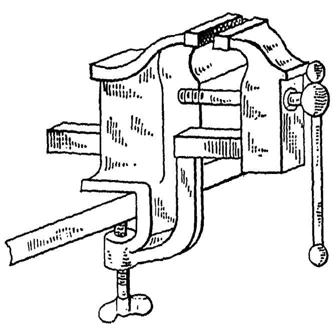


FIG. 60.—A machinist's vise combines the principle of the lever and the screw.

97. Combinations of Simple Machines.—Machinery is a more or less complicated combination of levers, pulleys, wheels and axles, and screws. The machinist's vise (Fig. 60) combines the principle of the screw and the principle of the lever. A builders' crane is made up of a movable pulley to which the weight to be lifted is attached. The force is applied through wheels and axles. Each of these wheels and axles has a mechanical advantage equal to the radius of the wheel divided by the radius of the axle. The

total mechanical advantage of the machine is found by multiplying together the mechanical advantage of the separate parts.

Problems

1. A nutcracker is 6 in. long, and the distance from the center of the nut to the fulcrum is 1 in. If the force required to crack the nut is 15 lb., how great a force must be applied at the end of the handle?

2. A beam balance has the knife edges supporting the pans placed 10 cm. away from the pivot supporting the beam. A rider weighing 10 mg. is placed 3.4 cm. to the right of the pivot; what mass placed in the left-hand pan will restore equilibrium?

3. A horse-driven capstan has a drum with a diameter of 15 in., and the lever to which the horse is hitched is 16 ft. long. What force is applied to the rope when the horse pulls 175 lb., assuming that friction can be neglected?

4. A grindstone has a diameter of 26 in., and it is driven by crank with a length of 8 in. If a force of 24 lb. is applied perpendicular to the crank, how much frictional force tangential to the rim of the stone can be overcome?

5. A windlass with a drum 6 in. in diameter has a crank arm 28 in. long. It operates with an efficiency of 80 per cent. What force must be applied to the crank in order to raise a load of 160 lb.?

6. A system of pulleys consists of a movable block with two pulleys and a fixed block with three pulleys. If one end of the rope is fastened to the movable block, what mechanical advantage is obtained? How far must the free end of the rope be pulled in order to displace the movable block through 1 ft.?

7. The axle of a capstan is 12 in. in diameter. There are six bars or levers projecting from the capstan, and a man exerts a force of 25 lb. at the end of each bar. How long must the bars be, in order that six men may raise a weight of 1.25 tons?

8. A jackscrew, which is to be used for lifting a weight of 3 tons, has a pitch of $\frac{1}{4}$ in., a lever arm 40 in. long, and an efficiency of 30 per cent. How much force must be applied perpendicular to the lever?

9. A jackscrew with a pitch of 0.25 in. has a handle 30 in. long. A force of 20 lb. must be applied when a load of 6,600 lb. is being lifted. Calculate the theoretical mechanical advantage, the actual mechanical advantage, and the efficiency.

10. Calculate the force available between the jaws of a vise with a screw of $\frac{1}{8}$-in. pitch, when a force of 35 lb. is applied to the lever at a distance of 1.25 ft. from the center of the screw, if the efficiency of the screw is 40 per cent.

11. In a differential pulley, the inner drum has a diameter of 8 in. The mechanical advantage is 15. What is the diameter of the outer drum?

12. Find the mechanical advantage of a differential pulley in which the radius of the larger pulley is 7 in. and that of the smaller pulley is 6 in.

CHAPTER VIII

MOLECULAR FORCES AND MOTIONS

98. Molecular Theory of Matter.—Any form of matter is made up of very small particles called molecules. These molecules are packed together in the substance and held together more or less securely depending on the nature of the substance. In between these molecules are spaces which are unoccupied. Even in solids, these spaces are so large that the molecules do not permanently touch each other. In liquids, these intervening spaces are still larger, and in gases, under ordinary conditions, they are very large in comparison with the size of the molecules. It is customary to assume that the molecules are spherical in shape, but this is not exactly true. They are in rapid motion and in the case of gases strike against each other and rebound much as steel balls when shaken vigorously in a closed vessel.

99. States of Matter.—There are three states in which matter may occur. It may be a **solid,** like ice, iron, or wax; a **liquid,** like water, mercury, or oil; or a **gas,** like air or hydrogen. Under suitable conditions, it is possible for a body to change from the solid to the liquid state, from the liquid to the gaseous state, and from the solid to the gaseous state. Thus, water is found in the form of ice when its temperature is sufficiently low; as water at higher temperatures; or as steam at still higher temperatures. A solid like camphor will pass directly from the solid to the gaseous state without first passing through the liquid state. This is evident from the odor which is directly received from it in its solid state. It has been found possible to change ordinary atmospheric air into a liquid by lowering its temperature and sufficiently increasing the pressure on it. Liquid air may also be solidified. Thus, substances which occur normally in nature in one state may under suitable conditions be a gas, a liquid, or a solid.

A solid is that state of matter in which the molecules strongly cling together and tend to keep the same relative positions.

Since the molecules of a solid are rather rigidly held together, solids resist any tendency to change their shape or size and therefore preserve their original form unless acted upon by some outside force.

In a liquid, the molecules tend to cling together but are free to move with respect to each other. **Liquids do not resist forces tending to change their shape** but resist forces tending to change their volume. Because the molecules of a liquid can be easily displaced with respect to each other, the layers of a liquid flow freely over each other, and a liquid assumes the form of the vessel in which it is placed.

In a gas the molecules are free to move about without restraint and thus tend to separate indefinitely. Gases do not resist forces tending to change their shape and only to a small degree do they resist forces tending to change their volume. Since the molecules of a gas can move freely and tend to separate indefinitely, a gas fills up entirely the space of the containing vessel and exerts a pressure on it.

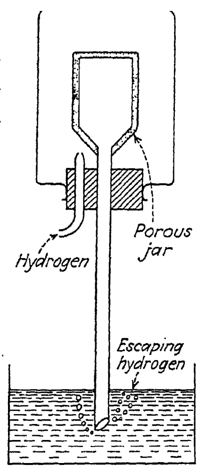


FIG. 61.—Hydrogen diffuses through the porous walls of the jar and increases the pressure in the interior.

100. Diffusion of Gases.—One of the evidences for believing that a gas is made up of a large number of molecules is found in the manner in which an odor penetrates to all parts of a room. Suppose that a bottle of ammonia water or some other substance with a pronounced odor is opened in a room. After some time the odor has penetrated to all parts of the room. Here the gas giving the odor, whether it comes from a solid or a liquid, consists of molecules which hit their neighbors and in turn are struck by other molecules. By this buffeting back and forth, the individual molecules are widely scattered and soon are present in all parts of the room. The pleasant odor of flowers or trees is a case of this scattering of the molecules of one gas among the molecules of the air. This process of scattering the molecules of one gas among the molecules of another gas is called **diffusion of gases.** If a porous jar (Fig. 61) is surrounded by another vessel into which hydrogen is introduced, the hydrogen diffuses through the porous jar and increases the gas pressure

inside the jar causing the gas to bubble up from the water into which the end of the tube leading from the porous jar is dipping.

It is diffusion of this kind which keeps the air, which is composed principally of oxygen and nitrogen, in such a state of uniform mixture.

101. Diffusion of Liquids.—In liquids as well as in gases, the molecules are free to move about with greater or less freedom according to the nature of the liquid. If a little vinegar is placed in a pail of water, all of the water soon becomes sour. When a lump of sugar is placed in a cup of coffee, the contents of the whole cup are sweetened by the distribution of the molecules of the sugar throughout the coffee. When a piece of fuchsin is dropped quickly to the bottom of a test tube containing water which is placed where it cannot be jarred, a diffusion of fuchsin from the bottom to the top of the tube is observed. This diffusion is made evident by the changes of color which take place in the tube.

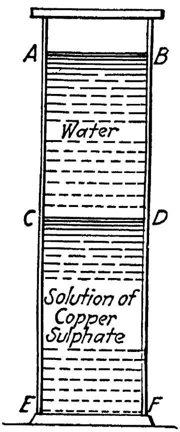


FIG. 62.—Diffusion of copper sulphate.

The diffusion of liquids may also be demonstrated by taking a tall glass jar (Fig. 62) filled with water and by means of a thistle tube extending to the bottom of the jar pouring into the jar a solution of copper sulphate until the bottom of the jar is filled to a height of 1 or 2 cm. If the jar is allowed to stand for some days without being disturbed, an upward diffusion of the copper sulphate can be observed. The force of gravity tends to keep the copper sulphate on the bottom, but diffusion overcomes this force, and the copper sulphate rises in the jar. The action which takes place in this case is very similar to that which takes place in the diffusion of gases except that it takes place more slowly in liquids than in gases. The motion of the molecules in diffusion is such that they seldom travel far without encountering neighboring molecules. Between collisions, their paths are a series of short straight lines. In liquids, the distance between collisions is less than in gases. Hence, the molecules diffuse more slowly in liquids than in gases.

102. Solutions.—If a crystal of some solid like salt or sugar is placed in a vessel of water, the substance distributes itself to every portion of the water. The solid is said to go into solution in the liquid. This process of solution arises from the fact that the molecules of liquid attract the molecules of the solid and pull them away from their neighbors. This will not go on indefinitely,

for after a while no more of the molecules of the solid will distribute themselves among the molecules of the liquid. The solution is then **saturated.** The amount of the solid which will go into solution depends on the nature of the solid, on the nature of the liquid, and on the temperature of the solution. Usually the amount of the substance which will go into solution increases as the temperature of the solution is raised.

Liquids like water will dissolve more or less freely all gases. When such a solution occurs, the gases are uniformly distributed throughout the liquid. The dissolved gas may be thought of as distributed between the molecules of the liquid. Thus, ammonia gas or hydrochloric acid gas is readily dissolved in water, and soda water is a solution of carbon dioxide in water.

The process of solution is very important in nature. Many of the fluids which have a part in the vital processes of the body are solutions of one sort or another. A considerable part of cooking, preserving, and canning is concerned with solutions and the changes in the concentrations of solutions. In the manufacture of maple sirup, a dilute solution of sugar is changed into a more concentrated solution by evaporating some of the water in which the sugar is dissolved. When salt is used to preserve food, it forms a solution in the water or juices of the food and then enters the food as a solution.

103. Osmosis.—When two miscible liquids or solutions are separated by a porous membrane, there is a tendency for one of the liquids to diffuse more rapidly through the membrane than the other liquid does. If water is separated from a solution of sugar by a suitable membrane, the water molecules on one side of the membrane diffuse through the membrane more rapidly than the water molecules on the other side do. This will cause the volume of the solution to increase and the volume of the pure water to decrease. If the membrane is fixed, a change in the level of the solution and the water takes place. This process of diffusion through a semipermeable membrane is called **osmosis.**

The following experiment illustrates osmosis. Take a carrot (Fig. 63) and cut out its interior, filling the space with a thick sirup. Insert in the mouth of the cavity a rubber stopper through which projects a long glass tube and immerse the carrot in a wide-mouthed jar filled with water. The water from the jar begins to flow through the pores of the carrot into the space filled with sirup. There is also a tendency for the sirup to flow out through the carrot into the jar. The tendency of the water to flow inward

is greater than the tendency of the sirup to flow outward. Hence, the volume of the sirup in the carrot increases and the solution rises in the tube.

104. Illustrations of Osmosis.—When dried fruits, such as prunes and raisins are cooked, they swell and may even burst if the pressure becomes sufficiently large. This swelling is due to the fact that the vegetable sac surrounding the fruit acts as a membrane through which the water diffuses from the outside to the inside. If marine animals like oysters are transferred from salt water to fresh water, more water flows into the animal through the membrane which serves as its covering. A dilation of the animal results since the conditions for osmosis are present. Another illustration of this kind is found in the boiling of sausages. The animal membrane which acts as a covering for the sausage is freely permeable to the water but does not so readily allow the passage of dissolved salts. Hence, the water flows in through the membrane, causing it to distend and usually burst during cooking. If the membrane is punctured before this rupture has occurred, a quantity of liquid squirts out.

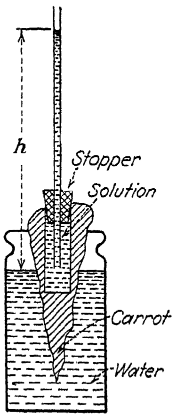


FIG. 63.—Osmosis in a carrot.

If fresh fruits are placed in strong sugar solution, they begin to shrivel and decrease in size. This is also due to osmosis. In this case, the stronger solution is on the outside and the weaker on the inside. Hence, the flow of water is from the inside to the outside causing the fruit to shrink. Such a shrinkage is observed in the canning of peaches. After the peaches have been placed in the cans and covered with a strong sugar solution, they shrink on being heated to the required temperature for canning.

105. Crystalloids and Colloids.—Those substances which, like cane sugar, glucose, etc., pass through animal membranes most readily have been called **crystalloids.** Substances like gums, starches, and albumens which do not diffuse through membranes are called **colloids.** The crystalloids in solution are in the molecular state; that is, the substance has been divided up into molecules. The colloids are less finely divided. Each colloidal particle contains a large number of molecules. The size of the colloidal particles varies greatly from one colloid to another. These particles are really in suspension, whereas the crystalloids are in solution. Milk is a good illustration of a colloid. Ruby glass owes its color to the presence of gold particles in the colloidal state. When crystalloids are dissolved, they produce marked changes in the properties of the solvent. If they are dissolved in water, they diminish its vapor pressure, lower its

freezing point, and raise its boiling point. When colloids are added to water, they produce scarcely any effect.

106. Osmotic Pressure.—It has been seen that if two solutions differing in concentration are separated by a semipermeable membrane, the solvent will migrate from the solution where the concentration is small to the solution where the concentration is larger. The force which is responsible for the flow of the solvent in osmosis is called **osmotic pressure.**

In order to get a measure of this pressure, the following experiment is satisfactory. Take a thistle tube (Fig. 64) over the mouth of which is fastened a piece of bladder or parchment. Fill the thistle tube up to the beginning of the glass stem with a strong solution of sugar. Now immerse the thistle tube in a beaker filled with pure water so that the level of the water in the vessel and the level of the solution in the tube are the same. A diffusion of the water takes place through the membrane into the solution, and the solution rises in the tube. This rise continues until the pressure due to the weight of the solution in the tube is sufficient to prevent any more water from flowing into the tube. The pressure necessary to prevent the flow of the water into the solution is equal to the **osmotic pressure.**

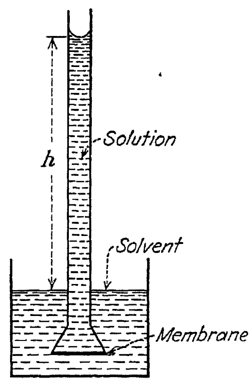


Fig. 64.—Osmotic pressure. The solvent diffuses through the membrane into the solution.

107. Laws of Osmotic Pressure.—Two important laws have been established concerning osmotic pressure. They are as follows:

1. In fairly dilute solutions in which the molecules are not dissociated, the osmotic pressure is proportional to the concentration of the dissolved substance.

2. In a solution of given concentration, the osmotic pressure increases as the temperature is increased, and the rate of increase is the same for all solutions.

108. Cohesion and Adhesion.—In liquids, the molecules move freely with respect to each other but are held together by the attractive forces of the molecules on each other. This tendency of the molecules to cling together is most noticeable in the case of the more viscous liquids like heavy sirups and least noticeable in the more mobile liquids like alcohol and water. Not only do the molecules of a liquid cling to each other, but they also

cling to the molecules of a solid as seen when a piece of glass is dipped into a vessel of water. The molecules of water adhere to the glass and form a thin film over its surface. The attraction of like molecules for one another is called **cohesion.** The attraction of unlike molecules for one another is called **adhesion.** The nature of the forces acting between the molecules is the same whether the molecules are alike or unlike.

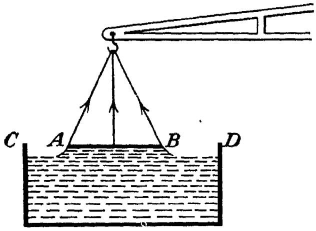


FIG. 65.—The cohesive forces hold AB to the surface of the liquid.

It is cohesion which causes a drop of water to form. It is also cohesion which holds together so firmly the molecules of iron, copper, and other solid substances. When glue or cement is used to fasten two pieces of wood or crockery together, the adhesion of the glue or cement for the wood or the crockery causes a firm joint to be formed. Sometimes the adhesion is so great that the joint thus formed is stronger than the material out of which it is formed. This force of attraction of molecules for one another is very large but acts only at small distances.

To get a measure of this force, suspend from one arm of a balance a circular glass plate (Fig. 65). Place weights enough on the opposite pan of the balance just to balance this circular plate. Having adjusted the plate so that it is horizontal, elevate a vessel of water so that the surface of the water just touches the under surface of the glass plate. Now find what weight must be added to the weights in the pan in order to raise the glass plate away from the surface of the water. This weight is a measure of the attractive forces of the molecules for each other. It is found that when the plate comes away from the water, its lower surface is wet. Thus, the molecules of water have pulled away from each other and not the molecules of water away from the molecules of glass. Hence, the force of cohesion between the molecules of water is less than the force of adhesion between the molecules of glass and water. A torsion balance (Fig. 66) may be used instead of an ordinary balance and a ring instead of a disk.

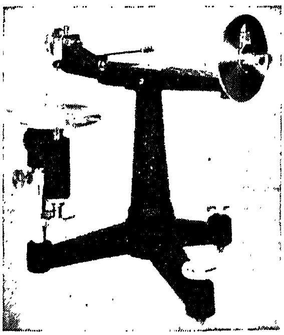


FIG. 66.—Apparatus for measuring surface tension. (*Courtesy Central Scientific Company.*)

109. Cohesion in Soils.—A good example of the action of cohesion is found in the behavior of soils. When the particles of soil are small, the

cohesion between the particles is small and the soils are easily blown about by the winds. The ease or difficulty of plowing or cultivating soil depends largely on the cohesive forces between the particles of the soil. When the cohesion is great, the soil is difficult to cultivate and easily forms clods. When the cohesion is small, the soil pulverizes easily and is easily cultivated. The cohesive forces in the soil may be measured either in the wet or in the dry condition, and its value will depend on the conditions under which it is measured.

110. Surface Energy and Surface Tension.—If a needle is greased and gently placed on the surface of water, it will float although the density of the needle is greater than that of the water. In like manner, some insects can walk on the surface of liquids like water. The surface of a liquid behaves as if it were

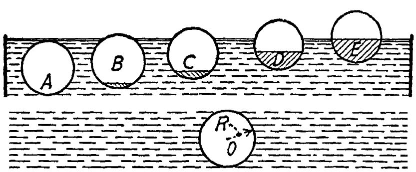


FIG. 67.—Forces on a molecule inside of a liquid. At the surface there is an unbalanced downward force on the molecule.

covered with a thin elastic film. It requires a force to break this film, and the amount of this force depends on the nature of the liquid. This surface film is due to the molecular attractions in the liquid.

A molecule in the mass of the liquid is in equilibrium, as it is attracted equally in all directions by the molecules on all sides. But molecules in the surface of the liquid are not attracted by molecules on the side away from the liquid. On account of this unbalanced force, the molecules at the surface are pulled toward the interior of the liquid with a force due to the attraction of the molecules lying below the molecules at the surface. The difference between the behavior of the molecules inside the liquid and those at the surface is seen by considering a sphere drawn around a molecule on the surface and around one in the interior of the liquid (Fig. 67). Let all the molecules which act on the molecule O at the center be included in this sphere. Now the forces on this molecule O are balanced, and there is no tendency for the molecule to move. But the forces on the molecule at the center of the sphere E are not balanced since there are no

molecules in the upper half of the sphere. This causes the molecule at the center to be pulled downward by the unbalanced attractions. This unbalanced force makes the liquid take the shape which has the least possible area under the conditions. Hence, a drop of mercury or water becomes spherical. If a drop of oil is placed in a mixture of water and alcohol of the same density, the drop of oil does not rise or fall, and under the action of the molecular forces it assumes a spherical form.

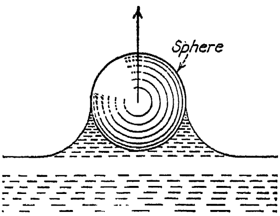


FIG. 68.—Withdrawal of a sphere from the surface of a liquid requires force to overcome cohesion.

If the molecules of a liquid have less attraction for each other than for the molecules of the solid with which they are in contact, the liquid adheres to the solid and wets it. The attraction of the molecules of a liquid for a solid is easily seen when a sphere (Fig. 68) is being withdrawn from a liquid which wets it. If, however, the molecules of the liquid cling to each other with a greater force than that with which they adhere to the solid, the solid is not wet by the liquid. Such is the case when mercury is in contact with glass, but when a drop of distilled water falls on a clean surface of glass, it spreads over the glass in a thin layer.

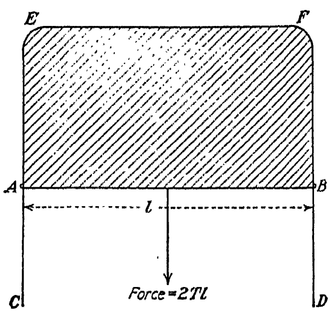


FIG. 69.—Force on a movable wire due to surface tension.

The relation between **surface energy** and **surface tension** can be obtained from Fig. 69, in which $AEFB$ is a wire frame covered by a soap film. The side AB is movable, and the force F which must be applied to it to keep the film from contracting is

$$F = 2 \cdot T \cdot l,$$

where l is the length of the wire AB, and T is the force per unit length. If the wire AB is drawn down a distance of x cm., the work done is,

$$W = F \cdot x = T \cdot 2 \cdot l \cdot x = T \cdot A,$$

where

$$A = 2 \cdot l \cdot x = \text{increase in area of both sides of the film,}$$

whence

$$T = \frac{W}{A} = \text{work per unit area.}$$

The work per unit area which is the surface energy is numerically equal to the force per unit length which is called the surface tension.

111. Capillarity.—If a piece of glass tube of very small bore has been thoroughly cleaned and is then dipped into water (Fig. 70), the water will wet the inside of the tube and rise in it. If the liquid does not wet the tube (Fig. 71), as in the case of mercury in a glass tube, the liquid is depressed in the tube. In general, liquids rise in tubes which they wet and are depressed in tubes which they do not wet. The smaller the bore of the tube, the greater the height to which the liquid rises or the greater the amount which it is depressed. This rise or depression of liquids in tubes of small bore is known as **capillarity.** It is caused by the molecular forces which are responsible for surface energy. The molecules of the liquid have an attraction for each other and also for the molecules on the surface of the wall of the tube, the former being a case of cohesion and the latter a case of adhesion. If the cohesive forces between the molecules of the liquid are greater than the adhesive forces between the molecules of the liquid and the molecules on the wall of the tube, the liquid is pulled away from the tube and depressed. Mercury in a glass tube is a case of this kind. If, however, the adhesive forces are the greater, the liquid wets the side of the capillary tube and rises in the tube. Water or alcohol in a glass tube rises in this manner.

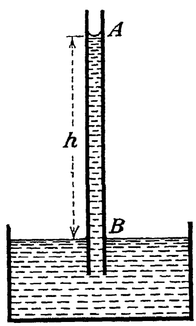


FIG. 70.—Rise of water in a capillary tube.

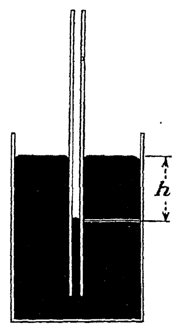


FIG. 71.—Depression of mercury in capillary tube.

112. Cause of the Variation in the Height of Rise.—The reason for the fact that liquids like water will rise higher in small tubes than in large ones is found in the relation between the volume of the tube and its internal circumference. When a glass tube is

thrust into water, the molecules in the surface of the wall just above the water pull up on the row of molecules lying nearest them and raise them above the level of the water in the vessel. This carries up the whole column of water until the weight of the column is equal to the force of attraction between the molecules of glass and the molecules of water. As each molecule of glass exerts a fixed attraction on each molecule of water near it, a tube of larger circumference will be able to lift more than a tube of small circumference. Now the circumference of the tube increases as the diameter so that a tube which is 0.1 mm. in diameter will lift one-tenth as much as a tube which is 1 mm. in diameter. But the volume and hence the weight of the water lifted increase as the square of the diameter. The larger tube will therefore lift its water only one-tenth as high as the smaller tube.

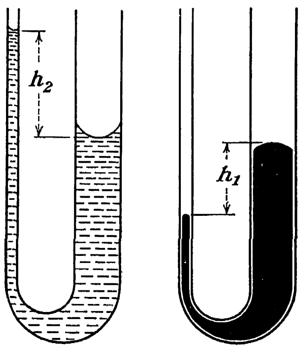


FIG. 72.—Unequal heights of liquids in tubes of unequal areas.

Let a tube of radius r be inserted in a liquid of density d (Fig. 70). Let the mean elevation of the liquid be h. Assume that the angle which the surface of the liquid makes with the wall of the tube is very small so that all the force of attraction is directed upward. The force due to the surface tension must be balanced by the weight of the column of liquid of height h.

The length of the film around the tube is $2\pi r$; the force upward is

$$2\pi r T. \quad (T \text{ being the force per centimeter.})$$

The mass of the liquid supported by this force $= \pi r^2 hd.$
The force in dynes to support this mass $= \pi r^2 hdg.$
Hence, for equilibrium,

$$\pi r^2 dhg = 2\pi r T.$$

$$h = \frac{2\pi r T}{\pi r^2 dg} = \frac{2T}{rdg}.$$

or

$$T = \frac{h \cdot d \cdot g \cdot r}{2} \text{ dynes.}$$

From this equation it is seen that the smaller the bore of the tube, the greater the height to which the liquid rises. Figure 72 shows the difference of level of liquids in tubes of unequal diameters.

Example.—The liquid in a capillary tube rises to the height of 7 cm. The radius of the tube is 0.1 mm. and the density of the liquid 0.8. If the angle between the liquid and the surface of the tube is zero, find the surface tension of the liquid.

$$\text{Surface tension} = \frac{\text{height} \times \text{density} \times \text{radius of tube} \times 980}{2}$$

$$T = \frac{hdrg}{2} = \frac{7 \times 0.8 \times 0.01 \times 980}{2} = 27.4 \text{ dynes per centimeter.}$$

Example.—In two capillary tubes of different diameters the liquid rises in one case to a height of 10 cm. and in the other case to a height of 20 cm. Find the ratio of the diameters of the tubes.

$$T = \frac{hdrg}{2}.$$

$$T = \frac{h'dr'g}{2}.$$

$$hr = h'r'.$$

$$\frac{h}{h'} = \frac{r'}{r}.$$

$$\frac{r'}{r} = \frac{10}{20} = \frac{1}{2}.$$

Hence, the diameter of the tube in which the liquid rises the higher is one-half the diameter of the other tube.

113. Illustrations of Capillarity.—There are many familiar illustrations of capillary action in nature. The oil in a lamp rises in the wick by capillary action. Ink spreads in a blotter, or water in a lump of sugar, by this same action. If one end of a towel dips into a bucket of water and the other end hangs over the bucket, the towel soon becomes wet throughout and drains the water out of the bucket. If the towel is covered with a preparation which prevents the water from adhering to its fibers, the towel does not become wet and the action does not take place. In this case, a surface film spreads over the cloth but does not enter it. By this means the cloth has been rendered waterproof. Cravenette cloth has been treated in this way.

The method of making cloth waterproof may be understood from the following experiment. Take a sieve made out of fine copper gauze which has been dipped into melted paraffin. By this means, each wire has been covered with paraffin and water will not adhere to it. Lay a piece of paper carefully on the under side of the sieve and fill the sieve with water. When the paper is carefully removed, the surface film of water will prevent the water from flowing through the sieve.

Small bodies floating near each other on a liquid are often seen to come together. If the bodies are wet by the liquid, the liquid between them rises higher than outside, and the hydrostatic pressure at C (Fig. 73a) is less than

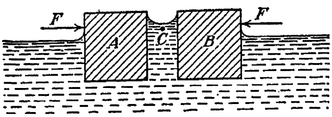


FIG. 73a.—Reduction of pressure between bodies due to surface tension.

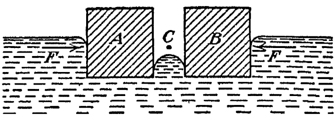


FIG. 73b.—The excess hydrostatic pressure causes the bodies to move together.

the atmospheric pressure acting on the outsides of the bodies, and the bodies are forced together. If neither body is wet by the liquid, the liquid surface is depressed between them. In Fig. 73b, the atmospheric pressure at C is less

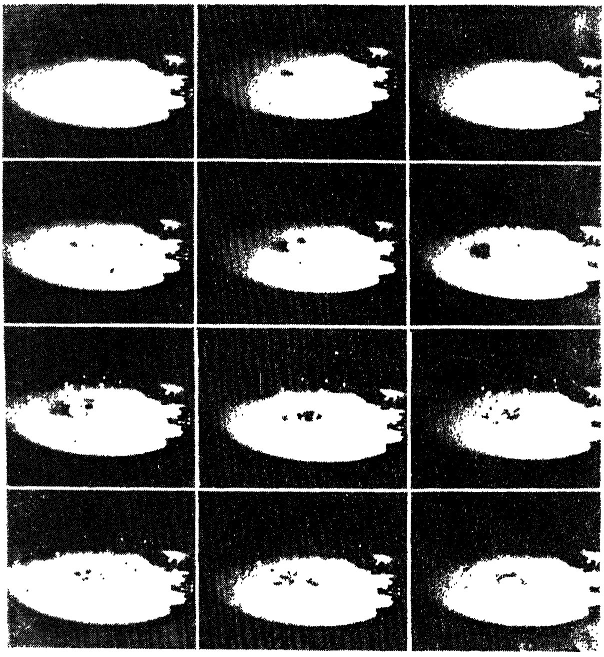


FIG. 74.—Splashes on the surface of milk showing effects of surface tension. High-speed motion-picture photography was used. (*Courtesy Edgerton, Germhausen and Grier, Massachusetts Institute of Technology.*)

than the hydrostatic pressure acting at the same level on the outsides of the bodies, and again they are pushed together.

The splashes formed (Fig. 74) when drops of milk fall on a surface of milk give a beautiful illustration of effects produced by surface tension.

The formation of bubbles is due to surface tension. The pressure inside of a soap bubble (Fig. 75) is greater than the external pressure. The forces across the equatorial plane $ABCD$ are in equilibrium. Hence

$$\pi R^2 p = 4\pi TR,$$
$$p = \frac{4T}{R},$$

where p is the excess pressure.

114. Capillary Action in Soils.—The distribution of water in soils depends largely on capillary action. The spaces left between the soil grains form imperfect triangular capillary tubes. The cross section of such a tube formed by four spherical grains is seen in Fig. 76. These tubes extend in all directions through the soil, and their effective diameters are roughly proportional to the diameter of the grains so that the effective diameter of these tubes is small in finely divided soils and large in coarse soils. Through these spaces the water rises as in capillary tubes, thus supplying water from the lower soil which is damp to the upper soil which is drier. If the soil is made too loose, the effective diameter of these tubes becomes too large and the water is prevented from rising any considerable height from the lower soil toward the upper soil. Constant cultivation keeps the soil loose and, therefore, lessens evaporation by preventing the rise of water from the lower layers to the surface.

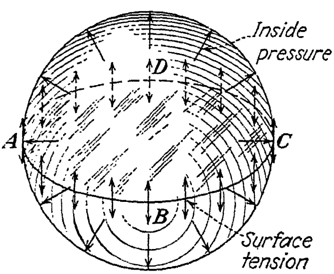


FIG. 75.—Excess pressure inside of the bubble $= p = 4T/R$.

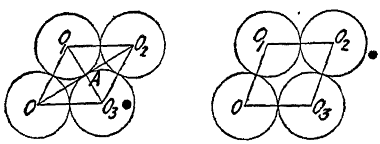


FIG. 76.—Capillary tubes between grains of soils.

Problems

1. How high will water (surface tension 73 dynes per centimeter) rise in a glass tube with a diameter of 0.14 mm. because of capillarity?

2. Find the surface tension of water, if the column supported by capillarity in a glass tube of 0.44 mm. diameter is 6.4 cm. high.

3. The surface tension of water is 0.075 g. per centimeter. Find the height to which water will rise in a capillary tube having a bore of 0.01 cm.

4. The pressure inside a soap bubble exceeds atmospheric pressure by 0.060 g. per square centimeter. If the radius of the bubble is 3 cm., find the surface tension of the soap solution.

5. How much will the surface of mercury stand below the normal level of the mercury in a tube which is 1 mm. in diameter? (Take the surface tension of mercury to be 470 dynes per centimeter.)

6. A tube (Fig. 92) is bent in the form of a U. The diameter of the large tube is 1.5 cm. and that of the smaller tube is 1.5 mm. Find the difference in levels when the tubes are filled with water which has a surface tension of 74 dynes per centimeter.

7. What is the diameter of a soap bubble when the excess pressure inside of the bubble is 60 dynes and the surface tension of the soap solution is 25 dynes per centimeter?

8. The surface of the water in a glass capillary tube which is 0.22 mm. in radius is at a height of 6.4 cm. above the level of the water in the beaker in which the tube stands. What is the surface tension of the water?

9. Find the work which must be done to increase the surface of a soap film by 250 sq. cm. Take the surface tension of soap solution as 25 dynes per cm.

10. One limb of a U-tube is 2 cm. in diameter and the other is 2 mm. in diameter. What will be the difference in level on the surface in the two tubes when mercury is poured into them? Take the surface tension of mercury as 470 dynes per centimeter and the angle of contact as 140 deg.

11. What is the excess pressure inside of a soap bubble which is 1 cm. in diameter, assuming 26 dynes per centimeter as the surface tension of the soap solution?

CHAPTER IX

LIQUIDS AT REST

115. Characteristics of Liquids.—The molecules of a liquid at rest are displaced by the slightest force, and for this reason a liquid has no shape of its own but takes the shape of the containing vessel. Hence, liquids yield to a continued application of force tending to deform them or to change their shape in any way. They, however, manifest wide differences in their readiness to yield to distorting forces. Water, alcohol, and ether are very mobile liquids which yield readily to forces tending to change their shape. Glycerine is less mobile, and tar is still less so. There is no sharp line of separation between liquids and solids. In warm weather, paraffin candles yield under their own weight and bend double. Although shoemaker's wax will break readily when cold, it behaves like a very viscous liquid at higher temperatures. All liquids offer large resistance to forces tending to change their volume. For example, it requires a pressure of 1,500 lb. per square inch to cause the volume of water to change 0.5 per cent.

116. Pressure in a Liquid under Action of Gravity.—If an empty can is pushed down into the water, the force of the liquid against the can tends to push it upward. When a block of wood is forced under the water, a force resists the action of the force tending to submerge the body, and the body when released returns to the surface. The fact that large ships float on the surface of the water shows that there is a force on their lower surface sufficient to overcome their weight.

Consider a cylindrical vessel partly filled with liquid of density d. Let R be the radius of the vessel and h be the depth of the liquid in the vessel. The volume of the liquid is

$$v = \pi R^2 \times h,$$

and the weight of the liquid is

$$W = vdg = \pi R^2 hdg.$$

This weight is sustained by the bottom of the vessel and will exert on each unit area of the bottom a force p, such that

$$p = \frac{\text{weight}}{\text{area of base}} = \frac{\pi R^2 h d g}{\pi R^2} = hdg \text{ dynes per square centimeter.}$$

This force per unit area is called the **pressure.** It is proportional to the depth of the liquid and to its density.

Example.—Find the force per unit area on the bottom of a tank which is 10 ft. in diameter and 25 ft. deep, if the tank is filled with water which weighs 62.5 lb. per cubic foot.

$$\text{Total force} = \text{height} \times \text{area} \times \text{weight per cubic foot}$$
$$= 25 \times \pi \times 5^2 \times 62.5 = 122{,}750 \text{ lb.}$$
$$\text{Force per unit area} = \frac{\text{total force}}{\text{area}} = \frac{122{,}750}{\pi \times 5^2} = 1{,}562 \text{ lb. per square foot.}$$

117. Pressure in Vessels of Different Shape.—Where a vessel has vertical sides, the pressure on the bottom is equal to the

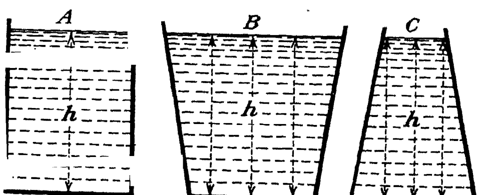


FIG. 77.—Pressure independent of shape of vessel.

height of the liquid times its density. If, instead of considering vessels with vertical sides, vessels are considered in which the sides flare out (Fig. 77), it might be expected that the pressure on each square centimeter of the bottom in case B would be greater than in case A because there is more water in B. The pressure on each square centimeter of the bottom is the same, since each square centimeter of the bottom holds up only the column of water above it. The extra water above the slanting sides is held up by these sides and does not press on the bottom. If the area of the base is the same in case A and case B, the total downward force on the base in the two cases is the same.

When the vessel is conical as in case C, the total force on the base is the same as in the preceding case. It is easy to see that in this case the pressure on the area directly under the top is the same as in the other cases. The slanting walls press down

on the liquid standing on the remainder of the base with a force which, when added to the weight of the liquid on that part of the base, makes the force on each square centimeter of the base equal to the force on each square centimeter directly under the top.

118. Liquid in Communicating Vessels.—It is a matter of common experience that liquids seek their own level in communicating vessels. For this reason, water stands at the same level in the spout of a tea kettle as in the tea kettle itself. If tubes of various sizes are connected, liquid poured into one of these tubes (Fig. 78) will come to the same level in all the tubes. This result is to be expected from the fact that the pressure in a liquid depends on the depth below the free surface. If points in the interior of the liquid are at the same level, the pressure at these points must be the same, or the liquid would flow from one point to the other until the pressure was equalized.

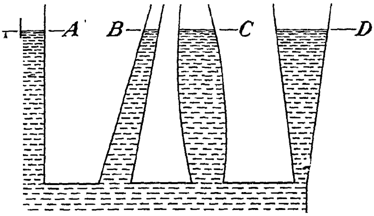


FIG. 78.—Liquids stand at the same level in communicating tubes.

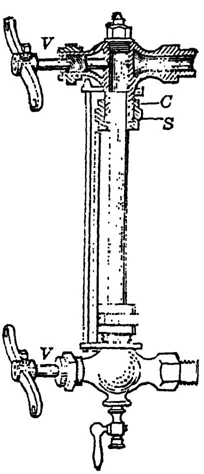


FIG. 79.—A water gauge shows the height of water in the boiler.

The water gauge in a steam boiler (Fig. 79) is an illustration of this principle. This gauge, which consists of a heavy-walled glass tube, is connected at the top with the steam space and at the bottom with the space filled with water in the boiler. The water in this glass tube stands at the same height as the water in the boiler and thus serves to indicate the height of the water in the boiler.

Artesian or flowing wells illustrate on a large scale the tendency of water to seek its own level, whatever the form or shape of the connecting vessel. The earth's surface in many places is made up of different strata. Some of these are porous to water and others are not porous. If a porous stratum happens to lie between two strata which are not porous, and the strata are then folded, the conditions are favorable for the formation of an artesian well. Rain sinks by gravity into the porous stratum. Since this stratum has an impervious stratum above and below it, the water fills the porous stratum until some opportunity of escape is provided. When a well is sunk into the porous stratum, the water from the well rises until it reaches the level of the water

in the stratum. Wherever the well is drilled, the water will rise to the same level.

From the discussion it is seen that the free surface of a liquid under the action of the forces of gravity is always horizontal. A particle in the surface is in equilibrium under the action of the gravity force downward and a hydrostatic force equal and opposite in direction. If, however, the liquid is rotating about a vertical axis (Fig. 80), the surface is no longer horizontal, since particles in such a surface would not be in equilibrium. The hydrostatic force, which is always perpendicular to the surface, and the gravity force together have a resultant that supplies the necessary centripetal force to keep the particle in its circular path. It can be shown that the surface of the liquid in this case assumes the form of a parabola.

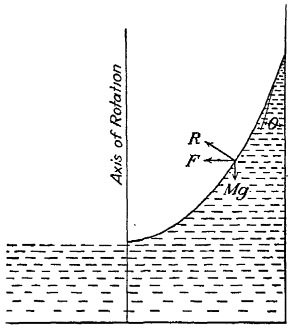


FIG. 80.—The free surface of a liquid revolving about a vertical axis is perpendicular to the resultant force.

119. Pressure Transmitted by Liquid.—Since liquids cannot resist forces tending to shear one layer of molecules over another, they have the power of transmitting pressure from one part of the liquid to all other parts. This characteristic of a liquid is illustrated in Fig. 81. The applied force is transmitted uniformly in all directions.

Liquids press against surfaces which are vertical or oblique as well as against those which are horizontal. The force is always found to be perpendicular to the surface against which it acts. Otherwise, owing to the mobility of the fluid, the component of the pressure along the surface would produce a flow of the liquid tangent to the surface. Liquids will be forced through a hole in the side of a pail near the bottom as well as through a hole in the bottom. The application of pressure to the piston (Fig. 81*b*) forces water from all the holes equally far in all directions. A comparison of the forces in different directions at any point in a liquid shows that when a liquid is at rest, the pressure at a

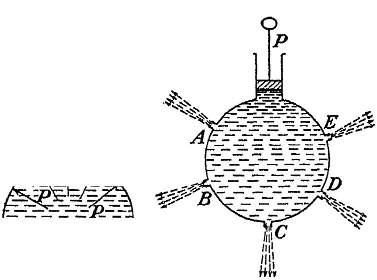


FIG. 81*a*. FIG. 81*b*.

FIG. 81*a*.—Pressure transmitted equally in all directions.

FIG. 81*b*.—Pressure uniform at all points on the surface of a sphere.

point is the same in all directions. Where a liquid is at rest and is acted upon only by gravity, the pressure is the same at all points on the same level.

120. Pascal's Principle.—Suppose there is a vessel filled with water (Fig. 82) and fitted with two equal pistons A and B, and suppose that the pistons carry equal weights so that there is no tendency for either of them to move. Now let a weight of 1 lb. be added to A. This will cause a pressure to be transmitted throughout the liquid, and it will be found that a weight of 1 lb. must be added to B in order to keep it from moving. This is a special case of a general principle discovered by Pascal and known as **Pascal's principle. This principle states that the pressure applied to an enclosed gas or liquid is transmitted without diminution in all directions.** If the pressure is increased or decreased at one point in the liquid or gas, it is increased or decreased uniformly throughout the liquid or gas. The expansion of a rubber balloon when it is being filled with air illustrates this principle. As the air is being forced into the balloon, an equal pressure is exerted in all directions so that the balloon expands equally in all directions and becomes a sphere.

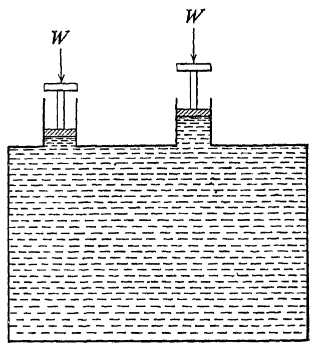


FIG. 82.—Pressure is transmitted in all directions without diminution.

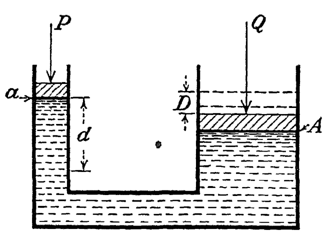


FIG. 83.—Multiplication of force by transmitted pressure: $Q/P = A/a$.

121. Multiplication of Force by Transmission of Pressure.—Consider two cylinders which are connected together and filled with water (Fig. 83). Assume that each cylinder is fitted with a piston which moves without friction. Let A be the area of the larger piston and a the area of the smaller piston. When a weight of w lb. is placed on the smaller piston, it produces a pressure of w/a lb. to the square inch on the water under this piston. This pressure is transmitted undiminished throughout the water and acts on the larger piston. The upward pressure

on the larger piston is then w/a lb. per square inch. Since the area of the larger piston is A, the total force exerted on this piston by the water is $\frac{w}{a} A$. Hence, by the application of a force of w lb., it has been possible to produce a force A/a times as large.

Let the smaller piston be displaced a distance d, and let D be the corresponding displacement of the larger piston.

$$A \times D = \text{volume swept out by larger piston.}$$
$$a \times d = \text{volume swept out by smaller piston.}$$

Since the liquid can be assumed incompressible,

$$A \times D = a \times d.$$

The work done on the smaller piston is

$$\text{Work} = \text{force} \times \text{distance} = \frac{w}{a} \times a \times d = w \times d.$$

Work done by the larger piston is

$$\text{Work} = \frac{w}{a} \times A \times D = w \times d.$$

Hence, work done on smaller piston = work done by larger piston.

Example.—The larger piston in Fig. 83 has an area of 100 sq. in. and the smaller an area of 5 sq. in. A force of 250 lb. is applied to the smaller piston. Find the force exerted by the larger piston.

Force per square inch on smaller piston = $^{250}\!/_5$ = 50 lb., and the force per square inch on the larger piston is also 50 lb.

Total force on the larger piston = force per square inch × area = 50 lb. per square inch × 100 = 5,000 lb.

122. The Hydrostatic Bellows.—This bellows consists of two disks of wood connected by waterproof canvas in such a way as to form a collapsible drum (Fig. 84). A small pipe passing through the lower disk opens into the drum and is then turned so that it is vertical when the disks are horizontal. If, now, the drum is filled with water and a man stands on the upper disk, water is forced into the vertical tube. The striking thing is that the height of the water is the same whatever the cross section of the tube. Hence, a small weight of water balances the weight on the disk.

The explanation of this fact is found in Pascal's principle. If the cross section of the tube is 1 sq. in. and the area of the upper disk is 500 sq. in., then the weight on the disk will be 500 times as large as the weight of the water in the tube.

123. Hydraulic Elevator.—Pascal's principle also explains the action of hydraulic elevators, which make use of hydraulic pressures for lifting heavy loads. At the bottom of the elevator (Fig. 85) is a well or pit. In this well is set a long hollow cylinder in which there moves a long plunger or piston which is firmly fastened at the top to the elevator cage. The admission of water under pressure acts against the lower end of the piston and lifts it and with it the elevator and its load. The weight of the elevator is partly counterbalanced by hanging a heavy weight over the pulley. The checking of the flow of water into the cylinder stops the elevator, and by allowing the water to flow out of the cylinder the elevator descends by its own

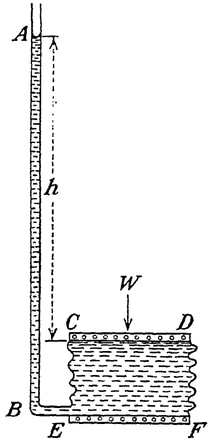


FIG. 84.—Hydrostatic bellows. Height of water in tube is independent of cross section.

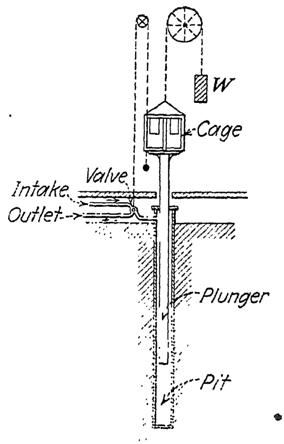


FIG. 85.—Hydraulic elevator is an application of Pascal's principle.

weight. In order to make the elevator move more rapidly than the plunger and thus get more speed, the motion of the plunger may be communicated to the elevator through a set of pulleys so that the cage of the elevator may be made to move several times as fast as the plunger.

Example.—The pressure in a water main is 50 lb. per square inch, and the diameter of the plunger of a hydraulic elevator is 10 in. How heavy a load can the elevator lift?

$$\begin{aligned}\text{Total force on the elevator} &= \text{pressure} \times \text{area of plunger}\\ &= 50 \text{ lb. per square inch} \times \pi \times 5^2 = 50 \text{ lb.}\\ &\quad \text{per square inch} \times 78.6 = 3{,}930 \text{ lb.}\end{aligned}$$

Example.—The pressure of water in the water mains is 35 lb. per square inch. How much work is required to pump 500,000 cu. ft. of water into the mains?

$$
\begin{aligned}
\text{Work} &= \text{force} \times \text{distance} \\
&= \text{pressure} \times \text{change of volume} \\
&= 35 \times 144 \times 500{,}000 \\
&= 252 \times 10^7 \text{ (ft.-lb.).}
\end{aligned}
$$

124. Liquids in Communicating Tubes.—Let two liquids which do not react chemically be placed in a bent tube (Fig. 86). When the liquids are at rest, the less dense liquid stands at a height h_1 above the junction of the two liquids. The pressure exerted by this column of lighter liquid is just balanced by the weight of the column of heavier liquid which stands above the junction of the liquids. Let d_1 be the density of the lighter liquid, d_2 the density of the heavier liquid, h_1 the height of the lighter liquid above the junction, and h_2 the height of the heavier liquid. Then,

$$h_1 \times d_1 = h_2 \times d_2.$$

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}.$$

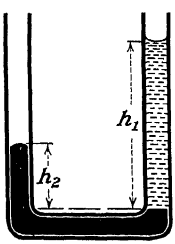


FIG. 86.—Density of non-miscible liquids by balanced columns.

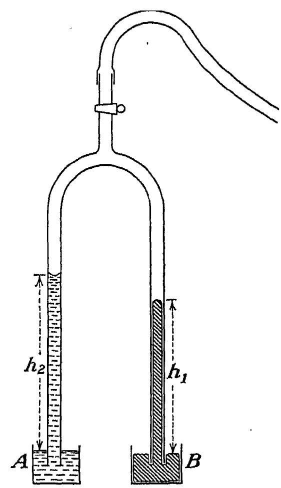


FIG. 87.—Densities of miscible liquids. The heights vary inversely as the densities

Hence, the heights of the two liquids above their surface of separation are inversely proportional to the densities of the liquids.

In case the liquids react chemically, the bent tube is inverted and the ends placed in cups containing the liquids (Fig. 87) whose densities will be denoted by d_1 and d_2. The air from the upper part of the bent tube is partially removed and the stopcock closed. The pressure above both liquids inside the tube is the same, and the atmospheric pressure on the liquids in the open vessels is the same. The difference between the pressure inside the tube and the atmospheric pressure is in each case

balanced by the rise of the liquid in the tube. These two differences in pressure are the same and

$$h_1 \times d_1 = h_2 \times d_2.$$

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}.$$

Example.—If one of the beakers in Fig. 87 contains sulphuric acid and the other contains water, and if the height of the column of water is 40 cm. when the height of the column of acid is 30 cm., find the density of the sulphuric acid.

$$\frac{\text{Density of acid}}{\text{Density of water}} = \frac{\text{height of water}}{\text{height of acid}}$$

$$\frac{d_2}{d_1} = \frac{h_1}{h_2} = \frac{40}{30}.$$

$d_2 = 1.33$ g. per cubic centimeter density of acid.

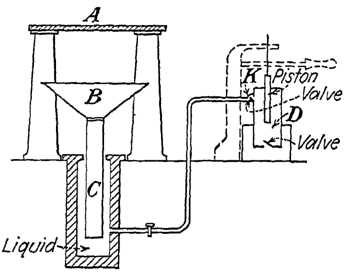


FIG. 88.—The hydraulic press produces large forces. Pressure is transmitted uniformly throughout the liquid.

125. A Hydraulic Press.—A hydraulic press consists of a strong cylinder (Fig. 88) in which works a cylindrical piston C. By means of a small pump D, oil is forced into the large cylinder through a check valve K which prevents its return. In consequence of Pascal's principle, whatever pressure is communicated to the liquid by the pump is transmitted undiminished to the walls of the containing cylinder and the piston C. If the large piston C has 100 times the area of the small piston D, the force exerted on C will be 100 times that

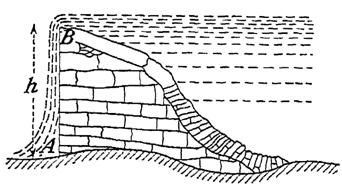


FIG. 89.—The pressure is determined by the height of the dam.

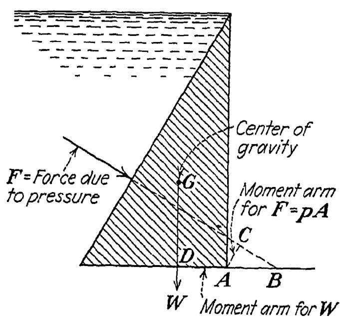


FIG. 90.—For equilibrium the torque due to the weight of the dam must exceed the torque due to pressure.

applied to D, and on the downward stroke of the small piston the large piston C will be moved only one-hundredth the distance through which the small piston moved. If the oil is incompressible, the work done on the large piston is just equal to that done on the small piston; that is, the input of the machine is just equal to the output. In order still further to increase the pressure exerted on the piston C, the small piston is

ordinarily forced down by means of a lever. Hydraulic presses are used in baling paper, cotton, etc., in punching holes through steel plates, and in extracting oil from seeds. By means of them, a small force operating through a large distance produces a large force operating through a small distance.

Problems

1. A water tank is located in the tower of a building, 550 ft. above the level of the street. If the city water supply has a pressure of 60 lb. per square inch at street level, how much additional pressure must be furnished by the pump used to fill the tank?

2. A diver works at a depth of 20 m. in sea water; density 1.03 g. per cubic centimeter. Find the pressure in excess of atmospheric pressure which he experiences at that depth. Express the result in kilograms per square centimeter.

3. A swimming pool 25 ft. wide and 75 ft. long has a depth of 3 ft. at one end and 10 ft. at the other. What is the total downward force of the water on the bottom of the pool? What is the average pressure on the bottom?

4. Find the force on the glass side of an aquarium containing salt water with a density of 1.03 g. per cubic centimeter, if the glass is 1.2 m. wide and the water behind it is 60 cm. deep.

5. A lock gate is 20 ft. wide and 30 ft. high. The depth of water on one side is 25 ft., and on the other side 12 ft. What is the net horizontal force on the gate due to water pressure?

6. Oil under a pressure of 150 lb. per square inch is applied to the piston of a rack used for lifting automobiles. The diameter of the piston is 9 in.; what is the total weight which can be lifted?

7. In a simple hydraulic elevator, the ram is 5 in. in diameter and 70 ft. long. Water is supplied at a pressure of 360 lb. per square inch. Neglecting friction, what total load can be raised, and how much work is done?

8. A small hydraulic press has a pump piston $\frac{1}{2}$ in. in diameter and a large piston 10 in. in diameter. If the efficiency of the machine is 85 per cent, find the actual mechanical advantage.

9. The lever of a hydraulic press gives a mechanical advantage of 6. The area of the small piston is 4 sq. cm., and that of the large piston is 100 sq. cm. A force of 20 kg. is applied to the handle. What force is exerted by the larger piston, if friction may be neglected?

10. A hydraulic elevator is operated from the city water mains. The maximum load for which the elevator is designed is 6 tons and the elevator and its plunger weigh 1.75 tons. The diameter of the plunger to which the pressure is applied is 1 ft. What pressure is necessary in the water mains?

11. Tin is being transferred by means of a siphon from a cauldron to a mold. The density of the molten tin is 7.3 g. per cubic centimeter and the atmospheric pressure is 76 cm. of mercury. What is the greatest height the rim of the cauldron can be above the surface of the molten tin?

CHAPTER X

ARCHIMEDES' PRINCIPLE

126. Buoyancy of Liquids.—It is a matter of common experience that bodies are lighter in water than they are in air. A fresh egg will sink in pure water but will float in water to which has been added a considerable quantity of salt. A piece of iron sinks in water but floats in mercury. This is because the density of the mercury is greater than that of the iron. When a diver lifts a stone under water and brings it to the surface, he finds that the stone is much heavier when it is brought above the surface. The diver also observes when he is in the water up to his neck that he is lifted up by the water so that it is difficult for him to retain his footing. In case of lighter bodies like wood or cork this lifting effect may be sufficient to keep parts of the body above water. This resultant upward pressure of a liquid on a wholly or partly immersed body is called **buoyancy.** It is a force acting vertically upward and counterbalancing in whole or in part the weight of the body.

That point through which the force of buoyancy acts is the **center of buoyancy.** This point lies at the center of gravity of the displaced liquid. The buoyant forces of all the displaced liquid might be replaced by a single force acting through the center of buoyancy without altering the behavior of the body.

127. Archimedes' Principle.—Suspend from one arm of a balance (Fig. 91) a hollow cylindrical cup and a piece of brass which has been nicely turned in the form of a cylinder so that it will just fit the cavity inside the cup. Now counterbalance the weight of the cup and cylinder by adding the necessary weights to the other pan of the balance. When a vessel of water is brought up in such a way that the cylinder C is completely immersed, it is observed that the side of the balance carrying the cylinder rises, showing that the water is pushing up on the cylinder. If water is now poured into the cup until it is just filled, the equilibrium of the balance is restored. Since the weight of a volume of water equal to that displaced by the cylinder is sufficient to compensate

for the lifting effect of the water on the cylinder, it is evident that the cylinder is lifted up by a force equal to the weight of the displaced water. If the experiment is repeated using kerosene or some other liquid instead of water, the same result is always found to be true. The loss in the weight of the immersed body is equal to the weight of the volume of liquid displaced by it. This principle which is known as Archimedes' principle may be stated as follows: **The loss of weight of a body immersed in a fluid is equal to the weight of the displaced fluid, or a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by it.**

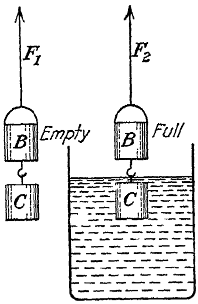


FIG. 91.—Test of Archimedes' principle.

128. Explanation of Archimedes' Principle. If a rectangular block (Fig. 92) is immersed in a vessel of liquid, the pressures on the vertical sides are equal and in opposite directions. These forces will not, therefore, tend to move the block in the liquid. Upon the upper face of the block, there is a downward force equal to the weight of the column of liquid having this face as a base and having a height h. On the lower face, there is an upward force which is equal to the weight of a column of liquid which has an area equal to the area of the lower base and a height H equal to the depth of this face below the surface of the liquid. The upward force exceeds the downward force by the weight of a column of liquid having a base equal to the area of the cross section of the block and a height equal to the height of the block.

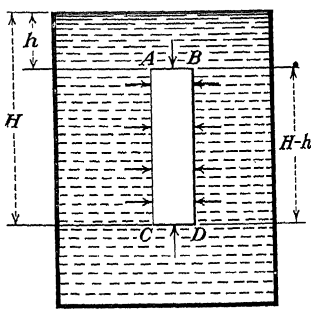


FIG. 92.—Upward forces on the submerged body equal weight of displaced liquid.

Now, the volume of this column is just equal to the volume of the liquid displaced by the immersed block, and the weight of this column is equal to the weight of the displaced liquid. The same sort of reasoning will hold for a body of any shape in any liquid. Hence, a body immersed in a liquid is lighter by the weight of the volume of liquid which it has displaced.

Example.—A piece of iron weighs 78 g. in air and 68 g. in water. What volume of water is displaced? What is the density of the iron?

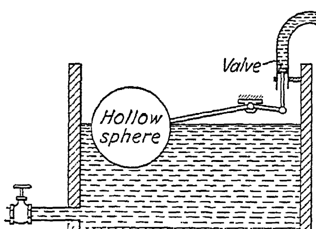
Weight of water displaced = loss of weight of iron in water

$$= 78\text{ g.} - 68\text{ g.} = 10\text{ g.}$$

Volume of water displaced = volume of iron = 10 c.c.

$$\text{Density of iron} = \frac{\text{weight of iron}}{\text{volume}} = \frac{78\text{ g.}}{10\text{ cu. cm.}} = 7.8\text{ g. per cubic centimeter.}$$

129. Archimedes' Principle Applied to Fish.—Fish are capable of moving toward the surface or into deep water by regulating the quantity of water which they displace and, therefore, the force which urges them upward. By a distension of the air bags in their bodies they can change their volumes and thus change the buoyancy of the water on them. This gives an upward force which causes them to rise. By a contraction of these air sacs the volume of the body is diminished and the fish sink. When a fish in a vessel of water is placed under the bell jar of an air pump and the air removed from the bell jar, these air sacs are ruptured and the fish sinks to the bottom no longer able to float.



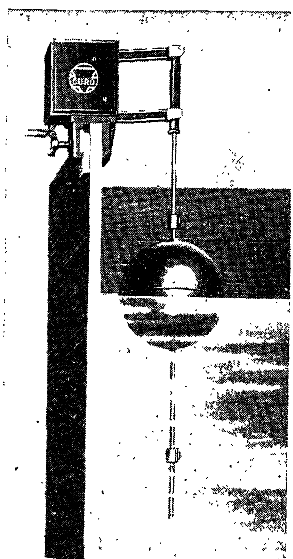


FIG. 93.—Float valves and automatic controls of water supply depend for their action on Archimedes' principle.

130. The Float Valve.—In many cases it is desirable to have an automatic cut-off for the water which flows into a tank in order to regulate the supply of water. The flow of water from a pump operated by a windmill is often controlled in this way. In such cases (Fig. 93), a float is provided to operate the cut-off valve when the tank is sufficiently full. This float is a light hollow brass or copper sphere which has less weight than an equal volume of water. This sphere is buoyed up by the water in the tank according to the principle of Archimedes. As the water rises in the tank, the float is buoyed with greater and greater force until the valve to which it is attached by means of a lever is closed. In the case of the windmill, the buoyant force on the float is also made to throw the pump out of action.

131. Floating Ship or Boat.—Since, according to the principle of Archimedes, every floating body displaces a weight of liquid equal to its own

weight, it is possible to calculate how much water will be displaced by a ship or boat when it is afloat. It is evident that the ship will sink until the lifting force of the water is sufficient just to balance the weight of the ship. For this reason, the ship will sink deeper when loaded than when empty. Since salt water is somewhat heavier than fresh water, the boat will sink deeper in fresh water than in salt water. A modern submarine can sink below the surface of the water by letting more water into certain tanks and thus making the boat heavier than it was when riding on the surface. It rises from below the surface by again pumping this water out of the tanks, thus making the boat lighter than the water which it displaces.

Example.—A barge is 30 ft. long and 16 ft. wide and has vertical sides. When two horses are driven on board, the barge sinks 2 in. farther into the water. How much do the horses weigh?

Volume of displaced water $= 30 \text{ ft.} \times 16 \text{ ft.} \times \frac{1}{6} \text{ ft.} = 80$ cu. ft.

Weight of water displaced $= 80$ cu. ft. $\times$ 62.5 lb. per cu. ft. $= 5{,}000$ lb.
Weight of horses = weight of displaced water = 5,000 lb.

132. Stability of Floating Bodies.—In order that a floating body like a ship be in stable equilibrium, it must tend to come back to its normal position when displaced from that position. The weight W of the body (Fig. 94) acts downward at G, the center of gravity of the body and the buoyancy of the displaced water acts upward through the center of gravity of the displaced water. When the body is displaced from its normal position, the center of gravity of the displaced water is shifted and the buoyancy of the displaced water acts upward through this point. These two forces, the weight of the floating body and the weight of the displaced water, must form a couple which tends to restore the body to its normal position if the body is to be in stable equilibrium. The condition for stability will be realized, if the **metacenter** M lies above the center of gravity of the body. The position of the metacenter is determined by the intersection of two lines—one drawn vertically through the center of buoyancy, B, and the other drawn vertically through the center of gravity, G, of the body before displacement. If as in Fig. 94 the metacenter lies above the center of gravity of

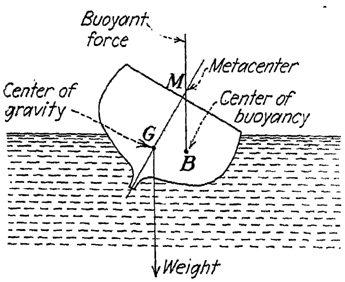


FIG. 94.—When the center of gravity is below the metacenter the boat is stable.

the body, there is a restoring couple and the body is in stable equilibrium. If, however, the metacenter lies below the center of gravity of the body as in Fig. 95 the couple which comes into play when the body is displaced from its normal position, tends to further increase the displacement and the body is not in stable equilibrium. The higher the metacenter is above the center of gravity of the body, the greater is the stability of the floating body.

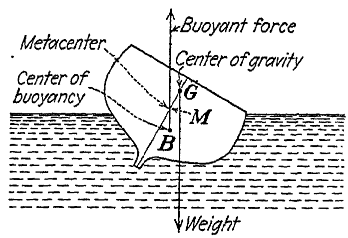


FIG. 95.—If the center of gravity is above the metacenter, the boat is unstable.

133. Density and Specific Gravity.—In order to determine the density of a body, it is necessary to determine its mass and its volume. The density is then found by dividing the mass by the volume. The mass of the body is easily determined by weighing, but it is sometimes difficult to find the volume, especially when the body has an irregular shape. In such cases, the volume may be determined by an application of Archimedes' principle. Since the body displaces a volume of water equal to its own volume and since each cubic centimeter of water weighs 1 g., the loss of weight in water is numerically equal to the volume of the immersed body.

The numerical value of the density of a body depends on the units in which the mass and the volume are measured. In the c.g.s. system, the density is the number of grams per cubic centimeter. In the British system, it is the number of pounds per cubic foot.

The specific gravity of a body is the ratio of its density to the density of water at 4°C. Since in the c.g.s. system a gram is defined to be the weight of a cubic centimeter of water at 4°C., the numerical values of the density and the specific gravity in this system are the same. In the British system, however, they are very different.

134. Density of Solids Heavier than Water.—When a body is heavier than an equal volume of water and is insoluble in water, its volume can be determined by finding its loss in weight when weighed in water. This loss of weight is equal to the weight of the water displaced, and if this loss of weight is expressed in

grams, it is numerically equal to the volume of the body in cubic centimeters. By dividing the mass of the body by this volume, the density is obtained.

135. Density of Solids Lighter than Water.—If the body is lighter than water but insoluble, its volume may still be determined by this method by fastening to the body a sinker large enough to force it below the surface of the water. In this case (Fig. 96), the combined weight of the body and the sinker is first determined when the sinker is immersed in water and the body above the surface of the water. The body is then also submerged and the combined weight redetermined. The change in weight is due to the buoyant force of the water on the body and equal to the weight of the water displaced by the body. It, therefore, gives the volume of the body in cubic centimeters. The density is then determined as in the preceding case.

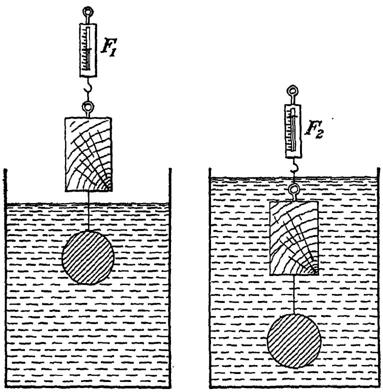


FIG. 96.—Densities of floating bodies determined by weighing them inside and outside of a liquid.

Example.—A piece of cork weighs 50 g. in air. When it is fastened to a sinker and the sinker alone immersed in water, the combined weight of sinker and cork is 200 g. When both sinker and cork are immersed, they weigh 75 g. What is the density of the cork?

Loss of weight due to submerging the body $= 200$ g. $- 75$ g. $= 125$ g.

Volume of water displaced by body $= 125$ c.c.

Density $= \dfrac{\text{mass of body}}{\text{volume}} = \dfrac{50 \text{ g.}}{125 \text{ c.c.}}$ 0.4 g. per cubic centimeter.

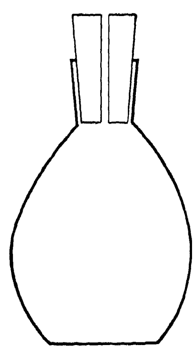


FIG. 97.—Specific-gravity bottle for determining densities.

136. Specific-gravity Bottle.—A specific-gravity bottle (Fig. 97) is often used to determine the specific gravity of liquids. It consists of a glass bottle of any convenient volume. This bottle is first weighed empty, then full of water, and then full

of the liquid the specific gravity of which is desired. The difference between the weight of the bottle empty and its weight filled with water gives the weight of the water in the bottle. Likewise, the difference between the weight of the bottle empty and its weight filled with the liquid gives the weight of the liquid in the bottle. The specific gravity is then found by dividing the weight of the liquid to fill the bottle by the weight of water to fill it.

$$\text{Specific gravity} = \frac{\text{weight of liquid}}{\text{weight of equal volume of water}}.$$

Example.—A specific-gravity bottle when empty weighs 240.30 g. When filled with water, it weighs 390.30 g.; and when filled with alcohol, it weighs 360.30 g. Find the specific gravity of alcohol.

Weight of water = 390.30 − 240.30 = 150.00 g.

Weight of equal volume of alcohol = 360.30 − 240.30 = 120.00 g.

$$\text{Specific gravity} = \frac{\text{weight of alcohol}}{\text{weight of equal volume of water}} = \frac{120.00\text{ g.}}{150.00\text{ g.}} \quad 0.80.$$

137. Hydrometer.—Another application of the principle of Archimedes is found in the hydrometer, which is extensively used for determining the specific gravity of liquids. It is usually made of a cylindrical glass tube (Fig. 98) which is provided with a narrow graduated glass stem. At the lower end of the hydrometer is placed a sufficient amount of mercury or shot to make the hydrometer sink to the desired level in the liquid. The depth to which the hydrometer sinks is determined by the density of the liquid, for the hydrometer sinks into the liquid until the buoyancy of the liquid is sufficient to overcome the pull of gravity on the hydrometer. The hydrometer then floats freely in the liquid, and the point at which the surface of the liquid touches the stem can be read and the density found directly from the graduated stem. Since the weight of the hydrometer remains the same, it will sink farther in light liquids like alcohol or kerosene than in heavier liquids like brine. For this reason the larger numbers are near the bottom of the scale and the smaller numbers near the top. Such hydrometers are now commonly used in finding the density of the acid in storage batteries (Fig. 99).

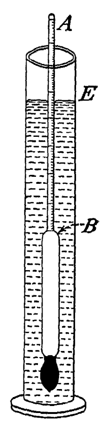


FIG. 98.—Hydrometer for measuring densities of liquids.

138. Lactometer.—A form of hydrometer is much used by dairymen in testing milk. For this reason it has taken the name of lactometer. Except for the way in which the scale is graduated, it does not differ from the hydrometer which has been just described. The specific gravity of cow's milk usually lies between 1.027 and 1.035. In order to determine the specific gravity of the milk, it is then only necessary to know these last two figures. It is sufficient to make the scale of the lactometer run from 20 to 40 which really means from 1.020 to 1.040. It is seen that the specific gravity of milk is greater than that of water. This is because the milk contains such substances as albumen, sugar, and various salts besides water. However, it contains other substances like butter fat which are lighter than water. The specific gravity alone does not give very definite information concerning the composition of the milk.

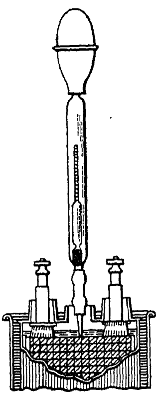


FIG. 99.—Hydrometer used to find density of acid in a battery.

Problems

1. A mixture of 4 l. of a liquid with sp. gr. 0.92 and 6 l. of another liquid with sp. gr. 1.18 shrinks to 9.5 l. Find the specific gravity of the mixture.

2. A truck loaded to a total weight of 8,000 lb. drives on to a ferry boat, causing the latter to sink ¼ in. deeper into the water. What is the area of the horizontal section of the boat at the water line?

3. A cylinder of nickel weighs 437.5 g. in air. What is the density of a liquid in which it weighs 322.3 g.?

4. A piece of brass weighs 1.5 lb. and has a density of 0.28 lb. per cubic inch. It is supported by a cord, in oil weighing 48 lb. per cubic foot. Find the pull on the cord.

5. The density of aluminum is 2.65 g. per cubic centimeter. Find the volume and the mass of a specimen of aluminum which weighs 1.6 kg. under water.

6. A piece of zinc weighs 42 g. in air and 37.2 g. when immersed in oil, with sp. gr. 0.8. Find the specific gravity of the zinc.

7. A piece of brass having a density of 7.8 g. per cubic centimeter weighs 858 g. when immersed in a liquid having a density of 1.25 g. per cubic centimeter. What is the volume of the piece of brass?

8. A piece of wood with a weight of 82 g. is immersed in water by using a sinker which weighs 42.5 g. in water. The combined weight of the wood and the sinker when both are immersed is 27.8 g. Find the density of the wood.

9. A diver with his suit weighs 320 lb. Blocks of lead with a volume totaling 60 cu. in. attached to his shoes just cause him to sink. How many cubic feet of water are displaced by the suit?

10. Find the volume of cork, sp. gr. 0.25, which must be employed in a life preserver if it is designed to support one-fifth of a man's body out of water, assuming a weight of 75 kg. and sp. gr. 1.00 for the body.

11. The density of rock salt is 2.18 g. per cubic centimeter. The mass of one salt molecule is 1.02×10^{-22} g. Find the number of molecules in 1 c.c.

12. Two masses, one of 80 g. and the other of 120 g., balance each other when suspended in water from equal arms of a balance. If the density of the 80-g. mass is 7.5 g. per cubic centimeter, what is the density of the other mass?

13. If the specific gravity of ice is 0.918 and that of sea water is 1.03, find the volume of the submerged part of an iceberg when 600 cu. m. are exposed.

14. A block of ice is floating in fresh water. What must be its least volume to support a man weighing 70 kg.?

15. A body was weighed in water, in oil, and in alcohol. Its loss of weight in water was 75 g., in oil 49 g., and in alcohol 60 g. What is the specific gravity (*a*) of the oil, (*b*) of the alcohol?

CHAPTER XI

MECHANICAL PROPERTIES OF GASES

139. Composition of the Air.—Just as water is the most widely distributed and most important of liquids, so the air is the most important and intimate of gases. It consists for the most part of two elements which are mixed together but not chemically combined. These elements are oxygen and nitrogen. In spite of the fact that there is no chemical union between them, the composition of the air is extraordinarily constant. Up to a height of 7 miles it always contains about 21 parts of oxygen to 79 parts of nitrogen. Besides oxygen and nitrogen, the air contains small parts of other gases, the most important of which are water vapor and carbon dioxide.

140. Weight of Air.—To an ordinary observer the air seems to have no weight and to offer little resistance to bodies moving through it. Yet smoke rises through the air and small balloons ascend out of sight. This is because the air is denser than the smoke or the gas with which the balloon is filled. The heavier air crowds the lighter gas upward as a piece of wood is forced to the surface of the water because it is lighter than water.

If a hollow glass sphere provided with a stopcock is weighed when the stopcock is open and then connected to an air pump by which as much of the air as possible is removed from the sphere, and if now the stopcock is closed and the sphere weighed a second time, it is found that the second weight is less than the first. The difference between these two weights is the weight of the air removed from the sphere. If the volume of the sphere is known and if it is almost completely exhausted, a fair approximation to the density of the air can be obtained by this method. A liter (1,000 c.c.) of air at the temperature of melting ice and under standard conditions weighs 1.293 g.

Example.—A hollow glass flask weighs 25.556 g. when it is empty. When it is filled with dry air at atmospheric pressure, it weighs 26.849 g. The volume of the flask is 1 l. What is the weight of 1 c.c. of air?

Weight of air = 26.849 g. − 25.556 g. = 1.293 g.

$$\text{Density of air} = \frac{\text{mass of air}}{\text{volume of flask}} = \frac{1.293}{1,000} = 0.001293 \text{ g. per cubic centimeter.}$$

141. Buoyant Effect of the Air.—Since air has weight, it produces a certain buoyant effect on bodies immersed in it just as liquids do on bodies immersed in them. As the density of air is so much less than that of liquids, its buoyant effect is also much less than that produced by liquids. That air exerts a lifting effect on bodies immersed in it may be shown by suspending a lead ball (Fig. 100) from one side of a small balance and a large hollow brass sphere from the other side. The hollow sphere is just heavy enough to balance the lead ball when both are in air. If the balance together with the suspended spheres is placed under a bell jar and nearly all the air removed by means of a pump, the lead ball will no longer balance the hollow sphere. This is because the lifting effect of the air on the hollow brass sphere is greater than its lifting effect on the lead ball for the reason that the hollow sphere displaces more air than the lead ball; and when this lifting effect has been removed, the true weights of the spheres become evident, and the hollow sphere is found to weigh more than the lead sphere.

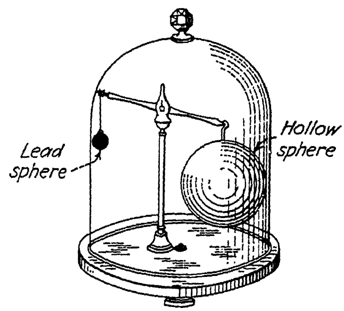


FIG. 100.—The greater the volume of the displaced air, the greater the lifting effect.

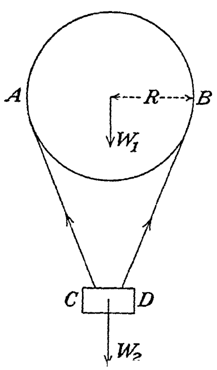


FIG. 101.—Buoyancy of air on a balloon.

The gross lifting capacity of a balloon (Fig. 101) is equal to the weight of the air which it displaces. The density of the air becomes less at higher elevations, and the lifting effect of the balloon decreases as it ascends. For this reason ballast is carried in the car or basket and thrown over when it is desired to make the balloon rise higher. By allowing some of the gas in the balloon to escape, the balloon can be made

to descend. Balloons used to explore the stratosphere show how great the lifting effect of air is in such cases.

Example.—A balloon has a volume equal to that of a sphere 15 yd. in radius. What is the gross weight which it will lift when the density of the air is 2 lb. per cubic yard?

$$\text{Volume of balloon} = \frac{4}{3}\pi(15)^3 \quad 14{,}132 \text{ cu. yd.}$$

$$\begin{aligned}\text{Weight of air displaced by balloon} &= \text{volume} \times \text{density}\\ &= 14{,}132 \times 2 = 28{,}264 \text{ lb.}\end{aligned}$$

142. Correction for Buoyancy of Air.—The buoyant effect of air is important in making accurate weighing on a sensitive balance. If the mass to be weighed and the brass weights used in making the weighings displace the same amount of air, there will be no error. If the density of the brass weights is greater than that of the body being weighed, the volume of the air displaced by the weights will be less than that displaced by the body, and the buoyancy of the air on the body will be greater than it is on the weights. This difference in buoyancy will make an error in the weighings so that the observed weight of the body will be too small. If, on the other hand, the density of the weights is less than the density of the body being weighed, the buoyancy of the air on the weights will be greater than it is on the body, and the observed weight will for this reason be too great.

To get the true weight from the apparent weight, a correction must be made for the difference between the buoyancy of the air on the weights and its buoyancy on the body.

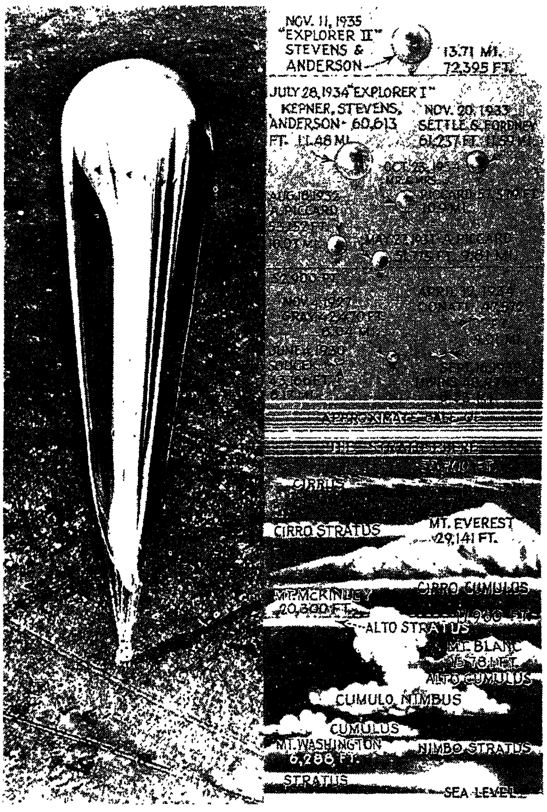
Let W = the true weight of the body.
w = the apparent weight of the body.
D = the density of the air.
d = the density of the object being weighed.
d_1 = the density of the weights being used.
W/d = the volume of the air displaced by the body.
W/d_1 = the volume of the air displaced by the weights.
DW/d = the weight of air displaced by the body.
DW/d_1 = the weight of air displaced by the weights.
DW/d = the buoyancy of air on body being weighed.
DW/d_1 = the buoyancy of the air on the weights.

True weight of body = observed weight + difference between the buoyancy of the air on the body and on the weights.

$$W = w + \frac{WD}{d} - \frac{WD}{d_1} \text{ or } W = w + \frac{wD}{d} - \frac{wD}{d_1} \text{ approx.}$$

143. Pressure of the Air.—Since a gas like air has weight, a column of it will exert a pressure just as a column of liquid exerts a pressure. Though a cubic foot of air weighs little, the height

of the atmosphere is large, and the weight of all this air pressing on the earth is large. That air exerts a pressure is shown by taking a thistle tube and covering the mouth with rubber dam so that it is air-tight. If the air in the thistle tube is partially



(*a*) (*b*)

FIG. 102.—Exploring the stratosphere with balloons. (*a*) Landing of the Explorer II in which Captain Stevens and Captain Anderson made a stratosphere flight, Nov. 11, 1935. (*b*) Diagrams showing how cloud forms mark altitudes in the lower atmosphere and the heights reached by the most important balloon flights. (© *N. G. S. Reproduced by special permission of the National Geographic Magazine.*)

removed by means of a pump, the pressure of the air on the outside of the rubber dam is greater than the pressure on the inside so that the rubber dam is stretched and forced inward.

If a fairly large sirup can containing a small amount of water is placed on the fire and the water boiled for a while, the escaping

steam carries most of the air out of the can. If the can is now removed from the fire and at the same instant closed by inserting a rubber stopper, the water vapor which filled the can soon condenses and leaves the can nearly free from air. This reduces the pressure on the inside of the can, and the atmospheric pressure on the outside causes the can to collapse.

144. Torricelli's Experiment.—An Italian named Torricelli first proved that the atmosphere exerts a pressure. He was able to measure this pressure in the following manner. A glass tube 100 cm. long was closed at one end and completely filled with mercury. Care was taken that no air was held between the mercury and the sides of the tube. The finger was then placed over the open end of the tube and the tube inverted in a basin of mercury. When the finger was removed from the open end of the tube under the mercury, the mercury in the tube sank until the level of the mercury in the tube was about 76 cm. (Fig. 103) higher than its level in the basin. The pressure of the atmosphere on the mercury in the basin supported the mercury in the tube so that it did not sink farther. Since each cubic centimeter of mercury weighs 13.56 g., the weight of this column is $76 \times 13.56 = 1{,}033$ g. for each square centimeter of cross section. This means that the atmosphere presses with sufficient force on each square centimeter to support a weight of 1,033 g. Since mercury is 13.56 times as heavy as water, the atmospheric pressure would be able to support a column of water which is 13.56 times as high as this column of mercury, or 1,033 cm., which is equal to a height of about 33.8 ft.

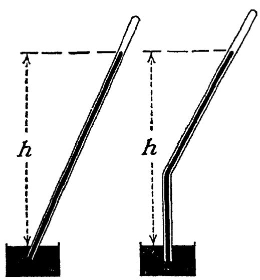


FIG. 103.—Torricelli's experiment. Height of mercury independent of shape or position of tube.

If this tube is carried up a mountain, the level of the mercury in the tube will drop because there is less air above the tube in the new position than there was at the base of the mountain. The greater the height to which the tube is carried, the greater is this drop. On the top of a mountain 1 mile in height the column of mercury stands 1.5 cm. lower than at the base of the mountain.

Example.—Assuming that the average density of the air at distances not greater than 1 mile above the surface of the earth is 0.001273 g. per cubic centimeter, find the change in the pressure on a balloon which has risen to a height of 1 mile.

Change in pressure = weight of column of air between balloon and earth.

Change in pressure = $5{,}280 \times 30.48 \times 0.001273 = 20.6$ g. per square centimeter.

145. The Mercury Barometer.—From this experiment of Torricelli has been developed the barometer, an instrument of the highest importance in modern life. There are two forms of mercury barometers. One of these forms (Fig. 104) is known as a cistern barometer and consists of a Torricellian tube. The pressure of the air on the mercury in the cistern supports the mercury in the tube, as in the experiment of Torricelli. As the pressure of the atmosphere varies from hour to hour or from day to day, the height of this column varies. These variations in the height of the barometer are made a basis for weather predictions.

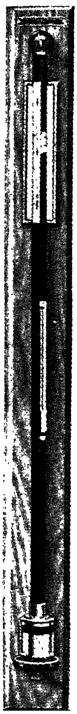


FIG. 104.—Cistern barometer.

The other type of mercury barometer consists of a U-tube partly filled with mercury as indicated in Fig. 105. If both ends of the tube were open, the mercury would stand at the same height in both parts of the tube. One end of the tube is closed, however, and there is a vacuum above the mercury in this part of the tube. The weight of the column of mercury which stands between the level of the mercury in the closed tube and the level of the mercury in the open tube is supported by the atmospheric pressure on the mercury in the open tube. The length of this column then gives a measure of the atmospheric pressure. At sea level, the length of this column is 76 cm. of mercury. Hence, it is customary to say that the atmosphere exerts a pressure which is equal to the weight of a column of mercury 76 cm. in height.

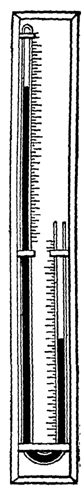


FIG. 105.—Siphon barometer.

146. The Aneroid Barometer.—The aneroid barometer consists of an air-tight box (Fig. 106) having a corrugated top. From this box most of the

air has been removed so that the pressure inside the box is much less than atmospheric pressure. The top of the box is a flexible diaphragm which moves inward and outward as the pressure on it is changed. As the pressure decreases, the diaphragm moves outward; as it is increased, the diaphragm moves inward. These movements of the diaphragm are slight but are magnified by a system of levers which are connected to a pointer which moves over a dial and thus indicates the change in pressure. The position of the pointer is marked when the atmospheric pressure is 76 cm. of mercury. By marking the points above and below this point, the variation of the pressure above and below normal can be determined.

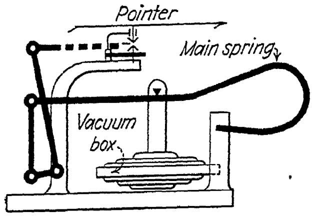


FIG. 106.—Aneroid barometer.

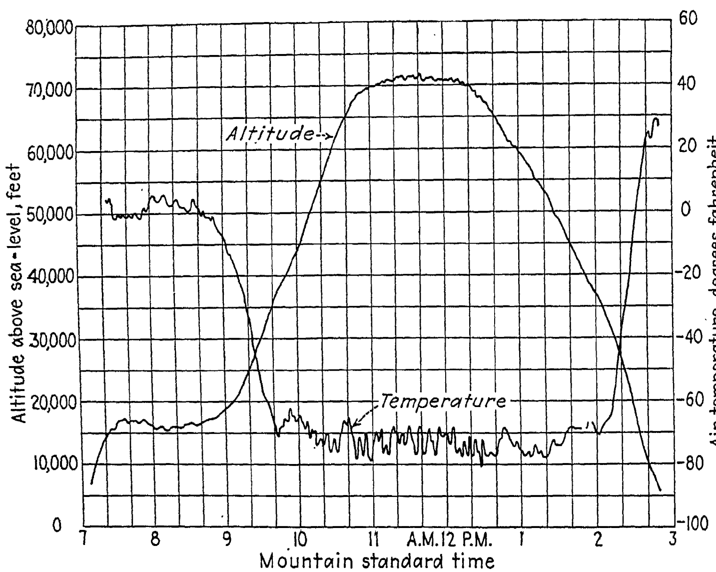


FIG. 107.—Chart showing relation between temperature and altitude in the stratosphere on the flight of the Explorer II. The lowest temperature was 81°F. below zero. (© *N. G. S. Reproduced by special permission of the National Geographic Magazine.*)

Example.—An aneroid barometer reads 75.35 cm. of mercury at sea level and 68.15 cm. on a near-by hill. How high is the hill?

$$\text{Change in pressure} = 75.35 - 68.15 = 7.20 \text{ cm. of mercury}$$
$$= 7.20 \times 13.56 = 97.92 \text{ g. per square centimeter.}$$

$$\text{Height in centimeters} = \frac{7.20 \times 13.56}{0.001293} = 76{,}000 \text{ cm.}$$

147. Depth of the Atmosphere.—Judging from the rate at which the pressure of the air decreases as a barometer is carried upward, it is thought that the depth of the atmosphere is about 30 miles. Since the atmosphere becomes less and less dense as we ascend, at the height of 30 miles the air must be very rarefied, but it cannot be said to have entirely disappeared. Figure 107 shows how the temperature of the air varies with the altitude and how the altitude can be found from changes in barometric pressure.

148. Applications of Atmospheric Pressure.—The breathing of animals is an application of atmospheric pressure. A reduction of pressure is made by a movement of the lungs and the diaphragm, and then the greater pressure of the outside air causes a fresh supply to fill them. Sucking and drinking animals take advantage of atmospheric pressure to aid them in this operation. By reducing the pressure in the mouth they really allow water to be forced into the mouth by the outside atmospheric pressure. Even in eating, the atmospheric pressure helps to force the food from between the teeth after it has been masticated.

149. The Siphon.—The siphon is an instrument for transferring liquid from a place of higher level to a place of lower level. It consists of a bent tube (Fig. 108) with the short end immersed in the liquid at the higher level and the other end immersed in the vessel into which the liquid is to be transferred. If the flow is once started, the liquid will continue to flow until it is at the same level in the two vessels.

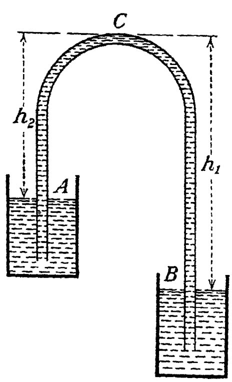


Fig. 108.—The siphon. The difference of pressure at C causes the flow.

Evidently, the water must flow through the tube if the force pushing it in one direction is greater than the force pushing it in the opposite direction. The pressure on the surface of the liquid is atmospheric pressure. The force on the water in the top of the tube pushing it to the right is atmospheric pressure less the weight of the column of liquid of height h_2. The pressure on the liquid in the top of the tube pushing it to the left is atmospheric pressure less the weight of the column of liquid of height h_1. Hence, the pressure toward the right exceeds the pressure toward the left by the difference

in the weight of these two columns of liquid. The pressure toward the right $= P - dh_2$ and that toward the left $= P - dh_1$.

$$\text{The net pressure} = d(h_1 - h_2).$$

This is the pressure responsible for the flow on the basis the liquid is at rest, but the liquid is in motion and this fact may change the pressure.

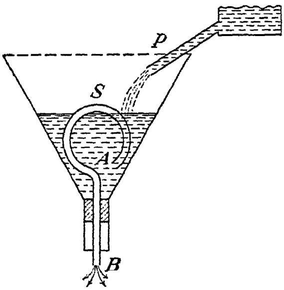


FIG. 109.—A form of siphon giving an intermittent flow.

If the water in the two vessels has the same level, the pressures balance and the flow ceases. The greater the difference in level, the greater the force and the more rapid the flow. Since the atmospheric pressure forces the water up to the bend in the tube, the water will not flow when the bend is higher than about 30 ft. above the level of the water in the vessel.

Example.—In the siphon shown in Fig. 108 the height of C above A is 1.5 in. and the height of C above B is 36 in. Find the force driving the water through the siphon.

Pressure driving water through the siphon

$$= (h_1 - h_2)d = \frac{(36 - 1.5) \times 62.5}{12} = 180 \text{ lb. per square foot}$$
$$= 1.2 \text{ lb. per square inch.}$$

150. The Intermittent Spring.—A siphon of the form indicated in Fig. 109 will give an intermittent flow. When the water fills the vessel so that

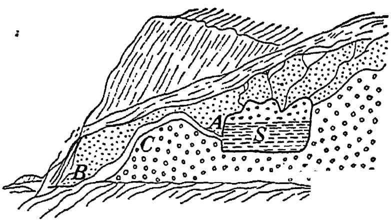


FIG. 110.—The intermittent spring.

the siphon is completely covered, the operation of the siphon begins, and the flow will continue until the vessel is empty. The vessel will then again fill and the process will be repeated.

An intermittent spring (Fig. 110) operates on the principle of such a siphon. The flow of water from the spring is periodic. After the spring has been once emptied, sufficient time must elapse for it to fill to the height at which the water will start to flow through the siphon.

151. Compressibility of Gases.—If we undertake to decrease the volume of a liquid by the application of pressure, it is found that it is necessary to apply enormous pressure in order to get appreciable changes in volume. The behavior of gases in this respect is quite different. It is easy to compress a body of air so that it occupies only one-third or one-tenth of its original volume. As soon as this pressure is removed, the air or other gas springs back to its original volume. The tires of automobiles are ordinarily filled with air. As more and more air is forced into the tire, the volume of the tire increases very little; but the air taken from the outside is forced to occupy much less volume than it originally occupied. As the air is forced into the tire, the pressure it exerts is more and more increased.

The tendency of air to expand is seen when a puncture occurs in an automobile tire and the air suddenly escapes. This tendency of a gas to expand also manifests itself when a toy balloon is placed under a bell jar and the air is removed from the bell jar by means of a pump. The balloon expands and bursts as soon as the pump is set in action. Before the pump is set in operation, the pressure of the air on the inside and on the outside of the balloon is nearly the same. That on the inside is only slightly larger than that on the outside. As the pressure on the outside is removed, the air on the inside expands as much as possible, causing the rupture of the balloon.

152. Boyle's Law. Relation between Pressure and Volume of a Gas.—The relation between the volume of any mass of gas and the pressure exerted by the gas upon the walls of the containing vessel was investigated by Robert Boyle and is known as Boyle's law. **This law states that at constant temperature the volume of a given mass of gas is inversely proportional to the pressure to which it is subjected.** Thus, if v_1 and p_1 denote the original volume and pressure and v_2 and p_2 denote the final volume and pressure,

$$p_1v_1 = p_2v_2 = \text{constant},$$

or for a constant temperature the product of the pressure and the volume is a constant. By pouring mercury into the open end of the tube (Fig. 111), the pressure on the air in AC is increased and its volume decreased. Since the density is inversely proportional to the volume, this law states that at constant temperature the density of a gas is proportional to the pressure.

$$\frac{p_1}{p_2} = \frac{d_1}{d_2}.$$

For high pressures and low temperatures this law is only an approximation. Gases which can be liquefied by the application of pressure do not obey this law near the temperature and pressure at which they begin to liquefy. Figure 112 shows the relation between the pressure and volume of a gas at different temperatures.

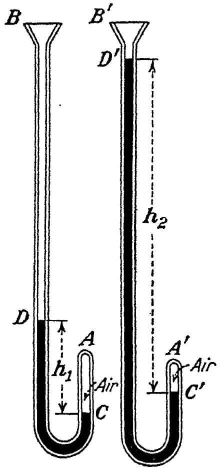


FIG. 111.—Boyle's law—pressure times volume is constant.

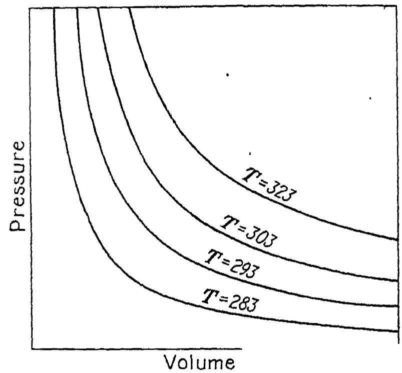


FIG. 112.—Relation of pressure to volume at different temperatures measured on the absolute scale. pv = constant.

Example.—Allow 100 cu. in. of air under a pressure of 400 lb. per square inch to expand until the pressure is 20 lb. per square inch. What is the new volume?

Original pressure × original volume = final pressure × final volume.

$$P_1V_1 = P_2V_2.$$
$$P_1 = 400 \text{ lb. per square inch.}$$
$$V_1 = 100 \text{ cu. in.}$$
$$P_2 = 20 \text{ lb. per square inch.}$$
$$400 \times 100 = V_2 \times 20.$$
$$V_2 = 2{,}000 \text{ cu. in.}$$

Example.—What percentage of the air leaves a room when the barometer changes from 75 to 72 cm. of mercury?

$$\frac{\text{Density at 75 cm.}}{\text{Density at 72 cm.}} = \frac{75}{72} = 1.04.$$

The density of the air decreases 4 per cent, and 4 per cent of the air leaves the room.

153. Applications of Boyle's Law.—There are many important applications of this law. When the faucet of a kerosene can (Fig. 113) is turned on, only a small amount of kerosene will run out of the can if the cap A is screwed down so that the can is air-tight. In this case, as the kerosene runs out of the can, the volume of air above the kerosene increases and its pressure decreases according to Boyle's law. On the kerosene flowing out at the faucet, there is, under normal conditions, a pressure of 1 atmosphere. When the pressure of the air in the can together with that due to the weight of the kerosene exerts a pressure of 1 atmosphere, the flow ceases. If the screw cap A is now opened so that atmospheric pressure acts undiminished on the upper surface of the kerosene, the flow begins again.

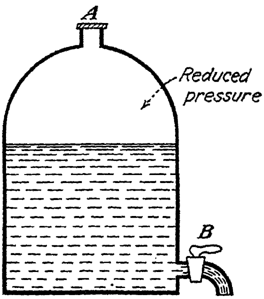


FIG. 113.—Flow of liquid from a can. Pressure over the liquid may be less than atmospheric pressure.

It is difficult to pour a liquid from a small-necked bottle without considerable sputtering. This fact is illustrated in Fig. 114. Suppose there is air at atmospheric pressure above the liquid at P. Then the weight of the column of liquid will be sufficient to cause a flow. While the flow is in progress, the air above the liquid expands and its pressure decreases. Before the flow ceases, the pressure will be decreased too far for equilibrium. This is because the liquid once in motion tends to keep

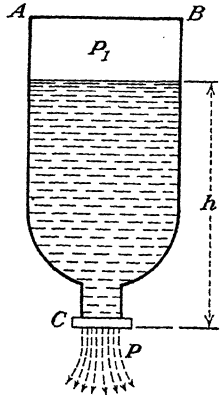


FIG. 114.—Flow of water from a bottle is intermittent.

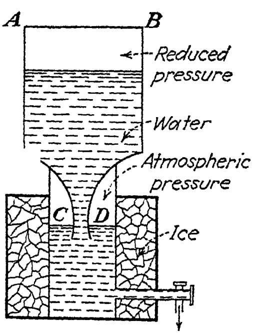


FIG. 115.—A form of water cooler.

moving until forced to stop. When the flow has ceased, more air will rush in through the mouth of the bottle. Too much air for equilibrium will rush in and the pressure above the liquid becomes too large. The flow will then begin again. This action causes the familiar gurgling which is noticed when liquids are poured from bottles. To prevent this gurgling and get a

steady flow, sometimes an auxiliary tube is inserted in the stopper. The air flows in through this tube continuously, and a steady flow of liquid is obtained.

The drinking fountain (Fig. 115) is also an illustration of the expansion of gases according to Boyle's law. The bottle containing some air is

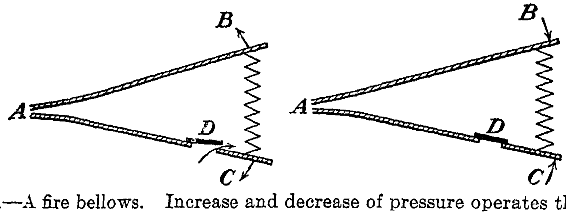


FIG. 116.—A fire bellows. Increase and decrease of pressure operates the valve.

inverted. The air at atmospheric pressure in the upper part of the bottle permits some water to run out of the faucet. As the water flows out, more air is sucked in through the mouth of the bottle as in the preceding case.

In the common fire bellows (Fig. 116), as the two sides *B* and *C* are pushed apart, the volume of the enclosed air is increased and its pressure decreased in agreement with Boyle's law. More air rushes in from the outside through the valve *D*. As the sides *B* and *C* are pushed together, the volume of the enclosed air is decreased and the pressure is increased. This additional pressure on the inside closes the valve, and air is forced out through the nozzle. The nozzle *A* is made small so that little air will come through it, in comparison with that which comes through the valve *D* when the volume of the bellows is being increased.

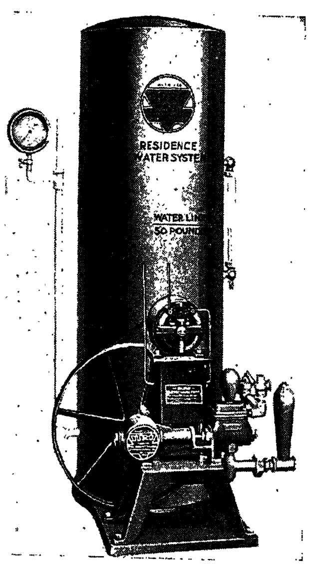


FIG. 117.—Air-pressure tank for water system. The lower the water in the tank, the less the pressure.

154. The Air-pressure Tank.—A modern and highly practical method of storing pumped water for use on a farm is the pneumatic or compressed-air water tank. The principle of this tank, which is an application of Boyle's law, is very simple. An air-tight steel tank (Fig. 117), similar to a steam boiler, is placed in the basement of a suitable building where it will not freeze. This storage tank is connected to a force pump. When the outlet pipe is closed, water is pumped through the inlet pipe. At the beginning, the tank is completely filled with air. As more and more water is forced into the tank, the air above the water is more and more compressed according to Boyle's law. When the tank is half full of water, the air occupies

one-half of its original volume and consequently exerts double its original pressure. As the pumping of the water into the tank continues, the pressure of the air above the water increases, and it becomes increasingly difficult to force more water into the tank. In this way, any desired pressure can be obtained in the tank. When a faucet is opened in any part of the house, water is forced from the tank through the faucet by means of the pressure of the air in the tank. As the water is allowed to flow, the volume of air above the water increases and the pressure of the air on the water decreases, so that when the tank is nearly empty, the pressure on the water is very small. When the tank is again filled with water, the pressure is again restored.

Example.—When the water in the pressure tank (Fig. 117) stands at the water line, the compressed air above the water exerts a pressure of 30 lb. to the square inch. When enough water has run out of the tank to allow the air to become 1.5 times its original volume, to what height will the water rise in the house?

By Boyle's law,

$$P_1V_1 = P_2V_2$$

since

$$V_2 = \frac{3}{2}V_1 \text{ or } P_2 = \frac{2}{3}P_1$$

Hence, the new pressure is two-thirds of the original pressure = 20 lb. per square inch. If the atmospheric pressure at the time = 15 lb. per square inch, the effective pressure for lifting water in the house is

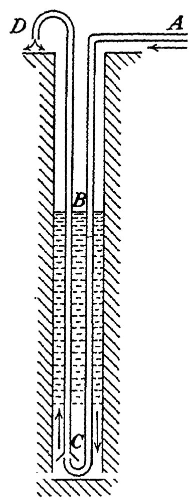


FIG. 118.—Compressed-air lift.

$$\begin{aligned} 20 \text{ lb.} - 15 \text{ lb. per square inch} &= 5 \text{ lb. per square inch} \\ &= 5 \times 144 = 720 \text{ lb. per square foot.} \\ \text{Pressure} &= \text{density} \times \text{height.} \\ 720 &= 62.5 \times h. \\ h &= 11.5 \text{ ft.} \end{aligned}$$

155. Compressed-air Lifts.—The force exerted by air when compressed is often used in carrying water from deep wells. If an air compressor is available, the process is quite simple. A water pipe DC (Fig. 118) open at both ends is submerged in a well, and a smaller pipe AC delivers compressed air into it at its lower end, which is considerably below the level of the water. The compressed air escaping from the smaller pipe rises through the pipe CD, carrying with it a column of water. This water is discharged into an open tank above the ground where the bubbles of air escape.

Problems

1. A balloon has a spherical bag with a diameter of 14 m. What is the weight of the air displaced when the density of air is 1.25 g. per liter?

2. An aneroid barometer used as an altimeter is sensitive enough to detect a difference of elevation of 1 m. at sea level. How small is the pressure change detected?

3. Air is pumped into a caisson at a pressure sufficient to keep out water at a depth of 28 ft. What must the pressure be, if the barometer reads 29 in. of mercury?

4. Assuming that the density of air at sea level is 0.001293, and the barometer reading is 760 mm. of mercury; what will be the barometer reading at an altitude of 50 m., neglecting the variation in the density of air?

5. The volume of the gas bags of an airship is 140,000 cu. m. The density of air at 0°C. and at atmospheric pressure is 1.29 g. per liter and that of the hydrogen with which the gas bags are filled is 0.092 g. per liter. Find the maximum weight which the airship will carry.

6. A tank with a volume of 2 cu. ft. contains oxygen at a pressure of 200 lb. per square inch. How much volume would be occupied by the gas at the same temperature at atmospheric pressure?

7. The tank of a water supply system contains 150 gal. of air at atmospheric pressure; 120 gal. of water are then forced in. How great is the pressure in the tank?

8. A cylindrical diving bell, open at the bottom, has a volume of 210 cu. ft. and a diameter of 5 ft. How high will the water rise in the bell when it is immersed to a depth of 28 ft., if the pressure of the air originally was 29 in. of mercury?

9. An air bubble released at the bottom of a pond expands to two times its original volume by the time it reaches the surface of the pond. How deep is the pond? (Barometer reading is 74 cm. of mercury.)

10. A vertical cylinder with a radius of 4 in. and a height of 21 in. is closed by a piston weighing 32 lb. When the piston is released, where will the gas pressure balance its weight, if the original pressure is 15 lb. per square inch and no gas leaks out?

11. A cylinder 15 in. long is closed at one end. A gas-tight piston is introduced in the other end and pushed in a distance of a 5 in. What is the ratio of the pressure in the cylinder to the external pressure of the air?

12. A man carries a barometer from the bottom to the top of a building which is 108 m. high. At the bottom of the building the barometer reads 76 cm. of mercury and at the top it reads 75.05 cm. of mercury. Find the average density of the air.

13. A barometer is filled with glycerine having a density of 1.25 g. per cubic centimeter. At what height does it stand when a mercury barometer reads 76 cm. of mercury?

14. How much lower does the barometer stand on the top of a building which is 150 m. high when the barometer at the base of the building reads 76 cm. of mercury? Take the density of air at 21°C. as 0.0012 g. per cubic centimeter.

CHAPTER XII

FLUIDS IN MOTION

156. Flow of Liquids or Gases.—When fluids, **liquids** or **gases,** are in motion, they assume new properties which cannot be predicted from the behavior of fluids at rest. The simple laws of hydrostatic pressure no longer apply without modification. The pressures in the fluid depend on the conditions under which it is moving as well as on the characteristics of the fluid.

When a liquid moves because of the application of a force, the shape as well as the position of the liquid may be changed. Because different layers of the liquid move with different speeds the internal friction between the various layers must be considered. For example, the velocity of the water in a river is greater in mid-stream than it is near the bank and the velocity of the wind is greater, the higher we go above the surface of the earth. Water flowing through a circular pipe moves with increasing velocity from the inner surface of the pipe to its center. Friction between the surface of the pipe and the water is greater and hence the velocity is smaller for the layer in contact with the surface of the pipe. For succeeding layers the velocity increases as the center of the pipe is approached. Each layer of water is pulled forward by the layer of water moving over it and backward by the layer of water over which it moves. The importance of these forces is seen in the difficulty of cranking an automobile on a cold day.

157. Viscosity.—If two beakers, one containing oil and the other water, are tilted from side to side, it is seen that the mobility of the water is greater than that of the oil or that the oil is more viscous than the water. This property of a liquid or gas, known as **internal friction** or **viscosity,** is due to the frictional forces between the molecules. In gases internal friction or viscosity is much less than it is in liquids. A body moving through air or some other gas experiences retarding forces just as if it were moving through a liquid. These forces in a gas are less than in a liquid, because of the increased distance between the molecules.

Suppose that a liquid flows over a horizontal surface AB (Fig. 119). The layer of liquid which is in contact with the surface remains stationary on account of adhesion, but each successive layer of liquid moves with respect to the layer directly below it. The speed of each layer increases with the distance of the layer from the solid surface AB. A more slowly moving layer tends to retard the motion of a more rapidly moving layer. To maintain this set of conditions each horizontal layer must be acted upon by a tangential force in the direction of motion of the liquid and by a retarding force in the opposite direction. These forces are due to frictional forces between the particles of the liquid when different layers are moving with different speeds. The dotted figures *abcd* and *efgh* show the distortion of the liquid when it is in motion. The upper layer *fg* travels faster than the lower layer *eh*. If the distance between these layers is l and the difference in their speeds is v, the liquid experiences a shearing stress and the force necessary to produce this stress is F/A, where F is the total horizontal force and A is the area of the upper or lower face. When the difference in velocity is not too great,

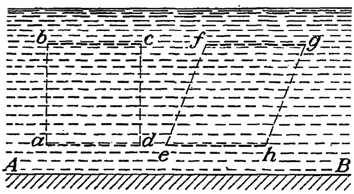


FIG. 119.—Displacement of layers of a moving liquid with respect to conditions at rest.

$$\frac{F}{A} \propto \frac{v}{l} \text{ or } \frac{F}{A} = \frac{\eta v}{l}$$

where η is a constant called the coefficient of viscosity. For liquids it decreases with rise of temperature.

158. Effect of Viscosity on Motions of Objects.—If an object is dropped from an airplane, the force of gravity at first exceeds the retarding force due to the frictional forces arising from the motion of the body through the air. As the velocity of the body increases these frictional forces also increase, but the force of gravity remains constant. By and by the frictional forces just balance the force of gravity. The motion then becomes uniform and there is no further increase or decrease in velocity.

The increase of internal friction with the velocity of the moving body, accounts for the excessive cost of operation when automobiles or trains are run at high speeds. At moderate speeds fluid friction increases directly with the speed. At higher velocities it varies with the square or even the

cube of the speed. Meteorites move through the air with such high velocities that they become incandescent and rapidly vaporize (Fig. 120).

"Streamlining" in airplanes (Fig. 121) is an attempt to reduce frictional forces of the air through which the plane is moving. The shape of the moving part of the airplane is so chosen that it reduces the eddies in the air to a minimum. To produce these eddies, a certain amount of energy is necessary in excess of what would otherwise be necessary to propel the airplane. By reducing these eddies to a minimum, the power necessary to drive the airplane at a given speed is decreased.

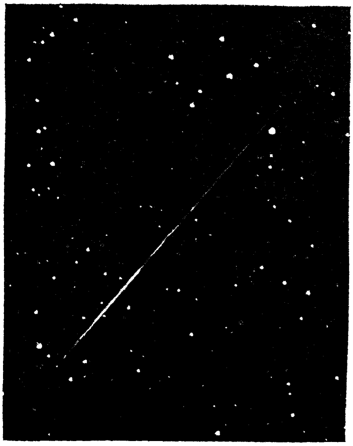


FIG. 120.—Meteor trail showing sudden increase in brightness. The resistance of the air is large for high velocities. (*Astronomical photograph from Yerkes Observatory. Reprinted by permission of University of Chicago Press.*)

Internal friction is also present in solids. The amplitude of a vibrating tuning fork decreases with the time. Energy is expended in displacing one layer of molecules in the prong of the fork with respect to another layer. A force is necessary to produce this displacement. The magnitude of this force varies with the characteristics of the metal and its temperature. Whenever a solid is deformed so that one layer of molecules is displaced with respect to another layer, the forces of internal friction must be overcome. Where belts are used to drive machines, or where driving shafts are twisted, internal friction appears, and work must be done to overcome it.

159. Lift Pump.—Water for household or farm purposes is usually lifted out of moderately deep wells by a lift pump. This pump (Fig. 122) consists of a cylinder which is connected to a pipe S. The lower end of the pipe S is immersed in the water in the well. At the bottom of the cylinder there is a valve B which opens upward. A plunger P containing a valve A opening upward is moved up and down in the cylinder by means of a pump handle. The valve B in the cylinder prevents any water above it from passing downward. As a handle is forced downward, the plunger is raised with the valve A closed. The water above the plunger is thus raised and flows out of the spout.

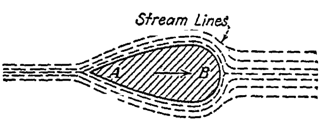


FIG. 121.—Streamlines to reduce friction.

The upward stroke of the piston reduces the pressure in the space below the plunger. The reduction of the pressure in this space allows the pressure of the air on the water in the well to force more water up the pipe S, through the valve B, into the cylinder.

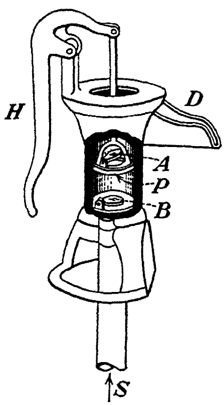


FIG. 122.—A lift pump.

When the piston makes its next downward stroke, the valve B closes and the water above the valve is entrapped in the cylinder. During the downward stroke the valve A in the piston opens and the water flows above the piston. The upward stroke is again repeated and the water flows out of the spout as before. In order that the pump may operate, the valve B must not be more than 30 ft. above the surface of the water in the well.

160. Force Pump.—In the force pump (Fig. 123) the suction pipe S with its valve A is just like this portion of the lift pump. An outlet pipe with a valve B is connected to the lower part of the cylinder. As the piston moves downward, the water in the cylinder is forced through the valve B into the delivery pipe D. Raising the piston allows the valve A to open and water to be forced through it by the atmospheric pressure on the water in the well. On the downward stroke of the piston, this valve closes and the valve in the delivery pipe opens.

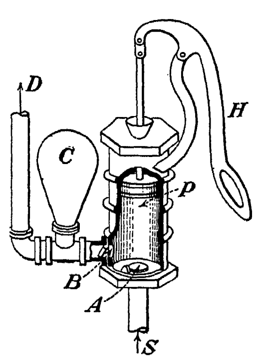


FIG. 123.—A force pump.

In order to obtain a steady stream of water from the pump, an air cushion C is provided. On the downward stroke of the piston, the air in this chamber is compressed by the water flowing into it from the delivery pipe. While the piston is making its upward stroke, the compressed air in this chamber expands and forces water through the delivery pipe. There results in this way a more or less steady stream of water through the delivery pipe. The compressed air in this chamber tends to prevent the jars and shocks which would accompany the starting and stopping of the water if it flowed only on the downward stroke of the piston.

161. Measuring Pumps.—Pumps are often used in measuring the volumes of liquids, especially in the sale of gasoline. The ordinary piston pump can be used for this purpose, if means are provided for defining the length of the stroke and insuring that each stroke of the piston will discharge the same volume of liquid. This requires that the valves be tight and the piston close fitting so as to prevent leakage or slippage of the liquid past the valves or the piston. Such pumps may discharge either on the up-stroke or they may discharge on both the upward and downward strokes. In the former case, they are said to be single-acting pumps and in the latter case they are known as double-acting pumps.

162. Circulation of the Blood.—In the circulation of the blood there is an interesting application of the principle of the force pump. In Fig. 124,

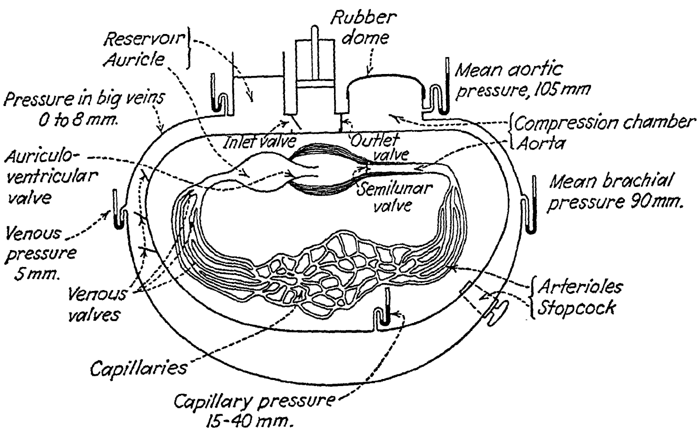


Fig. 124.—Principle of force pump in circulation of the blood.

a simple force pump and its circulating system are compared with the action of the left ventricle, aorta, etc. The manner in which the blood is forced from the left ventricle differs somewhat from the way in which the water is forced out by means of the pump. In the pump, a rigid piston descends within a rigid cylinder and forces the water through the connecting pipes. The force necessary to drive the plunger of the pump is derived from some outside source, external to the pump. In the heart, the elastic muscular walls of the ventricle contract as a whole and force the blood out through the arteries and capillaries. The energy necessary for this muscular work is derived from the potential energy of the materials brought to the heart by the blood.

163. Water Motors.—By employing rotary water motors or water wheels, the energy of water may be used to run various kinds of machinery. Common types of water motors are shown in Fig. 125. The stream of water issues with great velocity in the form of a jet and strikes against the blades or buckets on the rim of the wheel. After impact with these blades or buckets, the water which has lost most of its velocity runs away to the drain or sewer. The impact of the water against the blades or buckets

causes the wheel to rotate. By connecting the shaft of the water motor

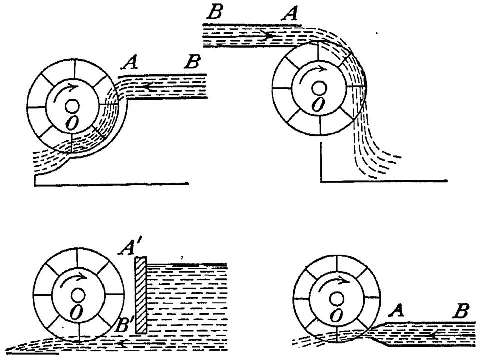


FIG. 125.—Types of water wheels. The energy of the moving water is transferred to the wheels.

to the machinery, the desired power can be developed. Bucket wheels (Fig. 126) developing great horsepower are obtained in this way.

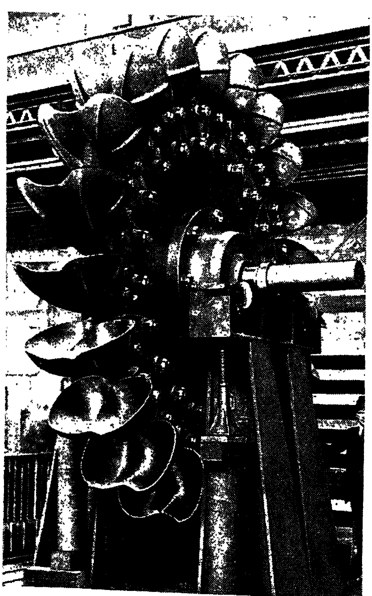


FIG. 126.—Bucket wheel for 30,000 hp. Weight of assembled wheel 25 tons. (*Courtesy Allis-Chalmers.*)

164. A Hydraulic Ram.—When there is present an abundant supply of water which has a fall of only a few feet, it is possible by means of a hydraulic ram to elevate some of this water to a much greater height than that of the source from which the water is obtained. The hydraulic ram operates on the principle that, by allowing a large amount of water to fall through a small distance, it is possible to raise a small amount of water through a much greater distance. The essential parts of a hydraulic ram are shown in Fig. 127. Through the pipe *DE* the water flows from the spring into the reservoir and then out at the waste valve *B* which is normally open. The water flowing through the pipe *DE* and out at the waste valve *B* soon attains considerable velocity. On account of the momentum which it thus possesses, it pushes up with sufficient force to close the waste valve *B*.

When the flow of the water is thus suddenly checked, the force exerted by the water lifts the check valve *A* and water enters the chamber *C*. As the

water continues to flow into the chamber C, the air in this chamber is compressed until the pressure of the air and the weight of the water are sufficient to stop a further flow of water into this chamber and force the water from C through the pipe FG. At this time, the pressure below the waste valve

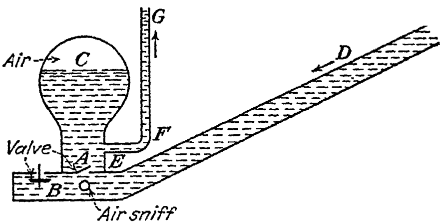


FIG. 127.—Hydraulic ram. The flow is intermittent.

B is sufficiently reduced to allow the valve to open, and the flow of water through the waste valve begins again. The water continues to flow with increasing velocity until it exerts sufficient force to close the waste valve again. The action is then repeated as before. The operation is automatic with an intermittent flow at the delivery pipe.

165. Force Exerted by a Jet.—Suppose that a jet of liquid impinges on a metal plate (Fig. 128) so that the direction of motion of the liquid is perpendicular to the plane of the plate, and assume that, after the impact of the liquid against the plate, the liquid flows off in a direction parallel to the plane of the plate. In such a case, the liquid is completely stopped at impact and its velocity reduced to zero.

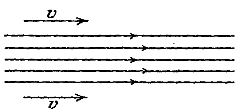


FIG. 128.—Impact of a liquid against a fixed wall.

Let v = the velocity of the liquid before impact.
d = the density of the liquid.
a = the area of the cross section of the jet of liquid.
$v \cdot d$ = the momentum of the jet per unit volume.
$v \cdot a$ = the volume of the liquid reaching the plate per second.
$d \cdot a \cdot v^2$ = the momentum of the liquid reaching the plate per second.

This momentum is destroyed at the plate by the impact of the liquid against it. Hence, $d \cdot a \cdot v^2$ = the rate of change of momentum. According to Newton's second law of motion, the rate of change of momentum is equal to the force which produces it. Hence, the force which the plate exerts on the water and also the force which the water exerts on the plate is

$$F = d \cdot a \cdot v^2.$$

$$\text{Force per unit area} = \text{pressure} = \frac{F}{a} = dv^2.$$

If the surface of the plate is curved as shown in Fig. 129, so that the direction of the motion of the liquid is reversed and the liquid leaves in the

opposite direction to that at which it came up and with a velocity equal to that with which it struck the plate, the momentum of the stream of liquid is at first destroyed by the plate, and then an equal amount of momentum is created in the opposite direction. The total change of momentum is then twice as much as in the preceding case and the force which the jet exerts on the plate is also twice as large as in the preceding case. The force in this case is,

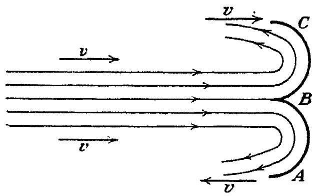


FIG. 129.—Force exerted by a stream of water when its motion is reversed.

$$F' = 2d \cdot a \cdot v^2,$$

and the pressure is

$$p' = \frac{F'}{a} = 2d \cdot v^2.$$

In some forms of water wheels the blades are made in the form of cup-shaped pockets. By means of these cup-shaped blades, the water is given a velocity in the reverse direction on leaving the blades. This reversal in the direction of the velocity of the water increases the force exerted on the water wheel and thus increases its efficiency.

166. Pressure in a Moving Fluid.—In a liquid at rest, the pressure at all points at the same level in connecting vessels is always the same, but in a moving liquid the pressure at two points at the same level may be quite different. In the case of fluid flowing through pipes, the internal friction between the layers of fluid also causes a decrease in the pressure of the fluid as the distance from the source of the fluid increases. Many phenomena of this type have great scientific and economic importance, and it is desirable to discuss the principles governing the flow of fluids in such cases.

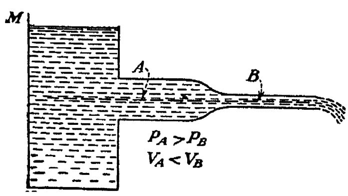


FIG. 130.—Velocity of a liquid in a pipe of variable cross section.

An interesting illustration arises in the case of a liquid flowing through a pipe in which there is a constriction (Fig. 130). Because the cross section of the pipe is smaller at B (Fig. 131) than it is at A or at C, the velocity of the liquid must be greater at B than it is at A or at C, in order that the flow of the liquid through each cross section of the pipe be the same. Consequently, the velocity of the liquid increases in going from the region A to the region B and decreases in going from B to C. The momentum of the liquid per cubic centimeter is greater at B than it is at

A or at C. Hence, there is an increase in momentum in going from A to B, and a decrease in going from B to C. Now according to Newton's second law of motion a force is always necessary to produce a change of momentum. In this case, this force must be the difference in pressure in the liquid at A and at B or the difference in pressure in the liquid at B and at C. Between

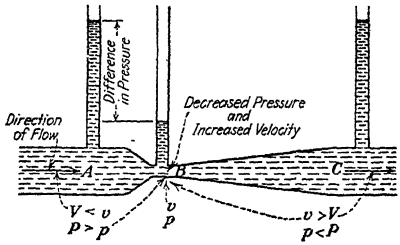


FIG. 131.—Pressure is least where speed is greatest.

A and B the momentum is increased. Hence, the pressure at A must be greater than the pressure at B, so that the difference of pressure will be positive and in the direction from A to B, producing a positive acceleration in the liquid. Similarly, the momentum per cubic centimeter is greater at B than at C, so that a net force must operate in the direction from C to B, producing a decrease in velocity. Hence, the pressure at C is greater than it is at B. In Fig. 131, the respective heights of the liquid at A, B, and C indicate the pressure at these points in the tube.

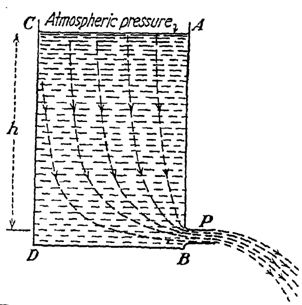


FIG. 132.—Efflux of liquid from an orifice.

167. Torricelli's Theorem.—This theorem deals with the velocity of flow of a liquid through an opening when the pressure on the liquid is due entirely to the weight of the liquid. In Fig. 132, the opening is at a distance h below the surface of the liquid. The kinetic energy per cubic centimeter with which the liquid emerges from the orifice is $\frac{1}{2}\rho v^2$ ergs. The liquid does not have any kinetic energy at the top of the tank. In going from the top to the bottom of the tank, the potential energy decreases by $h\rho g$ ergs per cubic centimeter. This potential energy has

been transformed into kinetic energy. By the law of conservation of energy, the loss in potential energy in this case must just equal the gain in kinetic energy.

$$\rho g h = \frac{1}{2}\rho v^2.$$
$$v^2 = 2gh.$$
$$v = \sqrt{2gh}.$$

Example.—Find the velocity with which water flows from an opening which is 64 ft. below the level of the water.

$$= \sqrt{2 \times 32 \times 64} = 64 \text{ ft. per second.}$$

168. Illustrations of Changing Pressure in Fluids. *a. A Ball in an Air Jet.*—A light ball can be held in position in an air jet as shown in Fig. 133.

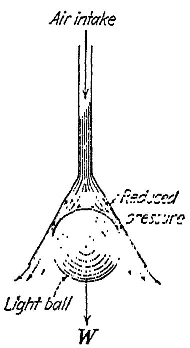


FIG. 133.—A light ball supported by a jet of air. Pressure above the ball is less than below it.

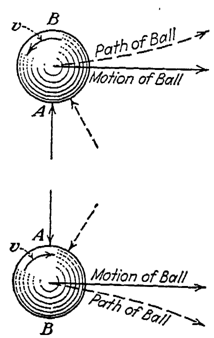


FIG. 134.—Motion of a rotating pitched ball. Pressure on one side less than on the other side.

In the space between the surface of the nozzle and the surface of the ball the pressure is less than atmospheric pressure, and this difference in pressure holds up the ball against the action of gravity.

b. Pitched Baseball.—Suppose a baseball pitcher throws a ball so that it spins around its own axis as it leaves his hand. The pressure on the side A (Fig. 134) is greater than it is on the side B. Because of this fact, the air exerts a force on the ball in the direction of the dotted arrow. As a result of this force, the ball moves in a curve instead of a straight path.

c. Force on Wings of Airplane.—The lifting effect on the wings of an airplane (Figs. 135 and 136) show a most important illustration of changes in pressure due to difference in speed of the air above and below the surface of the wing.

d. The Sprayer.—The forward stroke of the piston (Fig. 137) compresses the air in the cylinder, forcing a stream of air past the end of the tube *D*. The other end of this tube is immersed in the liquid which is to be sprayed. The stream of air flowing past the open end of the tube reduces the pressure

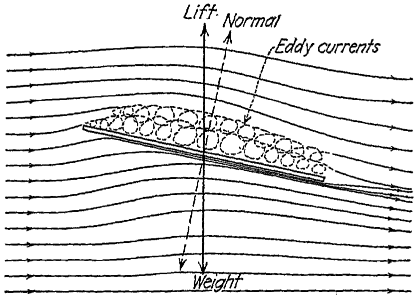


FIG. 135.—Eddy currents increase the frictional resistance.

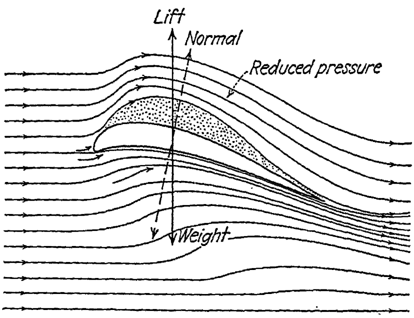


FIG. 136.—Reduced pressure above the wing produces the lifting effect.

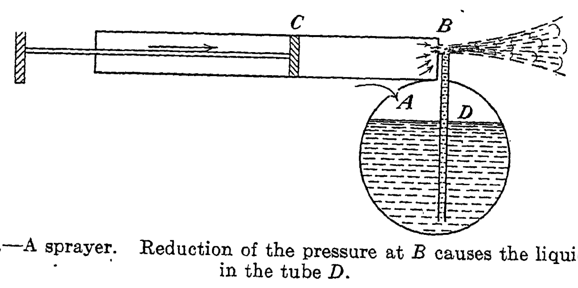


FIG. 137.—A sprayer. Reduction of the pressure at *B* causes the liquid to rise in the tube *D*.

on the liquid in the tube. Atmospheric pressure acting on the surface of the liquid in *A* forces more liquid up into the tube *D* from which it is carried away by the stream of air. A spray results from this mixture of air with the fine particles of liquid.

c. The Langmuir Diffusion Pump.—Mercury vapor diffusing through the constriction at O (Fig. 138) causes a reduction in the pressure which results in the diffusion of the gas from the vessel to be exhausted into the annular space about O. This gas is entrained with the mercury vapor. By means of a water jacket, the mercury vapor is condensed, and the mercury returns to the chamber from which it came and is there reevaporated. This diffusion pump must be backed up by some kind of a mechanical pump (Fig. 139).

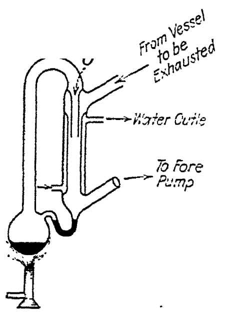


FIG. 138.—Mercury diffusion pump.

Problems

1. Water flows at the rate of 1 liter per second from a hole at the bottom of a tank in which the water is 2 m. deep. Find the rate at which the water would escape if an additional pressure of 10 kg. per square centimeter were applied to the surface of the water.

2. Two pipes are connected by a third pipe which has a smaller cross section. The pressure is 120 lb. to the square inch in the larger pipe and

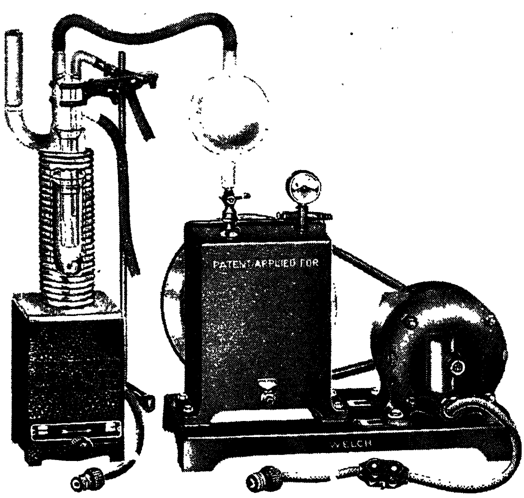


FIG. 139.—Oil fore pump used with diffusion pump. (*Courtesy Welch Manufacturing Company.*)

70 lb. per square inch in the smaller pipe. The pipes are horizontal. Neglecting friction, calculate the speed of flow in the smaller pipe if speed of flow in the larger pipe is 8 ft. per second.

3. Find the velocity with which water in a tank will be forced through an orifice in the side of the tank if the orifice is 1 ft. below the surface of the water in the tank, and there is a pressure of 125 lb. to the square inch on the surface of the water in the tank. Take atmospheric pressure as 14.7 lb. per square inch.

4. Find the velocity of efflux of water from a hole in the side of a tank when the water in the tank is 24 ft. above the hole. If the area of the hole is 1.5 sq. in., how many cubic feet will be discharged each second?

5. Compute the pressure in a tank of water which will cause the water flowing from an orifice in the side of the tank at a height of 7.5 m. from the ground to strike the ground at a distance of 22.5 m. from the foot of the tank.

6. At what rate will water discharge from a tank, which is 12 ft. deep, through an orifice which is 0.5 in. in diameter?

7. A tube having a cross section of 5 sq. cm. and a length of 400 cm. is inserted in the top of a cylindrical box 150 cm. high with radius of 25 cm. The tube and the box are filled with water. Find the force on the upper and lower faces of the cylindrical box.

CHAPTER XIII

ELASTICITY AND STRENGTH OF MATERIALS

169. Stress.—When a cable supports an elevator or a belt transmits power from one pulley to another, there is a pull applied to each end which tends to pull one part away from the neighboring parts. In such a case the belt or cable is sustaining a stress, and if this stress is sufficiently large the belt or cable breaks. When one end of a post is placed on a foundation and a load is placed on its top, the post resists a push at each end which tends to shorten it and may crush it. The first stress is a tensile stress and the second is a compressive or crushing stress. There is another kind of stress by which a screw is broken when trying to twist it into hard wood with a screw driver. In this case the molecules are so displaced with respect to each other that they no longer hold together. When a beam is bent or a rod twisted, one layer of molecules is sheared over a neighboring layer and made to occupy a new position. Stresses of this kind are called shearing stresses. When equal force is applied on all sides of a body, its volume is changed, but its shape remains the same. The force which produces the change in volume is also called a stress.

The stress is defined to be the force per unit area and is found by dividing the total force by the area to which the force is applied.

170. Strain.—The term strain is applied to any change occurring in the dimensions or shape of a body when forces are applied to it. A wire from which a weight is hanging becomes longer as the load is increased. The strain in this case is called longitudinal strain and is calculated by dividing the change in length by the original length. It is, therefore, the change in length per unit length.

Let L = the original length of the wire.

l = the change in length due to the added load.

l/L = the longitudinal strain = change in length per unit length.

Example.—A piece of brass wire 27.1 cm. long is found to be stretched 0.133 cm. by a load of 0.5 kg. What is the longitudinal strain?

$$\text{Longitudinal strain} = \frac{\text{increase in length}}{\text{original length}}$$
$$= \frac{0.133 \text{ cm.}}{27.1 \text{ cm.}} = 0.0049 \text{ cm. per centimeter of length.}$$

A body which is subjected to uniform normal stress all over its surface changes its volume. For example, when pressure is applied to an enclosed volume of a fluid, the volume of the fluid is decreased. This change of volume with increase of pressure is more noticeable in the case of gases than in the case of liquids. This kind of strain is known as **volumetric strain.**

Let V = the original volume of the liquid, and
v = the change in volume produced by the application of pressure.

Then v/V = volumetric strain or change in volume per unit volume.

Example.—A steel cylinder having a volume of 900 cu. in. is subjected to a hydrostatic pressure and its volume decreases 0.57 cu. in. Find the volumetric strain.

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$
$$= \frac{0.57 \text{ cu. in.}}{900 \text{ cu. in.}} = 0.000633.$$

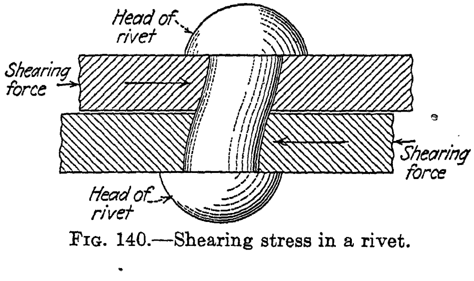


FIG. 140.—Shearing stress in a rivet.

Shearing strain occurs when a body is subjected to a shearing stress (Fig. 140). If one cover of a thick book is held firmly on the table, and a force parallel to the top of the table is applied to the other cover, the shape of the book is changed. Its thickness remains the same. If a shearing strain is applied to a rectangular

block, the shearing strain produces the result shown in Fig. 141. In cases of this sort, the shearing strain is measured by determining the angle through which a vertical line in the body has been rotated by the shearing force. This angle is denoted by x in Fig. 141, and it is measured in radians.

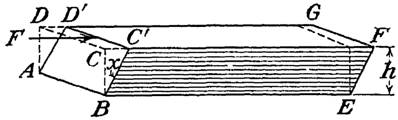


FIG. 141.—Shearing stress in a bar of rectangular cross section.

Since $\tan x = CC'/BC$ and since for small angles $\tan x$ is approximately equal to x,

$$\text{Shearing strain} = x = \frac{CC'}{BC}, \text{ approx.}$$

Example.—A rectangular block of metal 10 cm. high rests on a horizontal table. A force which is parallel to the surface of the table is applied to the upper surface of the block and produces a displacement of 1 mm. in the face $C'G$ (Fig. 141). What is the angle of shear?

$$\text{Shearing strain} = x = \frac{0.1}{10} \quad 0.01 \text{ radian.}$$

171. Modulus of Elasticity.—Elasticity is that property of a body by virtue of which it tends to return to its original shape and dimensions when the distorting forces have been removed. In many kinds of material, this return is virtually perfect if the material has not been loaded beyond a certain limit called the elastic limit. If the body is loaded beyond this limit, it does not completely recover its original shape and dimensions. In some cases, the return of the body to its original condition is delayed for some time especially after long-repeated distortions.

The modulus of elasticity is defined as

$$\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}}.$$

The stress as already defined is taken to mean the force per unit area producing the deformation, and the strain means the fractional deformation produced by the stress.

172. Hooke's Law.—If observations are made on the change in the length of a wire produced by hanging various loads on it,

it is found that the change in length is proportional to the load which produced it. When one end of a rod is held firmly and the other end twisted, the angle through which the end is twisted with respect to the fixed end is proportional to the moment of force which produced the twist. In like manner, experiments show that beams are bent and springs are stretched amounts which are proportional to the loads which were applied. These observations may be stated in the form of a single law which was discovered by Hooke and bears his name. Hooke's law states that **strains are proportional to the stresses producing them.**

$$\text{Stress} = \text{constant} \times \text{strain.}$$

This law is obeyed by many substances if they are not strained beyond their elastic limit.

173. Young's Modulus of Elasticity.—Where a rod or wire (Fig. 142) is pulled or pushed so that there is a longitudinal stress and a longitudinal strain, the modulus of elasticity is called Young's modulus of elasticity. It is defined as follows:

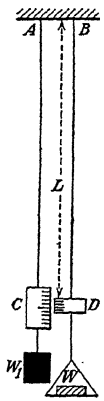


FIG. 142.—Young's modulus of elasticity. The elongation is proportional to the applied force.

Let W = the pull or weight applied to the wire.
a = the cross section of the wire.
L = the original length of the wire.
l = the change in length produced by weight W.

Then, the longitudinal stress $= W/a =$ force per unit area on cross section of the wire.

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{l}{L},$$

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \left(\frac{W}{a}\right) \div \left(\frac{l}{L}\right) = \frac{W}{a} \times \frac{L}{l}.$$

(For numerical values see table, page 767.)

Example.—A wire 120 in. long with a cross section of 0.125 sq. in. hangs vertically. When a load of 450 lb. is applied to the wire, it stretches 0.015 in. Find Young's modulus of elasticity.

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{W/a}{l/L} = \frac{450/0.125}{0.015/120} = 2.88 \times 10^7 \text{ lb. per square inch.}$$

174. Volume Elasticity.—When a body like a piece of iron is immersed in a liquid so that a uniform pressure is applied to its surface, there is a change in volume and its volumetric elasticity must be considered.

Let p = the increase of pressure applied.
v = the change in volume produced by this pressure.
V = the original volume.

$$\begin{array}{l}\text{Modulus of} \\ \text{volumetric elasticity}\end{array} = \frac{\text{volumetric stress}}{\text{volumetric strain}}$$

$$= \frac{\text{force per unit area}}{\text{change in volume per unit volume}}.$$

$$E = \frac{p}{v/V}. \quad \text{(Appendix E-2.)}$$

(For numerical values see table, page 767.).

Example.—The modulus of volume elasticity of copper is 6,600,000 lb. per square inch. A sphere of copper having a volume of 100 cu. in. is subjected to a pressure of 1,000 lb. per square inch. Find the change in volume which takes place.

$$E = \frac{\text{change in pressure per unit area}}{\text{change in volume per unit volume}} = \frac{p}{v/V}$$

$$E = 6{,}600{,}000 \text{ lb. per square inch.}$$

$$p = 1{,}000 \text{ lb.}$$

$$6{,}600{,}000 \quad \frac{1{,}000}{v/100}.$$

$$v = \frac{1}{66} = 0.016 \text{ cu. in.}$$

175. Limit of Elasticity.—A body which has been deformed and then released will return to its original size and shape unless the stress has exceeded a certain limit. This maximum stress for which a substance will completely recover its original size and shape is called the **limit of elasticity.** It differs widely for different substances. It is high for a substance like steel and low for a substance like lead. Figure 143 shows the relation between the tension and the elongation above and below the elastic limit.

When a metal rod is stretched beyond its limit of elasticity, and the stress is still further increased, a stage is reached at which the

rod begins to stretch rapidly even if the stress is somewhat decreased. This stage is called the **yield point.** Although the metal is cold, it behaves as if it were in a semifluid state.

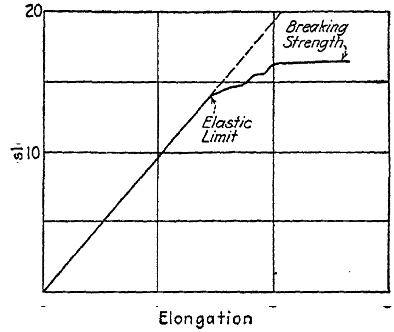


FIG. 143.—Relation between tension and elongation of a wire. Near the elastic limit Hooke's law fails.

176. Elasticity in Tissues and Bones.—The elasticity of connective tissues plays an important part in the body. It offers a means of resistance to permanent distorting force, such as muscular tension and the force of gravity. The elasticity of the disks between the vertebrae assists in keeping the body in an erect position. The elasticity of the ribs restores the chest wall to its normal position when the muscles relax in respiration. In the circulation of the blood an intermittent movement is transformed into a continuous movement of an elastic medium.

The head of the femur (Fig. 144) furnishes a good illustration of shearing stress and lines of stress in the bones of the body. Two kinds of stress, tension and compression, are present in this case. The outer or convex side of the bone has to resist tension, while the inner or concave portion below the head carrying the load sustains compression. The lines of stress which run along the outer convex side and curve downward as the head of the bone is reached are lines of tension. The other system of curves which start from the inner side of the shaft and spread outward so that they are concave downward are lines of compression. In the compact tissue of the shaft, the lines of compression and tension run parallel to each other. The central portion of the shaft bears no strain and is, therefore, hollow.

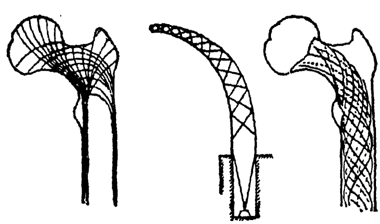


FIG. 144.—Stresses in bones.

177. Stiffness and Strength of Beams.—It is important to be able to calculate the bending (Fig. 145) which will be produced

in a beam by a given load. Experiments on beams of different size and shape have shown that the bending for a given load depends on the length of the beam, on its breadth, and on its depth. The relation between these dimensions and the stiffness of a beam of rectangular cross section can be expressed as follows:

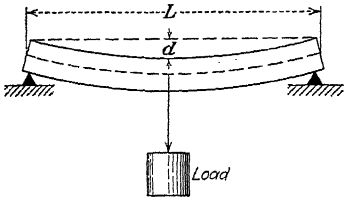


FIG. 145.—Bending of beams. The deflection is proportional to the applied force.

$$\text{Stiffness factor} = \frac{\text{breadth} \times (\text{depth})^3}{(\text{length})^3}.$$

Example.—Find the stiffness factor of a rectangular beam which is 20 ft. long, 4 in. broad, and 6 in. deep.

$$\text{Stiffness factor} = \frac{4 \times (6)^3}{(12 \times 20)^3} = \frac{864}{13{,}824{,}000} = 0.000063.$$

The stiffer of two beams is not necessarily the stronger. This means that the bending of a beam does not give at once the true measure of its strength. The strength does, however, depend on the same dimensions on which the stiffness depends, but it depends on these dimensions in a different way from that in which the stiffness depends on them.

$$\text{Strength factor} = \frac{\text{breadth} \times (\text{depth})^2}{\text{length}}$$

Example.—Find the ratio of the strengths of the following beams:

Beam A: length 20 ft., breadth 3 in., depth 6 in.
Beam B: length 30 ft., breadth 6 in., depth 4 in.

$$\frac{\text{Strength of } A}{\text{Strength of } B} = \frac{3 \times 6^2 \div 20}{6 \times 4^2 \div 30} = \frac{27}{16}.$$

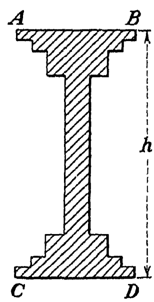


FIG. 146.—An I-beam gives greatest strength and stiffness for least weight.

178. Cross Sections of Beams.—Where rectangular wooden beams are used, they are placed so that the depth is greater than the breadth. Both the strength and the stiffness are proportional to the breadth, but the stiffness increases as the cube of the depth and the strength increases as the square of the depth. In order to place the beam so that the stiffness and the strength will be as large as possible, the depth must be made as great as possible. In steel beams, in order to get the greatest stiffness and strength for a given weight of beam, account is taken of the fact that, in bending, the top layer of a beam is shortened and must resist compression, and the

lower layer is lengthened and must resist a tension. The central layer, however, remains the same length. To make the beam as strong or as stiff as possible for a given amount of material, as much material as possible should be put in the upper and lower layers and as little as possible near the middle. For this reason, iron beams are designed as indicated in Fig. 146.

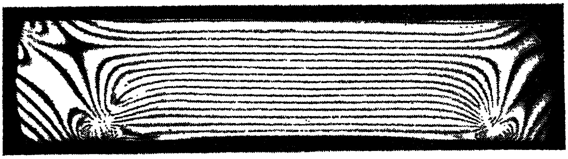


FIG. 147.—Distribution of stresses in a beam supported at both ends. Photographed with polarized light. (*Courtesy M. M. Frocht, Carnegie Institute of Technology.*)

They are called I-beams and give the greatest strength and stiffness for a given amount of iron.

The distribution of the stresses can be studied by making a model of the member out of transparent bakelite and making photographs by means of polarized light. These photographs show the distribution of the stresses. Figure 147 is a case of pure bending in a beam supported at both ends.

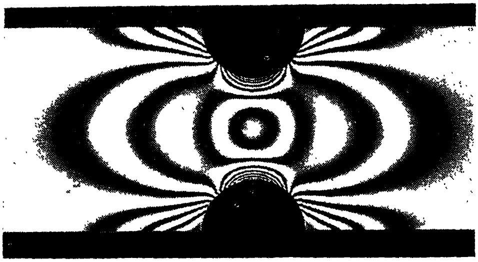


FIG. 148.—Tensile stresses in a member with semi-circular grooves. Photographed by means of polarized light. (*Courtesy M. M. Frocht, Carnegie Institute of Technology.*)

Figure 148 shows the stresses produced by tension in a member in which there are semicircular grooves.

Problems

1. The contraction per unit volume of water is 3.5×10^{-6} for a pressure increase of 1 lb. per square inch. Find the change in volume of the water in a tube 1 yard long and 2 sq. in. cross section, when a pressure of 600 lb. per square inch is applied to the water.

2. A brass wire with a cross section of 1.2 sq. mm. is 180 cm. long when supporting a load of 3 kg. How much longer will it be when the load is increased to 5 kg.?

3. A steel cable 200 ft. long is to be thick enough so that an additional load of 200 lb. on the cable will increase the length by no more than 0.1 in. What must be the diameter of the cable?

4. A wire 2 m. long, with a diameter of 0.8 mm., is elongated by 0.4 mm. when a 2-kg. weight is hung on it. What is Young's modulus for the material?

5. How much will a copper wire 15 m. long stretch when a load of 1.8 kg. is applied to it, if the cross section has an area of 0.025 sq. cm.? Young's modulus for copper is 1.2×10^{12} dynes per square centimeter.

6. An elevator must carry a maximum load of 4,000 lb., and its maximum acceleration is to be 6 ft. per second per second. Find the cross section of a cable which is necessary to operate this elevator, if the safe working stress of a steel cable is taken as 15,000 lb. per square inch.

7. If the density of water at the surface of a lake is 0.99, find the depth at which the density will be 1.00.

8. A hollow cast-iron pipe must support a load of 4,000 lb. without decreasing in length more than 0.02 cm. The length of the pipe is 90 cm. What must be the inner radius of the pipe if the outer radius is 8 cm.?

CHAPTER XIV

ROTARY MOTIONS

179. Circular Measure of Angles.—Instead of measuring angles in degrees, it is convenient in science to define a new unit of angular measure. **This unit is called a radian.** On the circumference of a circle (Fig. 149) lay off an arc AC which is equal in length to the radius OA. The angle AOC subtended by the arc AC is defined as a radian. **Or a radian is an angle which subtends an arc equal to its radius.** Since the radius can be applied to the circumference 2π times, it follows that

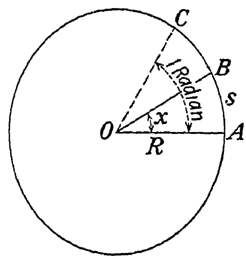


Fig. 149.—Definition of a radian. Angle at O is a radian if $OA = AC$.

$$360 \text{ deg.} = 2\pi \text{ radians.}$$
$$1 \text{ radian} = 57.3 \text{ deg., approx.}$$

From this definition of a radian we have the relation

$$x = \frac{s}{R},$$

where x = the angle in radians.
s = the arc.
R = the radius.

Example.—Find the number of radians in an angle of 45 deg.

$$\text{Radians} = \frac{\text{number of degrees}}{57.3}$$
$$= \frac{45 \text{ deg.}}{57.3} = 0.78 \text{ radian.}$$

180. Angular Velocity.—Let one end of a line OA of Fig. 150 be fixed and then let the line revolve in a plane about this fixed end. The rate at which the line rotates is called its **angular velocity.** (Appendix D-6.) The number of radians swept out by the line per unit of time gives the angular velocity. If the rate of rotation of the line is constant, the angular velocity

is constant and is equal to the angle through which the line turns in unit time. The angular velocity may also be measured in revolutions per second or per minute. The angular velocity in

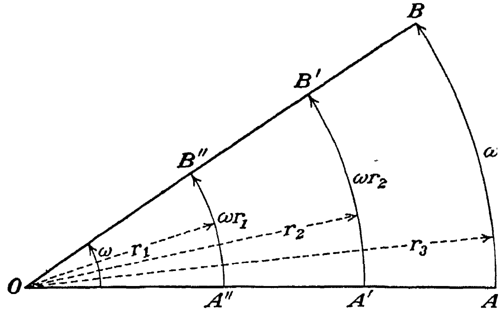


FIG. 150.—Relation between angular velocity and linear velocity. Angular velocity times radius gives linear velocity.

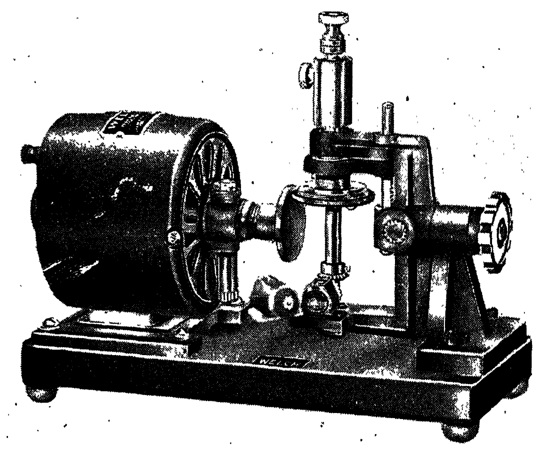


FIG. 151.—Change of speed along the radius of a wheel. The greater the distance from the axis, the greater the linear speed. (*Courtesy Welch Manufacturing Company.*)

radians is then given by multiplying the number of revolutions per unit of time by 2π, the number of radians in one revolution.

Example.—A wheel makes 5 revolutions per minute. What is its angular velocity?

Angular velocity $= 2\pi \times$ number of revolutions $= \omega$.

$$\omega = 2\pi \times 5 = \frac{220}{7} \text{ radians per minute.}$$

The linear velocity of a point on the line OA increases as the distance from the axis of rotation increases. Thus the linear velocity of A is largest and equal to ωr_3 while that of A' is less and equal to ωr_2. This fact is made use of in a number of devices for the regulation of speeds in machines. Figure 151 shows a device for regulating the rate at which a shaft revolves. It is an application of the fact that the linear speed increases as the distance from the axis of rotation increases.

181. Angular Velocities in Pulleys and Wheels.—When a belt is stretched around two pulleys, as in Fig. 152 or 153, the angular velocities of the pulleys

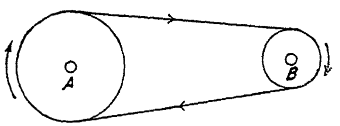


FIG. 152.—Angular velocity of pulleys.

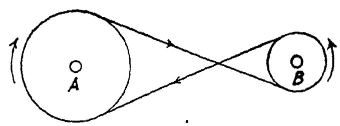


FIG. 153.—Reversal of direction of rotation of pulleys.

are not the same unless the pulleys have the same radii. In Fig. 152 the directions of rotation of the pulleys are the same, but in Fig. 153 their directions of rotation are opposite. The linear velocities of points on the circumferences of both pulleys are equal to the linear velocity of the belt.

If,

V = the linear velocity of the belt,
ω_A = the angular velocity of A.
ω_B = the angular velocity of B.
R_A = the radius of A.
R_B = the radius of B.

$$\omega_A = \frac{V}{R_A}.$$

$$\omega_B = \frac{V}{R_B}.$$

$$\frac{\omega_A}{\omega_B} = \frac{R_B}{R_A}.$$

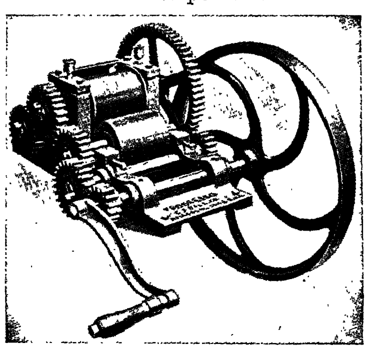


FIG. 154.—The angular velocities of toothed wheels are inversely proportional to the number of teeth.

Hence, the angular velocities of the pulleys are inversely proportional to their radii. In this way, it is possible to increase or decrease the angular speed at which one of the pulleys is driven.

By means of toothed wheels (Figs. 154 and 155) the same kind of alteration of angular velocities may be produced. In this case there can be no slipping, and the angular velocities of the wheels are inversely proportional to the number of teeth.

$$\frac{\omega_A}{\omega_B} = \frac{N_B}{N_A}.$$

182. Angular Acceleration.—**The rate at which the angular velocity changes is called the angular acceleration.** (Appendix D-7.) It is the increase or decrease in angular velocity per unit of time. It is related to the angular velocity in the same way in which the linear acceleration is related to linear velocity. In an angular acceleration as in linear acceleration it is necessary to specify two units of time. One of these units gives the unit of time in which the angular velocity is measured, and the other gives the unit of time used to measure the change in the angular velocity. Ordinarily, the same unit of time is used in both cases.

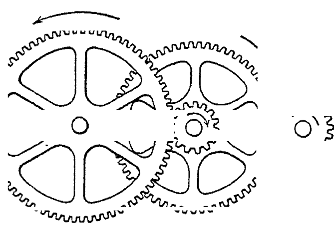


FIG. 155.—By means of toothed wheels angular velocities can be increased or decreased.

Example.—At a certain instant the angular velocity of a wheel is 30 radians per second. In 10 sec. the angular velocity has become 50 radians per second. What is the angular acceleration?

$$\begin{aligned}\text{Angular acceleration} &= \frac{\text{change in angular velocity}}{\text{time}}\\ &= \frac{50 \text{ radians per second} - 30 \text{ radians per second}}{10 \text{ sec.}}\\ &= 2 \text{ radians per second per second.}\end{aligned}$$

183. Equations of Angular Motion.—The equations for angular motion have the same form as the corresponding equations for rectilinear motion.

1. If a line revolves with a uniform angular velocity ω radians per second, the angle θ which it describes in t sec. is

$$\theta = \omega t \text{ radians.}$$

2. If the line starts to revolve from rest with a uniform angular acceleration A radians per second per second, the angular velocity ω at the end of t sec. is

$$\omega = At \text{ radians per second.}$$

The average angular velocity

$$= \frac{\text{initial angular velocity} + \text{final angular velocity}}{2},$$

$$\omega_{\text{ave.}} = \frac{0 + \omega}{2} = \frac{0 + At}{2} = \frac{1}{2}At.$$

Angle swept out in t sec. = average angular velocity × time.

$$\theta = \frac{1}{2}At \times t = \frac{1}{2}At^2 \text{ radians.}$$

Since $t = \frac{\omega}{A}$ and $t^2 = \frac{\omega^2}{A^2}$,

$$\theta = \frac{1}{2}A\frac{\omega^2}{A^2} \quad \frac{\omega^2}{2A}.$$

$$\omega^2 = 2A\theta.$$

3. If the line has an initial angular velocity ω_0 and an angular acceleration A, its velocity at the end of t sec. is

ω = initial angular velocity ± change of angular velocity.
$\omega = \omega_0 \pm At$ radians per second.

The average angular velocity is

$$\omega + \omega_0 \quad = \omega_0 \pm \frac{1}{2}At.$$

Angle swept out by the radius = average angular velocity × time.

$$= \left(\omega_0 \pm \frac{1}{2}At\right)t = \omega_0 t \pm \frac{1}{2}At^2.$$

Substituting for t,

$$\omega^2 - \omega_0^2 = 2A\theta.$$

Example.—A wheel starts from rest and acquires a speed of 450 radians per second in 15 min. What is the angular acceleration? Through how many radians did the wheel turn?

$$\text{Angular acceleration} = \frac{\text{change of angular velocity}}{\text{time}}.$$

$$A = \frac{\omega}{t} \text{ radians per second per second.}$$

$$\frac{450 \text{ radians}}{15 \times 60 \text{ sec.}} \quad 0.5 \text{ radian per second per second.}$$

Angle swept out by radius = average angular velocity × time.

$$\theta = \frac{1}{2}At^2$$

$$0.5 \quad \times (900)^2 = 202{,}500 \text{ radians.}$$

Example.—A wheel has an initial angular velocity of 60 radians per second and an angular acceleration of 0.2 radian per second per second. What is its angular velocity at the end of 50 sec.?

Final angular velocity = initial angular velocity + increase of angular velocity.

$$\omega = \omega_0 + At.$$
$$\omega = 60 + 0.2 \times 50 = 70 \text{ radians per second.}$$

184. Relation of Torque to Angular Acceleration.—In order to produce linear acceleration, it is necessary to apply a force to the body and the greater the force the greater the acceleration. To produce angular acceleration, it is necessary to apply a torque to the body. (For definition of *torque* see page 41.) The greater the torque, the greater is the angular acceleration which is produced. For a particular body rotating about a fixed axis, it is found that the angular acceleration is proportional to the torque which produces it. This factor of proportionality by which the angular acceleration must be multiplied in order to give the torque is a measure of the opposition of the body to being set in rotation. It is analogous to the mass of a body, which is a measure of the opposition of a body to being set in translation. The relation between the torque and the angular acceleration can be expressed by the equation

$$T = IA,$$

where A = the angular acceleration in radians per second per second.

T = the torque (force in poundals and distance in feet, or force in dynes and distance in centimeters).

I = the factor of proportionality called the rotary inertia.

This relation is known as Newton's second law for rotary motions. **It states that the angular acceleration is proportional to the torque.** (Appendix D-8.)

Example.—A grindstone has a rotary inertia of 300 lb.-ft.² Find the torque necessary to produce an angular acceleration of 3 radians per second per second.

Torque = rotary inertia × angular acceleration.

$$T = I \times A.$$
$$T = 300 \text{ lb.-ft.}^2 \times 3 \text{ radians per second per second.}$$
$$T = 900 = 900 \text{ poundals-feet.}$$

185. Rotary Inertia.—Experiment has shown that the opposition of a body to being set in translation is proportional to the mass of the body and does not depend on anything else. It is, however, found that the opposition which a body offers to being set in rotation or accelerated about a fixed axis depends not only on the mass but also on the way in which this mass is distributed about the axis. This opposition which is called the **rotary inertia or the moment of inertia is proportional to the mass and to the square of the distance of the mass from the axis of rotation.** For this reason, when it is desired to make the rotary inertia of a flywheel of given mass as large as possible, the mass

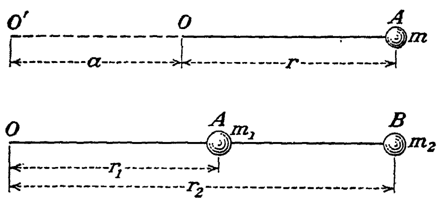


FIG. 156.—The rotary inertia about an axis is proportional to the product of the mass and the square of the distance from the axis.

of the wheel is concentrated in the rim. When the mass is concentrated near the axis, the tendency of the wheel to continue in motion, or its resistance to being set in motion, is much less. In order to calculate the rotary inertia, multiply each mass by the square of its distance from the axis of rotation and then add together all of these products.

Thus, the rotary inertia of a particle A of mass m about O (Fig. 156) is $I = mr^2$, where r is the distance of O, the axis of rotation from the center of the particle. If the axis of rotation is moved to O', a distance a, the rotary inertia of the particle about O' is

$$I' = (a + r)^2 m.$$

Since $(a + r)^2$ is greater than r^2, the rotary inertia of the particle is larger about the axis through O' than it is about the axis through O. Hence, the farther the axis is from the mass, the greater is the rotary inertia.

If two masses A and B (Fig. 156) are fastened together by a weightless rod, the rotary inertia of these masses about an axis

through O is

$$I = m_1r_1^2 + m_2r_2^2,$$

where r_1 and r_2 are the respective distances of A and B from O. In like manner, the rotary inertia of any number of masses is

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \text{etc.}$$

where r_1, r_2, r_3, etc., are the distances of the masses from the axis of rotation.

The rotary inertia of a disk of mass M about the axis through its center is

$$I = \frac{1}{2}MR^2, \text{ (Appendix E-3)}$$

where R = radius of disk.

The rotary inertia of a thin rod of mass M about an axis through its center, perpendicular to the axis of the rod, is

$$I = M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

where l = length, and r = radius of rod.

Example.—Find the rotary inertia of a mass of 10 lb. and another mass of 15 lb. about an axis of rotation which is 3 ft. from the 10-lb. mass and 4 ft. from the 15-lb. mass.

Rotary inertia equals sum of products of the masses by the square of the respective distances from the axis.

$$\begin{aligned} I &= m_1r_1^2 + m_2r_2^2. \\ m_1 &= 10 \text{ lb.}, r_1 = 3 \text{ ft.} \\ m_2 &= 15 \text{ lb.}, r_2 = 4 \text{ ft.} \\ I &= 10 \times (3)^2 + 15 \times (4)^2. \\ &= 90 + 240 = 330 \text{ lb.-ft.}^2. \end{aligned}$$

Example.—Find the moment of inertia of a circular disk which is rotating about an axis through its center perpendicular to the plane of the disk. The mass of the disk is 1 kg. and its radius is 5 cm.

$$MR^2$$

$$= \frac{1}{2} \times 1{,}000 \text{ g.} \times 25 \text{ cm.}^2 = 12{,}500 \text{ g.-cm.}^2.$$

186. Kinetic Energy of Rotation.—The work required to set a body in translation equals the kinetic energy of translation of the body. In like manner, work is required to set a body in rotation, and this work is stored up in the body as kinetic energy

of rotation. To find the expression for the kinetic energy of rotation in terms of the angular velocity and the rotary inertia of the body, consider Fig. 157, which represents a body rotating with an angular velocity ω about an axis through O perpendicular to the plane of the paper. Consider the particle at A having the mass m_1 and the linear velocity v_1. (Appendix D-9.)

$$\text{Kinetic energy of particle at } A = \frac{m_1 v_1^2}{2}.$$

Now,
$$v_1 = \omega r_1.$$
$$v_1^2 = \omega^2 r_1^2.$$

$$\text{Kinetic energy of particle at } A = \frac{m_1(\omega r_1)^2}{2} = \frac{\omega^2}{2} \times m_1 r_1^2.$$

For other particles m_2, m_3, m_4, etc., moving with velocities v_2, v_3, v_4, etc., similar expressions are found. Hence,

Total kinetic energy
$$= \frac{\omega^2}{2} m_1 r_1^2 + \frac{\omega^2}{2} m_2 r_2^2 + \frac{\omega^2}{2} m_3 r_3^2 + \cdots$$
$$= \frac{\omega^2}{2}(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots).$$

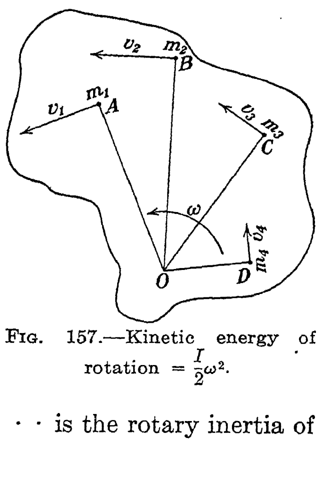


FIG. 157.—Kinetic energy of rotation $= \frac{I}{2}\omega^2$.

Now, $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots$ is the rotary inertia of the body, so that

$$\text{Kinetic energy of rotation} = \frac{\omega^2}{2} I \text{ (ergs or foot-poundals).}$$

(Appendix D-9.)

Example.—Find the kinetic energy of rotation of a wheel which has a moment of inertia of 10,000 lb.-ft.² when it makes 240 revolutions per minute.

$$\text{Kinetic energy of rotation} = \frac{I\omega^2}{2}.$$
$$I = 10{,}000 \text{ lb.-ft.}^2 \text{ units.}$$
$$\text{Kinetic energy} = \frac{10{,}000\left(\frac{240}{60} \times 2\pi\right)^2}{2} = 30.6 \times 10^5 \text{ (foot-poundals).}$$

Example.—What power is delivered by an electric motor which makes 60 revolutions per second and develops a torque of 9 lb.-ft.?

$$\begin{aligned}\text{Power} &= \text{torque} \times \text{angular velocity}\\ &= 9 \text{ lb.-ft.} \times 60 \times 2\pi\\ &= 3{,}394 \text{ ft.-lb. per second}\\ &= \frac{3{,}394}{550} = 6.1 \text{ hp.}\end{aligned}$$

187. Combination of Energy of Translation and Energy of Rotation.—Consider two cylinders of equal diameter and equal mass. Suppose that each of them is made of a certain mass of lead and the remainder of wood. In one case, the lead is put near the axis of the cylinder, and, in the other case, it is near the circumference of the cylinder as shown in Fig. 158. Since the lead is much denser than the wood, the cylinder with the lead near the axis has much less rotary inertia than the cylinder with the lead near its circumference.

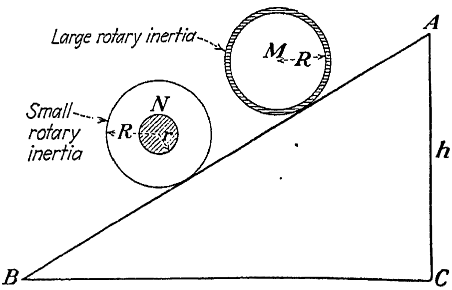


FIG. 158.—The angular velocity is greater for cylinder with lead near the axis.

Let the two cylinders be allowed to roll down an inclined plane, starting from rest at the same height on the plane. Since the cylinders have equal weights, they have the same amount of potential energy. On the way down the plane, this potential energy is changed into kinetic energy of translation and kinetic energy of rotation. At the bottom of the plane, all the potential energy has been transformed into kinetic energy. At the bottom of the plane, let

ω_1 = the angular velocity of the cylinder with the lead near its axis.
v_1 = the linear velocity of the cylinder with the lead near its axis.
ω_2 = the angular velocity of the cylinder with the lead near its circumference.
v_2 = the linear velocity of the cylinder with the lead near its circumference.
I_1 = the moment of inertia of the cylinder with the lead near its axis.
I_2 = the moment of inertia of the cylinder with the lead near its circumference.
M = the mass of each of the cylinders.
h = the height through which the cylinders have fallen.

$$Mgh = \frac{1}{2}Mv_1^2 + \frac{1}{2}I_1\omega_1^2.$$

$$Mgh = \frac{1}{2}Mv_2^2 + \frac{1}{2}I_2\omega_2^2.$$

If the two cylinders arrived at the bottom of the plane at the same time, that is, if $v_1 = v_2$, they would also have the same angular velocity because the radius is the same in each case; but, if the cylinders have the same angular velocity, the cylinder with the lead near the circumference would have the greater amount of kinetic energy of rotation because it has the greater moment of inertia. On the assumption that the two cylinders arrive at the bottom of the plane at the same time with the same linear speed, they would have the same kinetic energy of translation, but the cylinder with the lead near the circumference would have the greater energy of rotation. However, the energy of the two cylinders must be the same at the bottom of the plane, and this energy must be just equal to the potential energy at the top of the plane. Hence, the two cylinders cannot arrive at the bottom of the plane at the same time. The kinetic energy of rotation of the cylinder with the lead near its circumference is greater than the kinetic energy of rotation of the cylinder with the lead near its axis, and conversely the kinetic energy of translation of the cylinder with the lead near its circumference is less than the kinetic energy of translation of the cylinder with the lead near its axis. Hence, the cylinder with the lead on the outside should be expected to lag behind the one with the lead near its axis, a conclusion easily verified by experiment.

Problems

1. Express in radians the angle through which the earth rotates on its axis during 90 min.

2. A flywheel has a diameter of 12 ft. Through what distance does a point on the circumference travel while the wheel is describing 2.5 radians?

3. A small motor starts from rest and attains its rated speed of 1,800 revolutions per minute in 3 sec. Calculate the angular acceleration assuming it to be uniform.

4. An electric generator turning at the rate of 3,000 revolutions per minute is suddenly short-circuited so that its angular velocity drops 10 per cent within 0.25 revolution. Find the average angular velocity and the angular acceleration.

5. A ball rolling down a slope increases its angular speed at the rate of 0.4 revolution per second per second. If it starts from rest, what will be its angular velocity in radians per second at the end of 1.5 min.?

6. The rotary inertia of a body about its axis of rotation is 80,000 kg.-cm.^{2}. A torque of 300 kg.-cm. is applied to it; what will be the angular velocity at the end of 1.25 min.?

7. A turbogenerator with a rotary inertia of 10,000,000 English units rotates at the rate of 40 revolutions per second. The power and load are shut off simultaneously, and friction stops the rotation in 30 sec. Find the torque due to friction.

8. Calculate the moment of inertia of a wheel which has a kinetic energy of 14,000 ft.-lb. when it is making 600 revolutions per minute.

9. A flywheel has a mass of 4 tons and a radius of 5 ft. When it is turning at the rate of 90 revolutions per minute, how many horsepower can be

obtained from it during a quarter of a revolution if the velocity decreases 1 per cent in that time? Consider all the mass concentrated in the rim.

10. The flywheel of a motor has a radius of 1 ft. and makes 1,200 revolutions per minute. If the motor is just able to lift a weight of 100 lb. suspended from a rope wound around the circumference of the pulley, what is the horsepower of the motor?

11. A solid cylinder of steel is rotating around a horizontal axis through the axis of the cylinder. The cylinder is 40 ft. long and 18 in. in diameter. Its density is 450 lb. per cubic foot. Find the moment of inertia.

12. A hollow cylinder weighs 80 lb. and has a diameter of 3 ft. The mass is concentrated in the rim. It is rolling on a surface with a linear speed of 8 ft. per second. What is its kinetic energy of rotation?

13. What linear velocity is acquired by a solid iron disk which has a radius of 25 cm. and a thickness of 10 cm. when it rolls down an inclined plane which is 3 m. long. The plane makes an angle of 30 deg. with the horizontal?

14. A heavy flywheel has mass of 1,000 kg. and a radius of 120 cm. It is rotating with an angular velocity of 9 radians per second. If all the mass may be considered as concentrated in the rim, how much work was necessary to give the flywheel this angular velocity?

15. Find the kinetic energy of a system consisting of two masses of 1 kg. and 2 kg., respectively, connected by a rod of negligible mass 1 m. long, when the center of gravity of the system has a velocity of 20 m. per second and the system rotates about its center of gravity with an angular velocity of 60 radians per second?

16. What angular acceleration will be imparted to a wheel which has a radius of 1.5 ft. and a mass of 100 lb. by a weight of 40 lb. hanging from a cord wound around the axle of the wheel? Neglect the weight of the axle and assume its radius is 3 in.

17. A circular hoop weighs 2 kg. and has a radius of 75 cm. Find its moment of inertia about an axis through the center of the hoop and perpendicular to its plane and also about an axis through the hoop and parallel to the axis through the center.

CHAPTER XV

THE CONSERVATION OF MOMENTUM

188. Linear Momentum.—The principle of conservation of energy is not sufficient for the description of phenomena associated with colliding bodies. It is necessary to make use of an additional relation between the masses and the velocities of colliding masses. This relation involves what is known as the **momentum** of each of the colliding bodies. Here we must distinguish between two types of momentum—**linear momentum** and **angular momentum.**

The linear moment has already been defined as the product of the mass and the velocity—*i.e.*, linear momentum $= mv$.

Example.—A body having a mass of 50 g. is moving with a velocity of 50 cm. per second. What is its linear momentum?

Linear momentum = mass × velocity
= 50 g. × 50 (cm. per second)
= 2,500 (g.-cm. per second).

189. Angular Momentum.—In motions of rotation **angular momentum** appears in much the same way as **linear momentum** appears in motions of translation. In rotary motions the rotary inertia plays the same part that mass plays in linear motions and angular velocity the same part that linear velocity plays in linear motions.

The angular momentum of a body is defined as the product of its rotary inertia and its angular velocity around a given axis.

Example.—What is the angular momentum of a body which is rotating about an axis with an angular velocity of 20 radians per second, if the rotary inertia of the body about the axis is 40 lb.-ft.^{2}?

Angular momentum = rotary inertia × angular velocity
$= I \times \omega$
= 40 lb.-ft.^{2} × 20 radians
= 800 lb.-ft.^{2} radians per second.

190. Impulse.—A force acting on a free mass changes its momentum. This change in momentum depends upon the force

and the time during which it acts. For example, if a force F acts on a body of mass M, it imparts to it an acceleration a, such that

$$F = Ma.$$

If this force acts for a time t, the change in the velocity of the body is

$$v = at$$

and the change in momentum is

$$Mv = Mat = Ft.$$

There are cases in which the forces are so great and the time of application so short that it is impossible to measure either the force or the time. An illustration of such quick action is found when a baseball is struck by a bat. Forces of this kind which act for very short times are known as **impulsive forces.**

An impulse is defined to be the product of the average force and the time during which it acts.

$$\text{Impulse} = \text{force} \cdot \text{time} = Ft = Mv.$$

This impulse is of the same nature as change of momentum. The value of the impulse is equal to the total change of momentum experienced by the mass.

Example.—A ball weighing 6 oz. leaves the bat with a speed of 90 ft. per second. Find the value of the impulse, neglecting the change of momentum necessary to reverse the direction of motion of the ball.

Impulse $= Ft = Mv$

$$\frac{6}{16}\text{ lb.} \times 90\text{ ft. per second} = 33.7\text{ lb.-ft. per second.}$$

In the case of rotary motions an analogous relation holds, but the force F must be replaced by the torque T, the mass m by the rotary inertia I, and the linear velocity v by the angular velocity ω. The impulsive torque is measured by the product of the torque and the time during which it acts. It is numerically equal to the change of angular momentum.

$$\begin{aligned}\text{Angular impulse} &= \text{change of angular momentum}\\ &= \text{torque} \times \text{time} = T \times t = IAt = I\omega.\end{aligned}$$

Example.—Find the angular impulse when a flywheel having an angular velocity of 25 radians per second and a rotary inertia of 400 lb. ft.^{2} is suddenly stopped.

$$\text{Angular impulse} = \text{torque} \times \text{time} = I\omega$$
$$= 400 \times 25 = 10^4 \text{ lb.-ft.}^2 \text{ radians/sec.}$$

191. Conservation of Momentum.—By combining Newton's second law of motion with his third law it follows that the momentum, whether linear or angular, must be the same before and after collision. This law states that at collision there can be no gain or loss of momentum. More precisely, **the total momentum of any system of bodies is unchanged by any actions which occur between the different members of the system without the interaction of external forces.**

192. Experimental Illustration of Conservation of Momentum.—In Fig. 159 a man stands on a platform mounted on ball bearings. In his hands

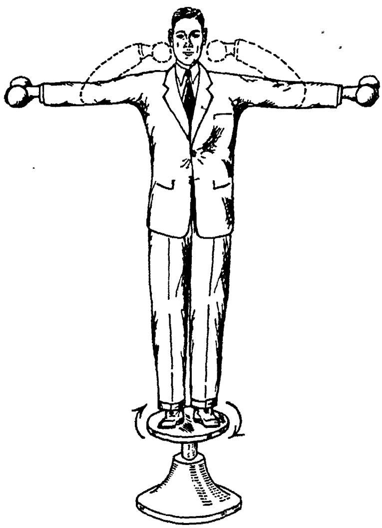


FIG. 159.—The angular momentum remains constant for different positions of the weights.

he holds heavy weights. When his arms are outstretched, his rotary inertia is greater than when they are folded up. If this man is set in rotation with his arms outstretched he will rotate at a constant rate. He has had imparted to him a certain amount of angular momentum which will, neglecting friction, remain unchanged so long as no additional torque acts on him. If now the arms are folded up as indicated in Fig. 159, the rotary inertia of the man and weights is decreased. Since the angular momentum

remains constant and since the angular momentum is equal to the rotary inertia times the angular velocity, the angular velocity of the man will increase in a way to keep the product of the rotary inertia and the angular velocity constant. This means that the man will spin faster when his arms are folded and more slowly when they are outstretched.

The same law of conservation of momentum applies when two parts of a body are thrown apart by internal forces. When a shell explodes, the individual fragments fly away with velocities which are in many cases greater than the velocity with which the unexploded shell was moving. Hence, the momentum of a particular fragment of a shell may be much greater than that which it had before the explosion. But the center of mass of the shell keeps on moving with the same speed it had before the explosion, *i.e.*, the momentum of the mass as a whole is not changed by the explosion. The sum of the momenta of the individual pieces after the explosion is just equal to the momentum of the shell before the explosion.

193. Inelastic and Elastic Bodies.—The motion of bodies after collision depends greatly on their degree of elasticity. If the bodies are without any elasticity, they do not recover their original form or dimensions after collision. For example, when two spheres of putty collide, they are permanently deformed and consequently do not exert any force on each other after collision and do not tend to separate again but move on as one body. On the other hand, when elastic or partly elastic bodies collide, they are only slightly deformed and tend to regain their original shape immediately after collision. While such bodies are regaining their original shape, they exert a force on each other and tend to separate. It is found by experiment that the relative velocity of the bodies after collision bears a definite ratio to their relative velocity before collision. This ratio of the relative velocity after collision to the relative velocity before collision is called the **coefficient of restitution.** In the case of elastic impact, all the energy is conserved, but in inelastic impact some of the energy is converted into heat.

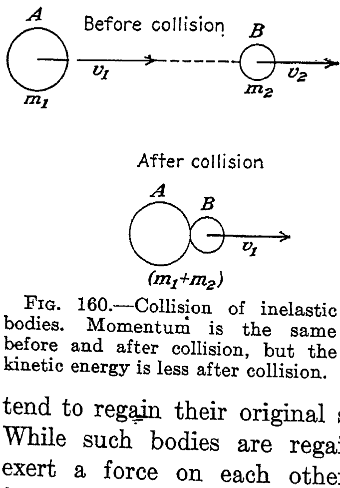


FIG. 160.—Collision of inelastic bodies. Momentum is the same before and after collision, but the kinetic energy is less after collision.

194. Impact of Inelastic Bodies.—In Fig. 160 two inelastic bodies of masses m_1 and m_2 and velocities v_1 and v_2, respectively,

are represented just before and after collision. After collision they move on as a single body of mass $(m_1 + m_2)$ and velocity v.

Since the two inelastic bodies were moving in the same direction with velocities v_1 and v_2, respectively. The velocity v after collision is obtained as follows:

Momentum before impact = momentum after impact.

$$m_1v_1 + m_2v_2 \quad (m_1 + m_2)v.$$

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}.$$

If the bodies were moving in opposite directions before collision,

Momentum before impact = momentum after impact.

$$m_1v_1 - m_2v_2 = (m_1 + m_2)v.$$

$$v = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}.$$

Example.—Two inelastic bodies of mass 400 and 800 g. are moving in the same direction with velocities of 50 and 20 cm. per second, respectively. Find their common velocity after collision.

Momentum before collision = momentum after collision.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v.$$

$$400 \times 50 + 800 \times 20 = (400 + 800)v.$$

$$v = \frac{20{,}000 + 16{,}000}{1{,}200} = \frac{360}{12} \quad 30 \text{ (cm. per second).}$$

195. Impact of Elastic Bodies.—The direct collision of two perfectly elastic spheres A and B, moving in opposite directions, is represented in Fig. 161. At collision the spheres are deformed, and during the recovery from this deformation they exert forces on each other which cause the spheres to separate and one or both of them to move in a direction opposite to the direction of motion before collision. Let m_1 and m_2 be the masses of the spheres, v_1 and v_2 their respective velocities before collision, and u_1 and u_2 the corresponding velocities after collision.

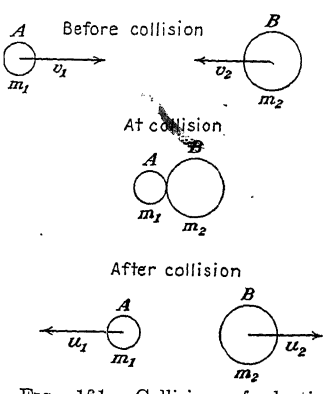


FIG. 161.—Collision of elastic bodies Momentum and kinetic energy are the same before and after collision.

Momentum before collision = momentum after collision.

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2.$$

If the bodies are moving in the same direction before collision, both v_1 and v_2 are to be taken as positive. If the bodies are moving in

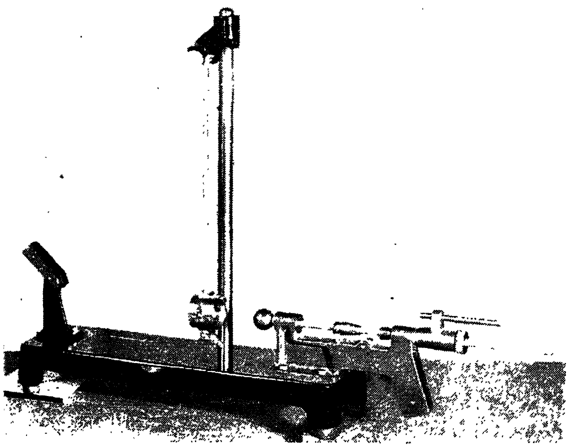


FIG. 162.—Ballistic pendulum. (*Courtesy Central Scientific Company.*)

opposite directions before collision, either v_1 or v_2 must be negative.

If the energies and momenta of the bodies are such that they move in the same direction after collision, both u_1 and u_2 are

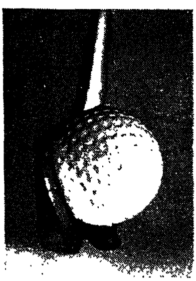


FIG. 163.—Deformation of golf ball struck with a club. Photograph taken with high-speed motion-picture camera. (*Courtesy Edgerton, Germhausen and Grier, Massachusetts Institute of Technology.*)

positive. If they move in opposite directions after collision, u_1 and u_2 have opposite signs.

Example.—Two elastic spheres of masses 40 and 60 lb. moving in opposite direction with velocities of 8 and 2 ft. per second, respectively, collide. If

the smaller sphere moves backward after collision with a velocity of 4 ft. per second, what is the velocity of the larger sphere?

$$\text{Momentum before collision} = \text{momentum after collision.}$$
$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2.$$
$$40 \times 8 - 60 \times 2 = -40 \times 4 + 60u_2.$$
$$= \left(\frac{40 \times 8 - 60 \times 2 + 40 \times 4}{60}\right) = 6 \text{ (ft. per second).}$$

For perfectly elastic bodies the kinetic energy before collision is always equal to the kinetic energy after collision. Hence

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

From the conservation of momentum and the conservation of energy it follows that for perfectly elastic bodies the velocity of approach is always equal to the velocity of separation. For inelastic bodies the kinetic energy before collision is greater than it is after collision. The difference is dissipated as heat.

Figure 162 shows a convenient form of ballistic pendulum for testing the laws of collision, and Fig. 163 illustrates what happens when a golf ball is struck with a golf club.

Problems

1. A freight car weighing 40 tons runs into another freight car having the same weight. If one car was stationary and the other running at the rate of 15 miles per hour and if the cars move off together after collision, with what velocity do they move?

2. Find the recoil velocity of a rifle weighing 9 lb. when it projects a 0.5-oz. bullet with a velocity of 2,400 ft. per second.

3. A machine gun fires eight bullets per second into a target. The mass of each bullet is 12 g. and the velocity 7×10^4 cm. per second. Find the force to hold the gun in position.

4. A mass of 15 lb. moving with a velocity of 6 ft. per second strikes a mass of 6 lb. moving in the same direction with a velocity of 2 ft. per second. Assume the masses are perfectly elastic. Find the velocity after impact.

5. Two perfectly elastic balls, one of mass 5 lb. and the other of mass 4 lb., are moving in opposite directions with velocities of 8 and 15 ft., per second, respectively. Find their velocities after impact.

6. A bullet weighing 25 g. is projected from a gun weighing 15 kg. with a velocity of 350 m. per second. What is the velocity with which the gun recoils?

7. A bullet weighing 5 g. is fired horizontally into a block of wood, with a velocity of 250 m. per second. The block of wood weighs 12 kg. The bullet is embedded in the block and the two move off together. What is their velocity?

8. Two ivory balls, each weighing 250 g., are suspended by two cords 60 cm. in length so that the balls are in contact when they are at rest. One ball is displaced until the angle between the cords is 30 deg. and is then released. Find the velocity of each ball after impact.

CHAPTER XVI

PERIODIC MOTIONS

196. Vibrations.—Periodic motions are very common. They occur in the vibrations of all sorts of mechanical structures, such as automobiles, bridges and buildings, in the different sources of sound, in waves, and in electrical oscillations. If a door slams, vibrations may be set up in the entire building. In the case of an earthquake, tremors are produced which travel over the world and may be recorded as vibrations on delicate instruments. If a mass on a string (Fig. 164) is pulled down and released, it moves back and forth in a straight line with a vibratory motion. When the pendulum of a clock or the balance wheel of a watch is displaced and subsequently released there is set up a vibratory motion which repeats itself at regular intervals.

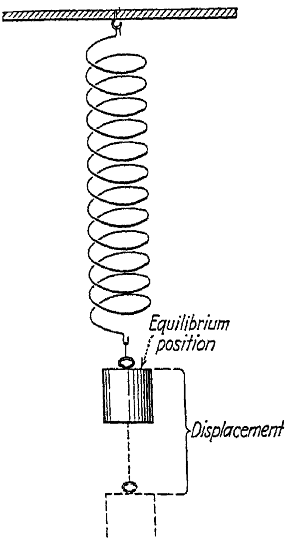


FIG. 164.—Vibratory motions in a stretched spring.

The cause of vibration is some kind of displacement which is resisted by an elastic force arising from the deformation of the body itself. A simple illustration is afforded by clamping one end of a strip of steel (Fig. 165) in a vise and displacing the other end by the application of a force. When the force is removed, the strip vibrates back and forth with a period which depends on the characteristics of the strip of steel. The elastic force tending to restore the strip to its normal position is proportional to the displacement. Hence the greater the displacement, the greater the restoring force. The restoring force arising from the displacement produces the motions in the strip. Because of inertia the strip does not come to rest at its position of equilibrium

but moves on until it has a displacement in the opposite direction. This displacement in the opposite direction again calls forth a restoring force which carries the strip back through its position of equilibrium to an opposite displacement. The motion thus repeats itself at regular intervals. The stretched spring (Fig. 164) behaves in a similar way. Fiction is generally present and modifies the motion of the vibrating body.

197. Types of Vibration.—The same body may vibrate in a number of different ways: A cylindrical rod clamped at one end

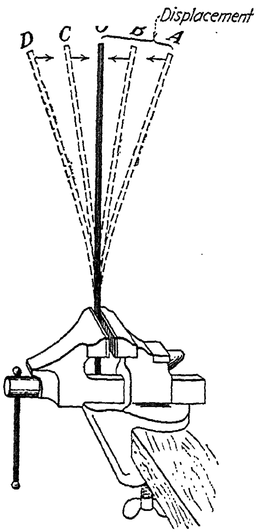


FIG. 165.—Restoring force is proportional to the displacement.

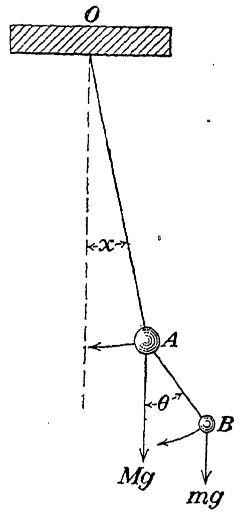


FIG. 166.—Complex vibratory motion of two connected pendulums.

may vibrate to and fro or the free end of the rod may be twisted around the axis of the rod. When this twist is relieved, the rod vibrates with a twisting motion. The rod also might be struck with a hammer so as to compress it in the direction of its length. The molecules of the rod would then oscillate to and fro. When the motion is perpendicular to the length of the rod, the vibrations are known as **transverse vibrations.** Where there is a compressional disturbance so that the motions of the particles are in the direction of the axis of the rod, the vibrations are known as **longitudinal vibrations.**

Vibratory motions are often complex. An illustration is afforded by suspending two pendulums as in Fig. 166. The bob of the upper pendulum is large in comparison with that of the lower pendulum. If the bob of the lower pendulum is displaced, the resulting motion will be the sum of the motions of the two pendulums. The resulting motion will thus appear complicated but can be analyzed into its components. In nature it frequently happens that a body can vibrate in a number of different ways at the same time, but these motions in many cases can be analyzed into their components.

198. Uniform Circular Motion.—If a body moves in a circular path (Fig. 167) in such a way that it always passes over equal distance in equal intervals of time, it is said to have uniform circular motion. The magnitude of its velocity is constant, but its direction of motion is always changing. Such a body has an acceleration, but this acceleration consists in a change in the direction, whereas the other accelerations we have considered have consisted in a change in the magnitude of the velocity. A force is necessary to produce a change in the direction of the motion of a body, just as a force is necessary to produce a change in its rate of motion. Since in circular motion this force produces no change in the magnitude of the velocity but only a change in the direction of motion, the force must act at right angles to the direction of motion. Otherwise it would act partly in the direction of motion and thus produce either an increase or a decrease in the magnitude of the velocity. Since the force is at right angles to the direction of motion and the motion is in a circular path, the force is along the radius of the circle and directed toward the center of the circle. The rate at which the direction of the motion is changing is constant, and the force which produces it must therefore be constant.

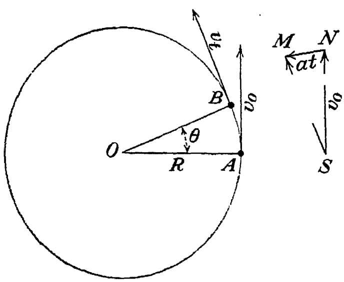


FIG. 167.—In uniform circular motion the acceleration toward the center $= v^2/R$.

199. Acceleration in Uniform Circular Motion.—In uniform circular motion (Fig. 167), there is at every instant an acceleration toward the center. To calculate this acceleration, suppose

that a particle passes over the arc AB with a constant speed v in the time t. The space passed over in this time is $AB = vt$. The direction of motion of the body has changed but it continues to move tangent to the circle. The angle θ between the radii OA and OB is equal to the angle between the velocity of the body at A and its velocity at B.

To find the rate of change of velocity, draw two vectors, v_0 and v_t each of the same magnitude v. These vectors differ only in direction. The velocity of the body at A is represented by the vector v_0 and the velocity at B by the vector v_t. The angle θ is the change in the direction of motion. In order to change the velocity from v_0 to v_t, an additional velocity must be added to the body. If a is the rate of change of the velocity,—that is the acceleration, and t, the time for the body to move from A to B, the change in velocity is at. This is the velocity which must be added to cause the velocity of the body to change from v_0 to v_t. To find this change of velocity, draw v_0 and v_t from a common point S, making an angle θ with each other. Close the triangle thus formed by drawing a third vector at which is equal to the change of velocity in the time t. When the angle θ is small, the chord AB, is nearly equal to the arc AB, and the two triangles AOB and MSN are similar.
Then

$$\frac{AB}{R} = \frac{MN}{v}$$

$$\frac{vt}{R} = \frac{at}{v}$$

and

$$a = \frac{v^2}{R}.$$

This acceleration is directed toward the center of the circle.
If ω is the angular velocity of the radius,

$$v = \omega R.$$

$$a = \frac{\omega^2 R^2}{R} \quad \omega^2 R.$$

This expression may be written in another form. If N denotes the number of revolutions per second,

$$v = 2\pi RN.$$
$$a = \frac{4\pi^2R^2N^2}{R} = 4\pi^2RN^2.$$

From this last expression it is evident that the acceleration increases when the radius of the path is increased, and it also increases when the number of revolutions is increased.

Example.—A machine is running on a circular speedway with a velocity of 120 ft. per second. The radius of the speedway is 1,000 ft. What is the acceleration toward the center?

$$\text{Acceleration} = \frac{v^2}{R}$$
$$\frac{(120)^2}{1{,}000} = \frac{14{,}400\ (\text{ft. per second})^2}{1{,}000\ \text{ft.}} = 14.4 \text{ ft. per second per second.}$$

200. Centripetal Force.—Newton's second law of motion states that wherever an acceleration is produced, a force must be applied. It has been seen that a body moving in a circular path with uniform velocity has an acceleration toward the center of the circle. To produce this acceleration and keep the body from flying off tangent to the circle, a force must be applied at right angles to the direction of motion. This force which is directed toward the center of the circle is called the **centripetal** force. To find its magnitude, recall that Newton's second law of motion is

$$F = Ma.$$

Now the acceleration in the case of uniform circular motion is

$$a = \frac{v^2}{R},$$

whence

$$F = Ma = M\frac{v^2}{R}$$

or

$$= 4\pi^2RN^2M.$$

This force is necessary to keep the body moving in a circular path. The reaction which the moving body produces in the restraint which holds it in its path is called the **centrifugal force.** It is equal in magnitude but oppôsite in direction to the centripe-

tal force. Thus, when a train rounds a curve, a centrifugal force is exerted on the track.

Example.—A mass of 10 lb. at the end of a string is being whirled in a circle of radius 3 ft. with a constant velocity of 10 ft. per second. What is the pull of the whirling body on the string?

$$\text{Centrifugal force} \quad M\frac{v^2}{R} \text{ poundals}$$

$$10 \text{ lb.} \cdot \frac{\qquad}{3 \text{ ft.}} = 333 \text{ poundals.}$$

This property of a body to move on in a straight line is of much importance in nature. The mud on a rotating carriage wheel or the water on a grindstone tends to fly off along the tangent. When a horse runs on a circular race track, he has some difficulty in keeping on the track. Where a train runs on a circular piece of track, the outside rail is elevated in order to overcome the tendency of the train to "jump the track." The greater the curvature of the track, the greater is this tendency for a given velocity of the train. The elevation of the outer rail causes the thrust of the track on the train, represented by R in Fig. 168, to have a component X toward the center of the circle, this component supplying the necessary centripetal force. An equal and opposite force, the centrifugal force, is exerted by the train on the track away from the center of the circle. For high velocities of the train, the centrifugal force exerted on the track increases, and to prevent a dangerous increase the train ordinarily slows down when running around a curve.

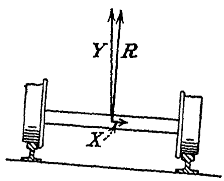


Fig. 168.—Forces on the rails when a truck goes around a curve.

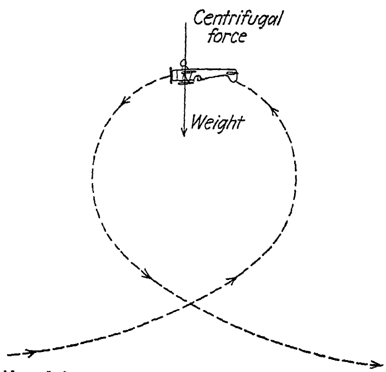


Fig. 169.—Centrifugal force on a pilot in an airplane making an inside loop.

When an airplane (Fig. 169) makes an inside loop, centrifugal force presses the pilot against the plane.

Example.—Find the force necessary to keep a locomotive weighing 125 tons on the track when it is running around a curve at a speed of 60 ft. per second. The radius of the track is 6,000 ft.

$$\text{Centrifugal force} \quad \frac{M \times v^2}{R}\ \text{lb.}$$
$$= \frac{250{,}000\ \text{lb.} \times (60\ \text{ft. per second})^2}{6{,}000\ \text{ft.}}$$
$$= 15.1 \times 10^4\ \text{poundals.}$$

201. Cream Separator and Centrifuge.—In a cream separator the difference between the tendencies of the milk and the cream to move in a straight

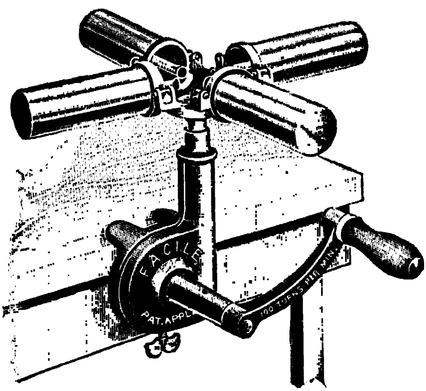


FIG. 170.—The centrifuge. Force per cubic centimeter tending to separate two liquids is proportional to the difference of their densities.

line makes it possible to separate the milk from the cream by rotating them about an axis. The heavier milk moves out farther than the lighter cream. This leaves the cream near the axis of the separator, and the milk finds a place nearer the outer edge of the separator.

The centrifuge, which is widely used in the separation of liquids of unequal densities, depends for its operation on centripetal force. One type of this machine (Fig. 170) consists of a wheel which rotates in a horizontal plane. To this wheel are attached buckets which are vertical when the wheel is at rest, but, when the wheel is revolving rapidly, the buckets assume a position so that their axes are horizontal. If a mixture of liquids of unequal densities is introduced into the buckets and the wheel is revolved rapidly, the liquids separate, the heavier liquids farther from the axis of rotation and the lighter liquids nearer to it. This means that the heavier liquids will be at the bottom of the buckets while the lighter liquids are at the top. When the centrifuge is stopped, the buckets return to their normal positions with their axes vertical. The lighter liquids are found on the top of the bucket and the heavier liquids at the bottom. A separation of the liquids has thus been brought about.

The forces developed in a high-speed centrifuge (Fig. 171) are very large.

202. Steam-engine Governor.—The governor of a steam engine depends for its action on centripetal force. The spheres C (Fig. 172) revolving about a vertical axis are acted on by the force of gravity tending to pull them into a

FIG. 171.—Rigid metal can not withstand large force developed by high-speed centrifuge. (*Courtesy J. W. Beams, University of Virginia.*)

vertical position and by the vertical and horizontal components of a force exerted on them by the governor arm. The vertical component of this force is equal to the weight of the balls. The horizontal component of this

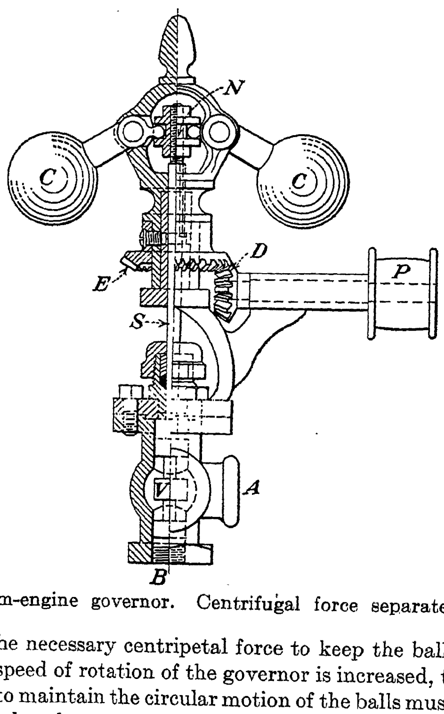


FIG. 172.—Steam-engine governor. Centrifugal force separates the spheres.

force supplies the necessary centripetal force to keep the balls moving in a circle. As the speed of rotation of the governor is increased, the centripetal force necessary to maintain the circular motion of the balls must be increased. This is effected by the governor arms becoming more nearly horizontal.

When this happens, a lever arrangement partially shuts off the supply of steam from the engine. This tends to make the engine slow down. The spheres C and C then descend again, opening the steam valve somewhat and increasing the supply of steam. The speed of the engine again increases. In this way, the speed of the engine is kept constant by regulating the supply of steam.

203. Centrifugal Force on the Moon.—Newton tested the law of gravitation by applying it to a study of the motions of the moon.

Let M = mass of the earth.
m = mass of the moon.
v = velocity of moon in its orbit.
r = radius of the orbit of the moon.
k = gravitational constant = 6.67×10^{-8} if masses are in grams, distances in centimeters, and force in dynes.
T = period of revolution of moon about the earth = about 28 days.

Since the pull of the earth on the moon kMm/r^2 just balances the centrifugal force Mv^2/r which makes the moon tend to leave its orbit,

$$m\frac{v^2}{r} = k\frac{Mm}{r^2}$$

and

$$v^2 = k\frac{M}{r}.$$

$$M = 6.1 \times 10^{27} \text{ g.},$$
$$r = 240,000 \times 5,280 \times 30 \text{ (cm.)},$$
$$k = 6.67 \times 10^{-8},$$

whence,

$$v = 1 \times 10^5 \text{ cm. per second approximately.}$$

Calculated directly from data on the motion of the moon in its orbit,

$$\frac{\times 240,000 \times 5,280 \times 30.4}{28 \times 24 \times 3,600} \quad 1 \times 10^5 \text{ cm. per second}$$

approximately.

Hence the velocity of the moon calculated from its period of revolution about the earth and its velocity calculated from the law of gravitation and the centrifugal force are in good agreement.

204. Rotation of Two Spheres about Their Common Center of Mass.—If two spheres (Fig. 173) are joined by a light rod and then projected with a motion of translation and free rotation, the motion of rotation will take place about an axis perpendicular to the line joining the two spheres. This axis of rotation passes through the center of mass of the two spheres.

Let M_1 = the mass of one sphere.
M_2 = the mass of the other sphere.
R_1 = the distance of M_1 from the center of gravity of the system.
R_2 = the distance of M_2 from the center of gravity of the system.
ω = the angular velocity of the line joining the spheres.

F_1 = the force on M_1.
F_2 = the force on M_2.

Then

$$F_1 = M_1 \cdot R_1 \cdot \omega^2.$$
$$F_2 = M_2 \cdot R_2 \cdot \omega^2.$$

Since the center of gravity (see Sec. 69) is given by the equation

$$M_1 \cdot R_1 = M_2 \cdot R_2,$$

we have

$$F_1 = F_2$$

Hence, the centrifugal forces exerted by the two spheres on their connecting rod are just equal in magnitude and opposite in direction when two spheres rotate about their common center of mass.

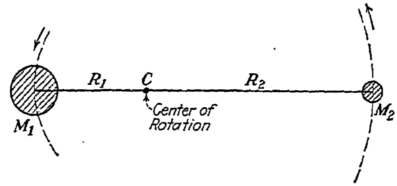


FIG. 173.—Rotation of two masses about their center of gravity.

205. Effect of Centrifugal Force on the Weight of a Body.—Because a force is necessary to keep a body moving in a circular path with uniform velocity, there is a tendency for bodies to fly away from the surface of the earth due to the rotation of the earth about its axis. At the equator a point on the surface of the earth has a velocity of about 1,460 ft. per second. To keep a mass of 1 lb. on the surface of the earth at the equator in its circular path necessitates a centripetal force.

$$M \cdot \frac{v^2}{R} \quad 1 \times \frac{1{,}460 \text{ (ft. per second)}^2}{21 \times 10^6 \text{ ft.}}$$
$$= 0.1 \text{ poundal.}$$

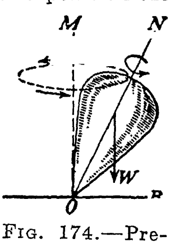


FIG. 174.—Precessional motion of a top.

The attraction of the earth on a mass of 1 lb. is equal to 32.2 poundals. Hence, the weight of a mass of 1 lb., which weight is found by measuring the force that supports the body, would, at the equator, be decreased by 0.1 poundal owing to the rotation of the earth about its axis. The acceleration of a body falling at the equator would also be decreased. If the earth were not in rotation and the acceleration were 32.2 ft. per second per second at the equator, the acceleration would be 32.1 ft. per second per second when the earth rotates once in 24 hr. about its axis.

206. Precessional Motion.—When a body is rotating and a torque is applied perpendicular to the axis about which it is rotating, the axis of rotation begins to move at right angles to the direction in which the torque is applied. This effect is shown by a top (Fig. 174) when it is spinning with its axis inclined to the vertical. The weight of the top exerts a torque tending to rotate the top about an axis through the point on which the top rests, but the top does not fall. Its axis moves around the vertical, keeping the inclination nearly constant. This type of motion is known as *precessional motion.*

207. Simple Harmonic Motion.—Consider a point moving around a circle (Fig. 175) with uniform speed. The projection of this point on the diameter of the circle moves with varying speed and describes a complete to-and-fro vibration on the diameter while the point moves completely around the circumference. If M is the projection of P on the horizontal diameter, the motion of M is called *simple harmonic* motion.

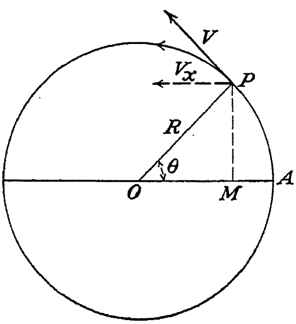


FIG. 175.—Projection of uniform circular motion on a horizontal axis gives simple harmonic motion.

Let R be the radius of the circle of reference, and θ the angle between OP and OA. Then, OM = displacement = distance of M from O, the position of equilibrium.

$$OM = x = R \cos \theta.$$

Let V = velocity of P, tangent to the circle.

$$\text{Horizontal velocity of } P = V_x = V \cos (90 - \theta)$$
$$= -V \sin \theta$$
$$\text{Velocity of } M = \text{horizontal velocity of } P = V_x$$
$$= -V \sin \theta$$

(the minus sign being used to indicate that the velocity is in the negative direction, *i.e.*, toward the left).

$$\text{Acceleration of } P \text{ toward center } O = \frac{V^2}{R}.$$

$$\text{Horizontal acceleration of } P = A_x = -\frac{V^2}{R} \cos \theta.$$

Acceleration of M = horizontal acceleration of P

$$= A_x = -\frac{V^2}{R}\cos\theta.$$

Dividing the acceleration of M by its displacement at the same instant,

$$\frac{A_x}{x} = \frac{-\frac{V^2}{R}\cos\theta}{R\cos\theta} = -\frac{V^2}{R^2}$$

whence,

$$A_x = -\frac{V^2}{R^2}x.$$

Since neither V nor R changes numerically during the motion, V^2/R^2 is a constant. This means that the ratio of the acceleration of M to its displacement is always a constant, whatever the position of M. The acceleration and the displacement are always opposite in direction. Both change but in such a way that their ratio remains unchanged. This, then, becomes the test of simple harmonic motion. **Whenever a body has a periodic motion in which the acceleration divided by the displacement is constant, it is said to move with simple harmonic motion.**

In simple harmonic motion there are, besides displacement and acceleration, three quantities to be considered. These quantities are *amplitude, period*, and *frequency*. For each of these quantities an explicit definition is necessary.

1. *The amplitude* is the distance between the mid-point and the end of the path over which the vibrating particle moves. It is equal to the radius of the circle of reference.

2. *The period* is the time necessary for the particle to make one complete vibration. It is equal to the time necessary for the reference particle to move around the circle of reference.

3. *The frequency* is the number of complete vibrations made by the particle in unit time. It is the reciprocal of the period.

With this agreement with respect to the definition of the period, it is possible to calculate the relation between the period, the acceleration, and the displacement in the following manner.

If T is the time for the particle to pass once around the circle, ω the angular velocity of the radius OP, and N the number of oscillations per second, we have

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi N = \frac{V}{R}$$

and

$$\frac{2\pi R}{T} = V,\ \omega = \frac{2\pi}{T},\ V = \omega R.$$

$$\frac{V^2}{R^2} \quad \frac{\omega^2 R^2}{R^2} = \omega^2 \quad \frac{2\pi}{T}$$

$$-\frac{A_x}{x} = \frac{V^2}{R^2} = \omega^2 = \left(\frac{2\pi}{T}\right.$$

Hence,

$$\sqrt{\frac{A_x}{-x}} \quad \frac{2\pi}{T}$$

and

$$T = 2\pi\sqrt{-\frac{x}{A_x}}. \quad \text{(Appendix E-4.)}$$

Also, if F_x denotes the force acting on the particle when its displacement is x, we get, by substituting the acceleration,

$$A_x = -\frac{V^2}{R^2}x = -4\pi^2 \cdot N^2 \cdot x$$

in Newton's second law of motion,

$$F = Ma,$$

the expression for the force acting on the mass at any instant in its simple harmonic motion. This force is

$$F_x = -4\pi^2 N^2 \cdot M \cdot x.$$

208. Simple Pendulum.—A simple pendulum affords an easy illustration of simple harmonic motion. When the pendulum is displaced from its position of equilibrium (Fig. 176), the force tending to bring it back to its original position is

$$F = -Mg \sin \theta.$$

Since $F = Ma$, the acceleration which this force produces is

$$a = -g \sin \theta.$$

The displacement of the pendulum bob from its position of equilibrium N is

$$S = l\theta.$$

Hence,

$$\frac{\text{Displacement}}{\text{Acceleration}} \quad \sim \quad -g \sin \theta$$

For small angles $\sin \theta = \theta$ when the angle is measured in radians and for small displacements of the pendulum,

$$\frac{\text{Displacement}}{\text{Acceleration}} = -\frac{l}{g} = \text{constant.}$$

Consequently the pendulum moves very nearly with simple harmonic motion. Its period is, therefore,

$$T = 2\pi\sqrt{-\frac{\text{displacement}}{\text{acceleration}}}$$
$$= 2\pi\sqrt{\frac{l}{g}}.$$

The longer the pendulum the greater is its period, and the greater the acceleration of gravity the shorter the period of the pendulum.

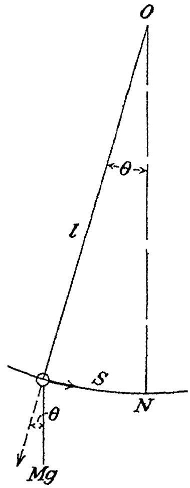


FIG. 176.—A simple pendulum moves with approximately simple harmonic motion.

From the preceding expression for the period of a simple pendulum, we may define a *seconds pendulum*, so that it requires just 2 sec. to make one complete oscillation or just 1 sec. to make one-half of one complete oscillation. The time to make one-half of a complete oscillation is

$$\pi\sqrt{\frac{l}{g}}$$

and, since $t = 1$ sec. in this case,

$$\pi\sqrt{\frac{l}{g}}.$$

$$l = \frac{g}{\pi^2}.$$

If the pendulum is at a point on the surface of the earth where $g = 980$ cm per second per second,

$$l = \frac{980}{\pi^2} = 99.2 \text{ cm.}$$

Problems

1. Assuming 10 ft. per second per second as the largest sidewise acceleration which is safe for a certain car, what is the smallest circle on which it can travel at the rate of 15 miles per hour?

2. A stone whirling on the end of a string 4 ft. long has an acceleration of 36 ft. per second per second toward the center of the circle. What is the velocity of the stone?

3. A thread 30 in. long which cannot support a force greater than 10-lb. weight is used to whirl a stone of 2.5-lb. mass in a horizontal circle. How fast will the stone be moving when the thread breaks? Consider only horizontal force.

4. What is the centrifugal acceleration of the moon in its orbit around the earth? The radius of the orbit is assumed circular and equal to 240,000 miles. The period of revolution of the moon about the earth is $27\frac{1}{2}$ days.

5. When a weight of 150 g. is suspended from a spring, the spring stretches 10.8 cm. What is the period of oscillation of the weight when it is given a small displacement?

6. A body moving in a circle with a radius of 2 ft. requires a force of 5 lb. to keep it in its circular path when making 18 revolutions per minute. Find the number of pounds in the body.

7. A stone weighing 4 lb. is whirled in a circle by means of a string which is 3 ft. long. The string breaks and the stone flies off with a velocity of 20 ft. per second. Find the pull in the string when it breaks. Assume that the effect of gravity can be neglected.

8. A child weighing 60 lb. is swinging in such a way that it describes an arc of 10-ft. radius. If the horizontal motion at the lowest point of the swing is 2.5 ft. per second, what is the total force which the ropes of the swing must sustain at that instant?

9. A pendulum has a length of 3.05 m. and executes 20 complete vibrations in 70 sec. Find the acceleration of gravity at that place.

10. A simple harmonic motion has a period of 0.005 sec. and an amplitude of 0.06 cm. What is the acceleration when the body has its maximum displacement?

11. The mass of the earth is approximately 6×10^{27} g. What centripetal force is necessary to keep it in its orbit, assuming a radius of 150,000,000 km., and a period of revolution of 365 days? What would have to be the dimensions of a steel cable with a strength of 10^{10} dynes per square centimeter, in order to sustain this force?

12. How many revolutions per minute would the earth have to make in order that the weight of a body at the equator become zero?

13. The governor of an engine has arms which are 25 cm. long and stand at an angle of 60 deg. with the vertical when the governor is in constant rotation. Find the angular speed of the shaft of the governor.

14. What is the angle at which a circular speedway must be banked for cars running at 90 miles per hour, if the radius of the track is assumed to be 900 ft.?

15. What is the speed of an airplane which is making a slow loop with a radius of curvature of 60 ft., when objects just begin to drop to the earth?

PART II.—WAVE MOTION AND SOUND

CHAPTER XVII

WAVE MOTION

209. Wave Motion.—One of the most important phenomena in nature is the transmission of energy from one point to another by **wave motion.** This kind of motion is illustrated in many ways. When a stone is dropped into a pool of still water, the surface of the water is covered with circular wavelets which widen out from the central point where the stone fell. The water does not really move outward from the central point, but it rises and then falls again. That such is the case is seen by observing a floating leaf or piece of wood. It does not move forward but returns again and again to its former position. Hence, the water on which the leaf rests must have this same kind of upward and downward motion rather than a forward motion.

The motion of the heads of wheat in a field on which the wind is blowing gives the impression that there is a forward motion. There is, however, only an upward and downward motion of each head of wheat. The steady onward motion of the waves in the wheat is not a forward motion of the wheat. It is a forward motion of a state of things, a shape, or a wave form.

In like manner when one end of a rope is fastened to a rigid wall and the free end moved up and down rapidly, each jerk travels along the rope, each portion of the rope communicating the jerk to the next portion. Each particle of the rope imparts its upward or downward motion to its neighbors. The jerk moves forward, but the particles of the rope move only up and down. Motions of this kind are **wave motions.** In all these cases it is evident that there is a vibrating center which produces motions in those portions of the medium immediately in contact with it, and that these portions impart their motions to the neighboring portions. In order that a medium should carry waves, there must be a force of restitution which is called into action when the parts of the medium are displaced with respect to each other.

210. Transverse Waves.—If part of a stretched string (Fig. 177) is drawn aside, the tension in the string tends to bring it back to its position of equilibrium. Since the string has inertia, the force which causes the displacement requires time to produce its full effect so that a wave can travel along the string with a definite velocity. Waves of this kind are easily produced in a rope fixed at A (Fig. 177) and held in the hand at B. If the rope is lightly stretched, a jerk imparted to the end B travels

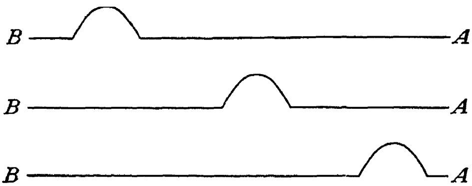


FIG. 177.—Waves in a stretched string.

down the rope as a wave. The successive positions of this wave are indicated in Fig. 177. The more tightly the rope is stretched, the more rapidly does the jerk travel down it. If a series of to-and-fro movements is imparted to the end B, a series of waves travels down the rope. Such waves are known as **transverse waves,** because the particles of the medium in which the waves travel move perpendicular to the direction of the wave motion. They can be easily represented by plotting the displacements on the vertical axis and the distances from the source in a given direction on the horizontal axis. Light and other forms of electromagnetic waves are excellent illustrations of **transverse waves.**

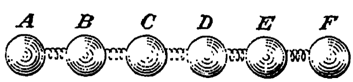


FIG. 178.—Waves in compressed springs.

211. Compressional or Longitudinal Waves.—In Fig. 178 is represented a series of equal masses resting in a frictionless groove. They are joined together by springs. If the mass A is moved toward B, the latter will begin to move because the spring between A and B is compressed. But B will begin to move a little later than the time at which A starts, for B does not begin to move until the spring has been somewhat compressed. The forward motion of B produces a compression in the second spring which in turn sets the mass C in motion. In like manner each mass in turn is set in motion. Similarly, if the mass A

is pulled away from the mass B, the spring between them is stretched and B starts to move. The motion of B causes the second spring to be stretched and this process continues through the whole series of balls.

If a number of steel balls (Fig. 179) are suspended so that they are in contact, and if now K is drawn aside and allowed to fall against J, the shock is transmitted through the other balls, and the ball A on the farther end of the row moves away from B into the position A'. When the ball K strikes the ball J, the ball J is momentarily compressed, and this compression is handed on to the other balls. In this way, a wave of compression travels along the balls and forces A away from its neighbor B. This wave of compression is not instantaneous. Each ball takes a little time to be compressed and then to recover its former shape. The velocity with which the compression is transferred is, however, great.

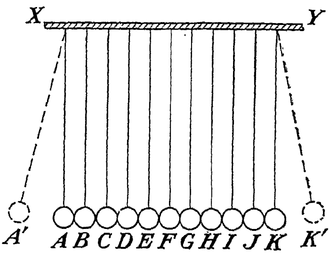


Fig. 179.—Elastic waves transmitted through spheres in contact.

Waves of this type are known as **compressional** or **longitudinal waves.** The particles of the medium are displaced to and fro in the direction of the wave motion. Sound waves are **longitudinal waves.** The molecules of the medium, in which the sound travels, move back and forth in the direction of the propagation of the sound.

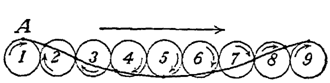


Fig. 180.—Waves in water.

212. Water Waves.—If the surface of a pool is disturbed at one point by the upward and downward motion of a stick dipping in the water, waves spread out from the stick. These waves arise because the force of gravity tends to maintain a level surface. If a vertical cord like a fishing line is moved sidewise through the surface of a pond, short waves called ripples may be observed on the side toward which the cord is moving. These waves are due to the restoring force which arises out of the surface tension of the water.

In the case of a wave in deep water, the particles of water describe circles which lie in a vertical plane (Fig. 180). When a

particle is on a crest A, it is moving in the direction in which the wave is moving; but when it is in the trough B, it is moving in the opposite direction to that in which the wave is moving. All the particles move in a clockwise direction with the same uniform velocity. The radii of the circles in which the particles move become smaller and smaller as the distance below the surface is increased, and the effect of the wave is felt only a short distance down from the surface. In shallow water the paths of the particles are ellipses with their major axes horizontal.

213. Other Kinds of Waves.—There are many other kinds of waves besides those in material media in which the disturbances arise out of the displacement of particles. If the temperature of one end of a metal rod is first raised gradually, then lowered, raised again, etc., there is set up a succession of changes in the temperature of the rod. These changes travel forward in the rod as a wave of temperature. The daily heating and cooling of the surface of the earth, as it is turned toward and away from the sun, produce waves of temperature which go down into the earth for a short distance.

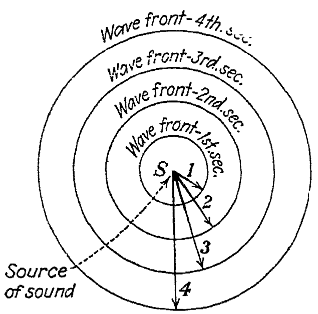


Fig. 181.—Wave fronts from a point source.

If one end of an ocean cable or telephone wire is suddenly joined to a battery, the change in potential thus produced in the wire is gradually felt along the conductor. If the potential of the battery is varied systematically, an electric wave is transmitted along the conductor.

214. Wave Front.—When waves spread out from a center of disturbance, a surface can be described that marks at any instant the points to which the disturbance has reached. This surface (Fig. 181) is called the **wave front.** The wave front is really a surface on which all the particles are in the same phase of vibration. When a drop of water falls on the surface of a pond, the wave front is a circle which continuously expands. When a small balloon bursts in the air, the wave front of the sound produced is a sphere with the balloon as a center. The waves of light from a distant star have a spherical wave front of such large radius that it may be considered as the portion of a plane.

215. Intensity of Spherical Waves.—The intensity of spherical waves or the energy per cubic centimeter decreases rapidly as the distance from the source is increased. If at the point S (Fig. 182)

there is a source which produces spherical waves, **the relative intensities of these waves at different distances from the source are inversely proportional to the squares of the distances from the source.**

Let I_1 = the energy per cubic centimeter in a spherical shell of radius R_1.

I_2 = the energy per cubic centimeter in a spherical shell of radius R_2.

Then the energy in a spherical shell of radius R_1 and of unit thickness is

Volume of shell $\times$ energy per cubic centimeter $= 4\pi R_1^2 I_1$.

The energy in a shell of radius R_2 and of unit thickness is

Volume of shell $\times$ energy per cubic centimeter $= 4\pi R_2^2 I_2$.

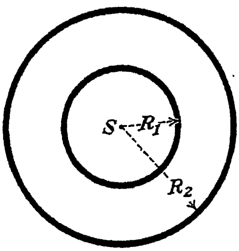


FIG. 182.—The intensity is inversely proportional to the square of the distance from the source.

If the rate at which energy is sent out by the source is constant, the amount of energy in shells of equal thickness is constant. Therefore,

$$4\pi R_1^2 \times I_1 = 4\pi R_2^2 \times I_2.$$

$$\frac{I_1}{I_2} = \frac{R_2^2}{R_1^2}.$$

$$\frac{I_1}{I_2} = \frac{\text{intensity at distance } R_1}{\text{intensity at distance } R_2} = \frac{R_2^2}{R_1^2}.$$

Hence, the intensity varies inversely as the square of the distance from the source.

When the energy cannot spread out freely in all directions, the intensity will not vary inversely as the square of the distance.

216. Velocity of Waves.—The rate at which the wave front advances is called the **velocity of the wave.** The velocity depends on the physical characteristics of the medium in which the waves travel.

1. *Velocity of Transverse Waves in Strings.*—The velocity of a transverse wave along a flexible stretched string is constant for a given cord under a definite tension. Whatever the nature of the

wave in the cord, its velocity of propagation is the same so long as the tension is unchanged. The expression for the velocity in this case is

$$v = \sqrt{\frac{\quad}{m}},$$

where v the velocity of the disturbance in centimeters per second.

T = the tension in the string in dynes.

m = the mass of the string per unit length in grams per centimeter.

From this formula it is seen that the velocity of a disturbance in a stretched string is proportional to the square root of the tension in the string and inversely proportional to the square root of the weight of the string per unit length. Hence, the velocity will be large when the tension is great, and small when the weight per unit length is great.
(Appendix E-5.)

2. *Velocity of Compressional Waves.*—The velocity of a compressional wave depends on the density and the elasticity of the medium. The greater the elasticity and the less the density, the more rapidly the compressional wave travels. The relation between the velocity of the wave, the density, and the elasticity of the medium is expressed in the formula,

$$v = \sqrt{\frac{e}{\rho}},$$

where v = the velocity of the compressional wave.

e = the volume modulus of elasticity of the medium.

ρ = the density of the medium.

Where the modulus of elasticity divided by the density is large, the velocity of the compressional wave is great. For this reason, the velocity of a compressional wave in steel is greater than it is in air. Although the density of steel is greater than that of air, its modulus of elasticity is so much greater than that of air that the velocity in steel is much greater than it is in air.

217. Amplitude and Frequency.—A vibrating body, such as the prong of a tuning fork, passes back and forth through its

position of equilibrium. The extreme displacement on either side of the position of equilibrium is called the **amplitude.** Except for the fact that the vibration dies away slowly, this displacement is the same on the two sides of the position of equilibrium.

The number of complete vibrations made by the body in 1 sec. is called the **frequency of vibration.**

218. Relation between Wave Length, Frequency, and Velocity.—The distance between successive crests or between successive troughs in a water wave is called the **wave length** (Fig. 183). It is, in general, the distance between successive points in

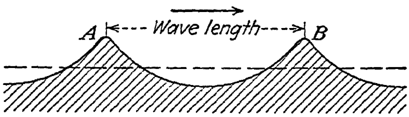


FIG. 183.—Definition of wave length.

same phase of vibration. Let n be the number of vibrations made by a particle of the medium in 1 sec.; v, the distance traveled by the wave in 1 sec.; and λ, the length of each wave. Then,

$$\lambda = \frac{v}{n}.$$

Velocity of the wave = number of waves per unit time $\times$ wave length.

$$v = n\lambda.$$

Example.—The velocity of a disturbance in a steel rod is 5,000 meters per second and the frequency of the vibrations is 2,500 per second. What is the wave length?

$$\text{Wave length} = \frac{\text{velocity}}{\text{frequency}} = \frac{5{,}000}{2{,}500} = 2 \text{ ft.}$$

Example.—A siren having 50 holes at one radial distance revolves with a speed such that it makes 450 revolutions per minute. Find the wave length of the tone which is emitted if the temperature of the air is 20°C.

$$\text{Revolutions per second} = \frac{450}{60} = 7.5 \text{ per second.}$$

$$\text{Frequency of sound wave} = n = 50 \times 7.5 = 375 \text{ per second.}$$

$$\text{Wave length} = \frac{\text{velocity}}{\text{frequency}}.$$

$$\text{Velocity of sound in air at 20°C.} = 343 \text{ m.}$$

$$\text{Wave length} = \frac{343}{375} \quad 0.91 \text{ m.}$$

219. Representation of Waves.—Each particle of the medium in which the waves are traveling will be set in simple harmonic motion, but the phase of vibration varies from point to point. The simplest way of representing these waves graphically is to choose two axes (Fig. 184) and plot on the vertical axis the displacement of the particles at a given instant and on the horizontal axis the distance of the point from the source of the disturbance. Such a curve gives the displacement at any instant for all the particles in the medium along a line in the direction in which the waves are traveling.

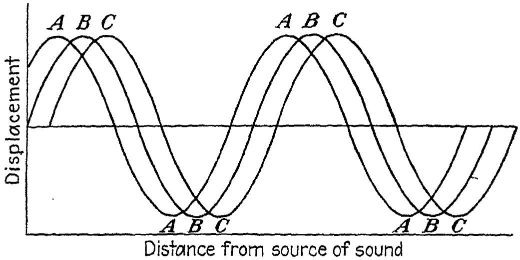


FIG. 184.—Displacement of medium at successive intervals of time.

The positions of these points for successive instants are represented in the three curves A, B, and C of Fig. 184.

220. Standing Waves.—A long, flexible rubber tube or an elastic cord is fixed at one end (Fig. 185), while the other end is held in the hand. If the cord is stretched fairly tight and the free end is moved sidewise with a simple harmonic motion, waves will be set up in the cord which will travel to the fixed end where they will suffer reflection and travel back to the hand. At any

FIG. 185.—Waves in a string reflected at a wall.

instant, two trains of waves will be traveling in the cord in opposite directions. If the frequency of the motion is properly chosen, the cord ceases to have the appearance of being traversed by trains of waves but vibrates transversely in one, two, or more segments.

The explanation of the behavior of the cord can be seen from Fig. 186. The dotted line represents a wave traveling from right to left. The waves represented by the broken line are traveling from left to right. The former are the waves originally produced in the string, and the latter are the waves set up by the

reflection of the original train of waves at the end of the string. The resultant disturbance which arises from the combination of these two trains of waves is represented by the continuous line. This resultant is at any instant in the form of a wave, but it

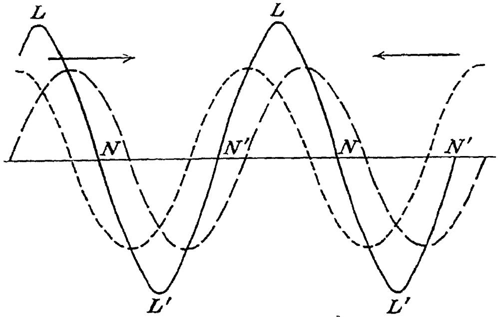


FIG. 186.—Standing waves in strings. Two trains of waves are superposed on each other.

does not move along. Each particle vibrates, but the amplitude of vibration varies from particle to particle. When the frequency is high, all the eye sees is a characteristic blur which appears to remain motionless. This appearance has given rise to the name **standing waves.** At L, the crests of the two component waves are approaching each other. When the two crests coincide, the resulting displacement is a maximum. A quarter of a period later, the two components will exactly neutralize each other. The crest of one wave will then be just above the trough of the other wave. At that instant, the cord will be straight. As the waves travel still farther in opposite directions, the portion of the string $N'LN$ will be depressed below the horizontal and after another quarter of a period will have its maximum displacement in the negative direction. At the points N, N', there will never be any displacement. These points are

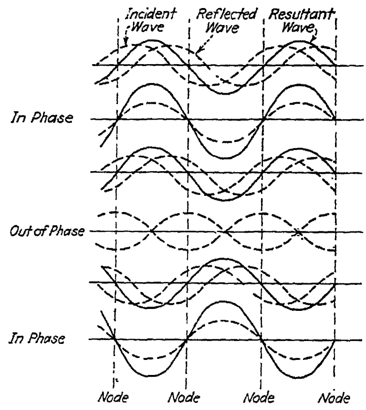


FIG. 187.—Successive positions of incident and reflected waves producing standing waves.

called **nodes.** All the particles of the cord between two adjacent nodes are moving in the same direction at any instant, but two adjacent segments of the cord are always displaced in opposite directions. The length of a segment between two consecutive nodes is equal to one-half of a complete wave length. (See Appendix, note E-6.)

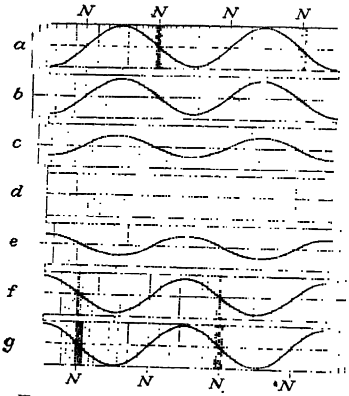


FIG. 188.—Diagrammatic representation of different stages of standing sound waves.

In Fig. 187 are represented successive positions of the incident and reflected trains of waves and the resultant standing wave. At the nodes, the two waves are always in opposite phase, and they neutralize each other. Midway between the nodes, the two waves are always in the same phase, and the resultant displacement goes through its greatest variation. These points having the maximum amplitude of vibration are called **antinodes.**

Standing waves may result with any kind of wave motion. (Appendix E-6.) The essential condition for their production is that two similar wave trains travel in opposite directions in the medium. Figure 188 represents standing waves in a column of air such as is in an organ pipe. The short vertical lines represent layers of air, displaced as shown. It can be seen that at the places marked N in the figure, the air is alternately compressed and rarefied.

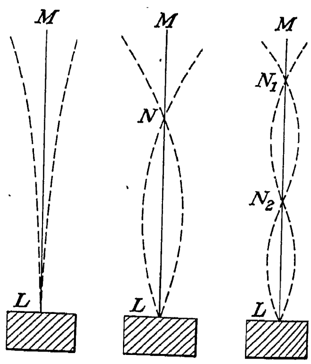


FIG. 189.—Vibrations in a rod. Fundamental A, second overtone B, and fourth overtone C.

221. Tuning Forks.—When the free end of a rod is pulled aside and released, a wave travels to the fixed end where it is reflected and travels back again to the free end. There is thus set up in the rod a set of stationary waves. The modes of vibration possible for such a rod are indicated in Fig. 189.

A tuning fork is essentially a rod bent in the form of a U. When the prongs vibrate, the stem moves up and down with the same period. There are two nodal points, one on either side of the middle point. By giving a tuning fork the form indicated in Fig. 190 and making the prongs close together, the secondary or harmonic vibrations of high frequency present in vibrating rods are nearly excluded, and the tuning fork gives a pure musical tone.

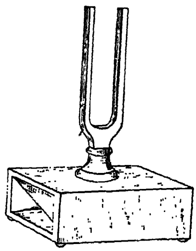


Fig. 190.—A tuning fork mounted on a resonator.

The prongs set so little air into vibration that the loudness of the sound due to their vibrations is not sufficiently great. By resting the stem on a board or on the top of a table, the air in contact with the board is set in vibration and the loudness of the sound very much increased. For this reason, tuning forks are ordinarily mounted on the top of a box which is open at one end. If the air chamber enclosed by the box is chosen of the proper dimensions, the air column will resonate and greatly augment the sound. Since the resonator reinforces most strongly the fundamental of the fork, the sound emitted by the fork on the resonator box will be free from overtones.

Problems

1. A timer sets his watch by the report of a gun 220 yd. away. What is the error due to the time required for the sound to travel from the gun to the ear?

2. The vertical walls of a canyon are 9,300 ft. apart. A man in the canyon fires a gun and hears the echo from the farther wall 6 sec. after the echo from the nearer wall. How far is he from each wall? Assume sound travels 1,080 ft. per second.

3. A sounding source with a frequency of 612 cycles per second sends out waves which travel from air into water. Find the wave length in each medium if the velocities are 330 m. per second and 1,450 m. per second in air and water, respectively.

4. A tuning fork with a frequency of 500 cycles per second sends out waves which travel 1,080 ft. per second. How many vibrations does the fork make in the time required for the sound to travel 810 ft.?

5. Water waves are observed passing a certain point at a velocity of 20 miles per hour, with a distance of 12 ft. between crests. What is the frequency of the waves?

6. Sound travels in water at the rate of 1,450 m. per second. What is the modulus of elasticity of water in c.g.s. units?

7. What is the velocity of a transverse wave in a string 180 cm. long with a mass of 100 g. and subject to a tension of 98 kg.?

CHAPTER XVIII

PRODUCTION AND TRANSMISSION OF SOUND

222. Nature of Sound.—The source of sound is always in a state of vibration. As the vibration dies down, the intensity of the sound diminishes. If a ringing bell is touched with the fingers, the sound ceases because the vibrations are stopped by the fingers. When a weight falls to the floor, the weight as well as that part of the floor which is struck is set in vibration, and sound waves are produced. If a stretched guitar string is plucked, it gives a musical note owing to the vibrations set up in it. These vibrations take place too fast for the eye to follow them, and the string seems to be drawn out into a ribbon in the middle. In a vibrating tuning fork the prongs alternately approach and recede from each other. These movements of the prongs (Fig. 191) can be felt by touching the prongs with the fingers. They produce compressions and rarefactions in the surrounding air. These disturbances travel forward and are the sound waves.

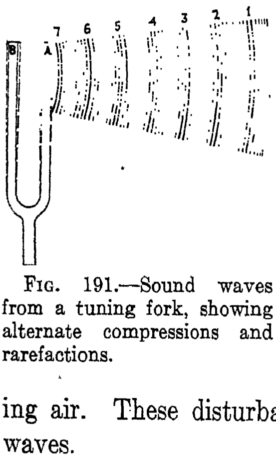


FIG. 191.—Sound waves from a tuning fork, showing alternate compressions and rarefactions.

223. Velocity of Sound.—Since sound is a compressional wave, its velocity depends on the density and elasticity of the medium in the manner discussed in Sec. 216. That is,

$$v = \sqrt{\frac{e}{\rho}},$$

where v = the velocity.
e = the modulus of elasticity.
ρ = the density of the medium.

Where the modulus of elasticity divided by the density is large, the velocity of the sound is great. The velocity of sound

in water is greater than it is in air. In water, the velocity of sound is 1,435 m. per second. In air at 0°C. and at atmospheric pressure, it is 331 m. per second.

If the temperature of the air is changed, the ratio of the modulus of elasticity to the density is changed, and consequently the velocity of the sound changes. If the temperature is increased but the pressure kept constant, the modulus of elasticity remains constant, but the density decreases. Hence, the velocity is increased. The velocity of sound in air is increased about 2 ft. or 0.60 m. per second for each degree centigrade rise in temperature.

224. Speed of Sound in Warm and Cold Air.—Since the speed of sound in warm air is greater than it is in cold air, the direction of propagation of the sound will change as it passes from a layer of air at one temperature to a

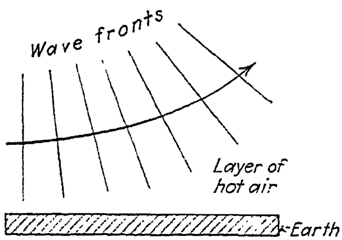


FIG. 192.—Upward deflection of wave front by hot air.

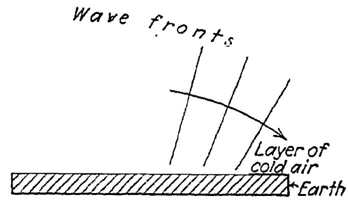


FIG. 193.—Downward deflection of wave front by cold air.

layer of air at a different temperature. If the air is at rest and the temperature and density are uniform, a wave front from a point source on the surface of the earth is spherical. In such a case the direction in which the sound travels is normal to the spherical surface; and this means that it travels along the surface of the ground in all directions. If the air at the surface of the ground is warmer than it is at higher altitudes, the speed of the sound is greater at the surface than at higher altitudes. The wave front will then be as in Fig. 192. The wave front will no longer be perpendicular to the surface of the ground, and, since the direction of propagation is perpendicular to the wave front, the sound will not travel along the surface of the ground but will be deflected upwards. As a result of this upward deflection, the sound cannot be heard to as great distances as if this distortion did not take place. When the air at the ground is colder than at the higher altitudes, the sound travels more slowly near the ground than in the region above it. Since the direction of propagation is normal to the wave front, the sound will be deflected downward (Fig. 193) and travel more nearly along the surface of the earth. Its intensity will decrease less rapidly with the distance, and the distance at which the sound can be heard will be increased. This condition is sometimes noticed on a lake at the end of a hot day.

225. Effect of the Wind on the Wave Front.—When the wind is blowing, the speed of the sound with respect to the earth is decreased in the direction from which the wind comes and increased in the direction in which the wind is blowing. The higher the altitude, the greater is the velocity of the wind

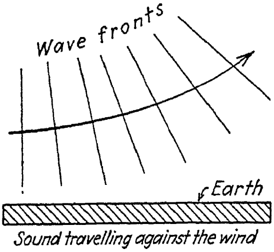


FIG. 194.—Upward deflection of wave front by wind in opposite direction.

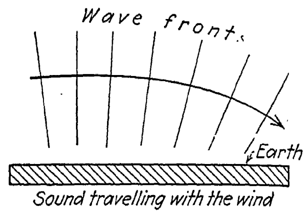


FIG. 195.—Downward deflection of wave front by wind in same direction.

and the greater the change in the speed of the sound with respect to the earth. This unequal change in the speed of the sound waves causes a distortion in the wave front. On the windward side of the source of sound, the speed of the sound is greater at the ground than at points above the

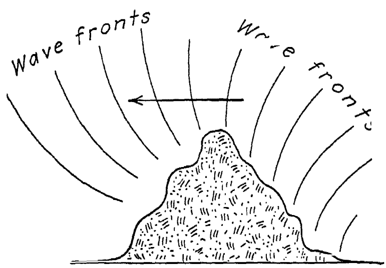


FIG. 196.—Change in wave front by mountain.

ground (Fig. 194). This inequality of speed causes the wave front near the ground to be inclined to the vertical and the line of propagation to be directed upward from the earth. On the side of the source of sound toward which the wind is blowing, the speed of the sound near the ground is less

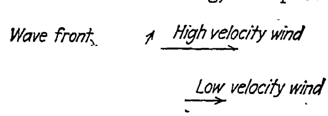


FIG. 197.—Change in shape of wave front by a high-velocity wind.

than at higher altitudes. In this case, the direction of propagation is bent toward the ground (Fig. 195), making it possible for the sound to be heard at greater distances.

If the wind blows against a hill (Fig. 196), the wave fronts are distorted in passing over the hill so that there will be a space behind the hill which the sound does not reach. Beyond this point, there will be sound again.

The change in the wave front due to winds of high and of low velocities is evident in Fig. 197.

226. Pitch and Frequency.—If a sound recurs at regular intervals, it is heard as a series of separate impulses. If the number of these impulses per second is increased, the separate impressions disappear and a continuous note is heard when the number of impulses is about 30 per second. As the frequency of the impulses is increased beyond this number, the pitch of the sound is raised. That there is a direct relation between the pitch and the frequency can be shown by aid of the siren (Fig. 198), which consists of a disk having in it a number of holes distributed on concentric circles. If a draught of air from a tube *A* is directed against the holes while the disk is in rotation, a puff of air passes through each hole and comes out on the other side. When the number of these puffs is increased either by increasing the speed of the disk or by directing the stream of air against the outer holes, the frequency of the sound is increased and the pitch becomes higher. **The greater the frequency, the higher the pitch.**

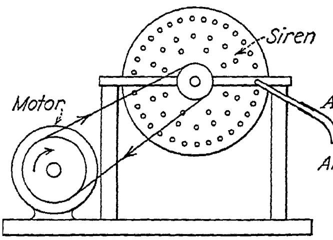


FIG. 198.—The siren. Frequency proportional to the number of holes in a ring.

227. Loudness.—The loudness of a sound depends on the energy of the vibrating source, and the energy in turn depends on the amplitude of the vibrating body. The distance through which the vibrating body moves determines the displacement of the medium through which the sound travels, and the amount of this displacement determines the energy of the vibrating body and the medium surrounding it. The vibrating body is continually under the action of a force which is directed toward the center from which the body was displaced. The greater the displacement, the greater this force and the greater the work to produce the displacement. Since the force is proportional to the displacement, and the work to produce the displacement is equal to the force times the displacement, the work to produce the displacement is proportional to the square of the displacement; that is, the energy of the vibration is proportional to the square of the amplitude. This is true for the sound waves as well as for the vibrating body. Consequently, the intensity of a sound is **proportional to the square of the amplitude of the vibration of the medium.**

228. Quality.—Sounds which have the same loudness and pitch may differ in the complexity of the vibrations and may, therefore, produce a very different effect on the ear. It very rarely happens that a sounding body produces a simple harmonic vibration. A tuning fork on a resonance box produces such a simple tone, but in most cases the tone is not pure. Besides the fundamental vibration, there may be present one or more superposed notes with frequencies which are integral multiples of the frequency of the fundamental. These frequencies are

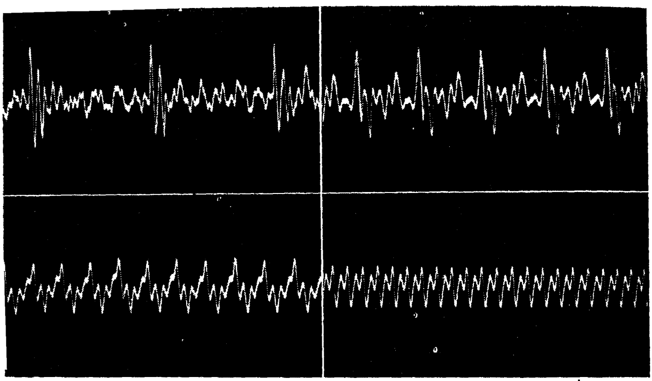


Fig. 199.—The characteristics of the sound emitted by a trumpet are changed by blowing it in different ways.

known either as harmonics or as overtones. The second harmonic is twice the frequency of the fundamental; the third harmonic is three times the frequency of the fundamental; etc. The second harmonic is also called the first overtone, etc. They are very simply related to the frequency of the fundamental, the frequency of the overtones being 2, 3, 4, etc., times that of the fundamental. The number of overtones present and the complexity of the sound will depend on the way in which the body is set into vibration. The ear resolves unconsciously these complex sounds and thus distinguishes differences which are not due to either pitch or loudness. These differences due to the complexity of the sound are differences in **quality.** The characteristics of the sound emitted by a trumpet change according to the way the musician blows it (Fig. 199).

229. Photography of Sound Waves.—By means of suitable apparatus it is possible to obtain photographs of sound waves which are produced in a variety of ways. The method depends

essentially on properly timed spark illumination. The apparatus is so arranged that the illuminating spark occurs while the sound wave to be photographed is between the spark and the photographic plate. There would be a shadow of any opaque object in front of the photographic plate. Now the variations in the density of the air through which the sound waves are traveling cause the light to bend in such a way that *shadows* of the sound waves are produced on the photographic plate.

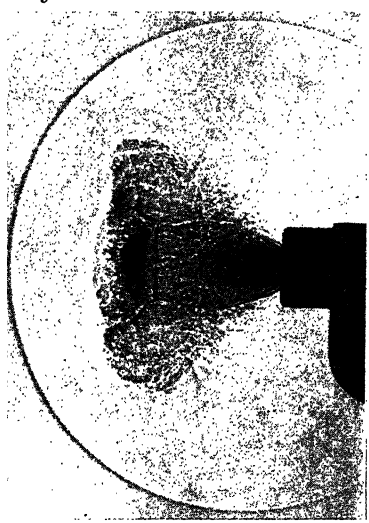


FIG. 200.—Wave front from a revolver which has just been fired. (*Quayle, Bureau of Standards.*)

In Fig. 200 the cartridge has just been fired from an automatic revolver. The mushroom-shaped mass of gas just in front of the muzzle consists chiefly of air which is being pushed out of the barrel. The spherical sound wave was generated by the unseating of the bullet from the cartridge case. A later stage when the bullet is well out of the muzzle but is still being accelerated is shown in Fig. 201. At a somewhat later time (Fig. 202) the projectile has outdistanced all the other effects of the discharge, for it is traveling with a velocity greater than that of the sound waves. It is seen that the projectile has two waves associated with it. One of these starts from the head of the projectile and the other from its base. The angle between these wave fronts and the direction in which the projectile is moving depends on the velocity of the sound wave and the velocity of the projectile. The greater the velocity of the projectile in comparison with the velocity of the sound wave, the less the angle between the wave front and the direction of motion of the projectile.

Figure 203 shows the way in which the wave front is modified when the projectile is fired through a soap bubble filled with a mixture of hydrogen and air. The velocity of sound in this mixture of gases is greater than it is in ordinary air. Hence, the

wave of compression travels faster while the bullet is in the bubble, and the wave front gets ahead of the position it would have occupied if the bullet had not passed through the bubble. The

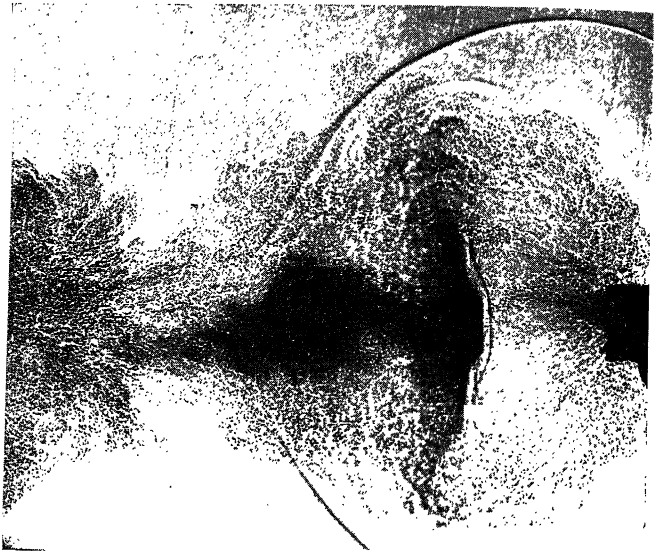


FIG. 201.—Progress of wave front after shot has been fired. (*Quayle, Bureau of Standards.*)

FIG. 202.—Compressional wave traveling with bullet, having a speed of 2,600 ft. per second. (*Quayle, Bureau of Standards.*)

forward curvature of the wave front after leaving the bubble is evident.

By measuring the angle between the direction of motion of the bullet and the direction of the wave front, the velocity of the

bullet can be computed, provided the velocity of sound in the medium is known. This is one of the satisfactory methods for determining the velocities of projectiles.

230. The Intensity of Sound.—When sound waves spread out in every direction from a source of sound, the intensity varies inversely as the square of the distance from the source. In this case, the sound waves spread out as spheres just as the waves discussed in Sec. 215. The same amount of energy is transmitted across every spherical surface having its center at the source of sound. The larger the surface of these spheres, the smaller the energy that goes through each square centimeter

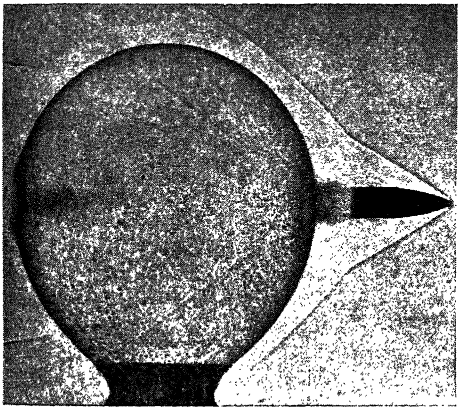


FIG. 203.—Bullet fired through a soap bubble. The bubble has not had time to collapse. Note change in shape of wave front. (*Quayle, Bureau of Standards.*)

of surface. The surfaces of these spheres increase as the squares of their radii. Hence, the energy which passes through unit area decreases as the squares of the radii increase.

1. *Speaking Tubes.*—In a speaking tube the waves are prevented from spreading out, and the intensity of the sound does not decrease as the wave advances. For this reason the sound is heard with only slightly diminished intensity at the other end of the tube.

2. *Ear Trumpet.*—In an ear trumpet the wave entering the wide end is gradually diminished in area until it reaches the small end. In this way, the whole energy of the wave is concentrated in a small area and the intensity is increased. The energy per cubic centimeter may be many times as great as it was in the large end.

3. *Megaphone.*—In a megaphone the sound waves coming from the speaker's mouth are limited by the walls of the megaphone, so that the wave which emerges from the end has all the energy of the speaker's voice.

231. Sound Ranging.—The location of a source of sound can be determined by measurements on the sound waves which spread out from the source. If the sound originates at the point S (Fig. 204), it spreads out in the form of a circle $EFGA$ with S as a center. When observations have been made at the three stations A, B, and C, the position of F can be determined by a geometrical construction. If the sound arrived at B in t_2 sec. after reaching A, and at C in t_1 sec. after reaching A, circles are drawn with B and C as centers and with radii equal to Vt_2 and Vt_1 where V is the velocity of sound in air. The center of that circle which passes through A and is tangent to the circles drawn about B and C as centers coincides with the point S which is the origin of the sound. This method cannot be applied to a continuous sound. It can be used only where the sound has a definite beginning.

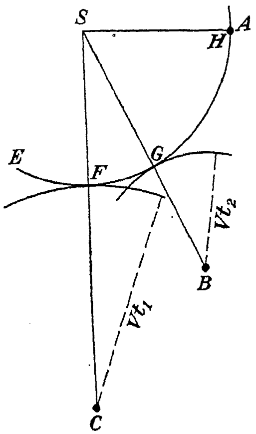


FIG. 204.—Finding the range by sound. Observations at three stations are necessary.

232. Detection of Aircraft by Sound.—It is often important to be able to locate aircraft at night or in foggy weather. This location can be made by means of the sound from the engine or the propellers. Two large conical trumpets are fixed on a framework with their axes parallel. The framework can be rotated about a vertical axis. The trumpets, which are a few feet apart, have their small ends connected by rubber tubes to the ears of the observer. If the trumpets with their axes horizontal are kept stationary, and a source of sound, as some one blowing a horn, moves past them from left to right, the effect on the listener is as if the source of sound were directly out on the left until it is nearly in front of the trumpets, and then it seems to move rapidly across until it appears to be straight out on the right. The instant when the source of sound was directly in front of the observer can be determined with considerable accuracy.

In locating an airplane, the trumpets are turned around a vertical axis until the observer judges that they are pointing directly at the source of sound. A small movement of the trumpets from this position causes the source of sound to seem to move far to the left or to the right. The direction from which the sound is coming can then be found with accuracy. This method gives the direction from which the sound was traveling when it finally reached the observer. If the sound has been reflected or refracted on its path, the true direction of the sound may be different from this observed direction.

233. Location of Sound under Water.—It is frequently important to transmit signals under water or to locate the position of a sounding body which is submerged, as a submarine in the water. One method of locating such a body is as follows: A thin sheet of metal framed by a heavy ring has a small microphone at its center and is hung in the water. One side of this metal disk is screened by a baffle plate made of some non-resonant material. This screening makes it impossible for sound to reach the microphone except through the unscreened face. If the metal disk is caused to vibrate, sounds will be heard in a telephone which is connected to the microphone. If the plane of the metal disk faces the direction from which the sound waves come, the intensity of the sound in the telephone is greatest. If, on the other hand, the disk is turned around so that the sound waves travel in a direction parallel to its plane, no vibrations will be produced in the microphone. By suspending the disk in the water with its plane vertical and then turning it so that it faces various directions of the compass, it will be possible to find a position from which the intensity of the sound seems to be a maximum, and a second position for which it is a minimum. The latter position is at right angles to the former. The sounding body must lie in the direction from which the sound appears to be a maximum.

Problems

1. The disk of a siren has 48 holes. Find the frequency of sound produced at 1,800 revolutions per minute.

2. An air-driven turbine breaks up the stream of air into 16 pulses per revolution. What frequency of sound will be heard when the turbine is rotating at the rate of 8,100 revolutions per minute?

3. Determine the pitch produced by the exhaust of a 16-cylinder engine at 2,800 revolutions per minute, if each cylinder fires once for every two revolutions.

4. What is the modulus of elasticity for air, when the density is 0.001293 g. per cubic centimeter and the velocity of sound is 330 m. per second?

5. What is the velocity of a compressional wave in a steel rod for which Young's modulus of elasticity is 2.2×10^{12} dynes per square centimeter? Assume the density of the rod is 7.3 g. per cubic centimeter.

6. The velocity of sound in steel is 5,000 m. per second and the density of steel is 7.8. What is the coefficient of elasticity of steel?

7. The velocity of sound in air at atmospheric pressure and 0°C. is 332 m. per second. What is the velocity of sound in hydrogen at the same temperature and pressure? The density of hydrogen is 0.0896 g. per liter and that of air is 1.293 g. per liter at 0°C. and atmospheric pressure.

CHAPTER XIX

REFLECTION, REFRACTION, ABSORPTION, AND INTERFERENCE

234. Reflection of Sound.—If an observer stands at some distance in front of a cliff or large building and produces a sound, the sound is returned to him. The sound travels to the wall or cliff and then back without having its characteristics essentially changed. If the observer were about 1,100 ft. from the cliff, it would take the sound about 2 sec. to return to him. The rolling of thunder is due to the reflection of the original sound by clouds at different distances from the observer. These reflections reach the observer at different times and produce the rolling or continuation of the sound. Sounding boards are often placed behind pulpits in order to reflect the voice of the speaker toward the audience. This prevents the sound from traveling upward and backward and thus being lost. In large halls and churches, these reflections often make it difficult to hear distinctly the words of the speaker. In such cases, precautions must be taken to reduce the reflections and the resultant echoes to a minimum. Whispering galleries are so constructed that they reflect the sound to a particular point without much loss in intensity.

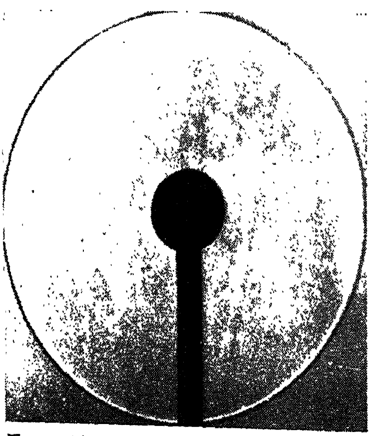


FIG. 205.—Spherical sound waves. (*Foley, University of Indiana.*)

Figure 205 shows the way in which an undisturbed spherical sound wave spreads out from its source. In Fig. 206*a* a spherical wave has been reflected by a plane surface. When the source of the sound is at the focus of a parabolic mirror (Fig. 206*b*), the reflected wave front is a plane. Figure 207 shows the simul-

taneous reflection and transmission of sound waves. The transmitted system of waves has come through four rectangular apertures, and the reflected system has come from three rectangular bars and the surface above and below them.

235. Angle of Reflection.—When a sound wave meets a reflecting surface, the direction of propagation is changed. If a spherical wave is spreading out from O (Fig. 208) in the direction OP, it will be reflected from the surface AB so that after reflec-

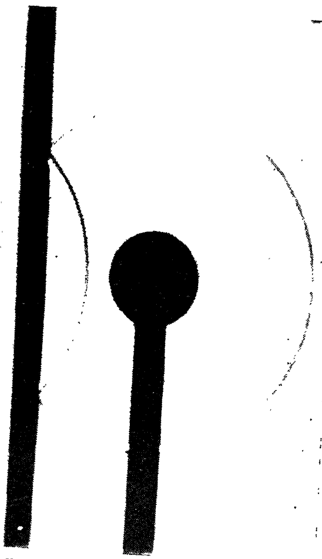


FIG. 206*a*.—Reflection of spherical sound waves by a plane surface. (*Foley, University of Indiana.*)

FIG. 206*b*.—Reflection of spherical sound wave by a parabolic mirror. (*Foley, University of Indiana.*)

tion it will travel in the direction PD. The reflected wave front W_2 will have O' for its center, and the reflected waves will behave as if they had come from O' instead of from O. The angle i which the direction of the incident wave makes with NP, the normal to the reflecting surface, is called the **angle of incidence**. The angle which the reflected ray PD makes with NP is called the **angle of reflection. Reflection always takes place so that the angle of incidence is equal to the angle of reflection.** Figure 209 shows the reflection of spherical sound waves originating at O by the plane surface AB. The reflected wave front AMB is spherical.

236. Refraction of Sound Waves.—When sound waves pass from one medium into another, there is in most cases a change in velocity. This change in velocity causes the direction of propagation of the wave to be changed in case the waves meet the surface of separation obliquely. Let AC (Fig. 210) represent the advancing wave front. When it meets the surface AB and enters the second medium, its velocity becomes less. The other end of the wave front at C travels forward with its normal velocity. When the entire wave front has entered the second medium, it has been rotated so that the direction of propagation makes a smaller angle with the normal to the surface of separation of the two media. When the sound leaves a medium in which the velocity is greater and enters one in which the velocity is less, the direction of propagation of the wave is bent toward the normal. If, however, the sound goes from a medium in which the velocity is less and enters a medium in which it is greater, the direction in which the wave is traveling is bent away from the normal.

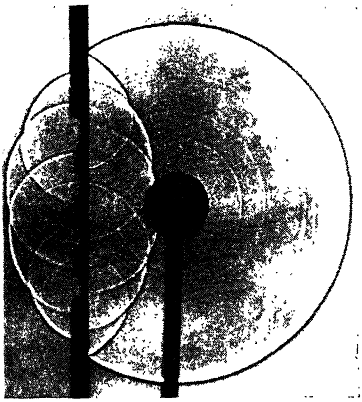


FIG. 207.—Transmission of spherical sound waves by rectangular apertures. (*Foley, University of Indiana.*)

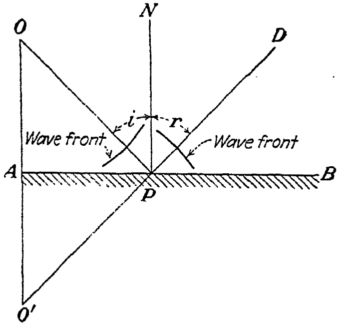


FIG. 208.—Reflection of sound waves. The angle of incidence is equal to the angle of reflection.

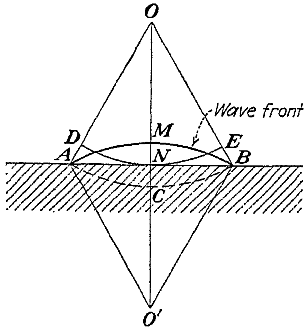


FIG. 209.—Reflection of spherical sound waves. The curvature of the wave front is reversed by reflection.

The angle between the normal to the surface of separation of the media and the direction of the incident sound is called the **angle of incidence.** The angle between the normal to the surface and the direction of the refracted sound is called the **angle of refraction.**

Figure 211 shows how a spherical sound wave coming from a point source S is bent by a spherical gas-filled lens so that,

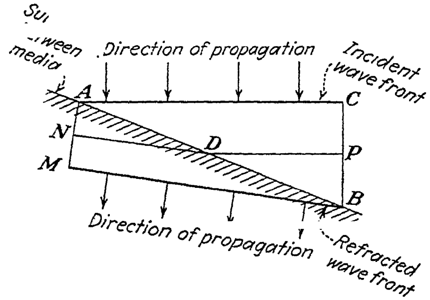


FIG. 210.—Refraction of sound waves. Direction of propagation is changed.

instead of diverging, it is made to converge to a point P. The lens is here a rubber bag filled with a gas in which the velocity of sound is less than it is in air.

237. Sound Waves in a Room.—When waves of sound are produced in a room, they spread out until they strike the walls, ceiling, or floor of the room. Here they are partly reflected

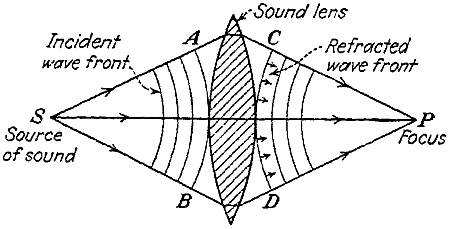


FIG. 211.—Refraction of sound waves by a lens. The curvature of the wave front is reversed.

(Figs. 212 and 213), partly absorbed, and partly transmitted. The amount of absorption or reflection depends on the character of the walls. A hard smooth wall reflects most of the sound and absorbs little of it. On the other hand, a porous or yielding wall built of a substance like felt absorbs most of the sound and reflects little of it. In such cases, after several reflections,

the energy of the sound waves is much decreased and the intensity of the sound dies out.

If a sound is maintained for some time in a room, the way in which it builds up is shown in Fig. 214. After a time a state of

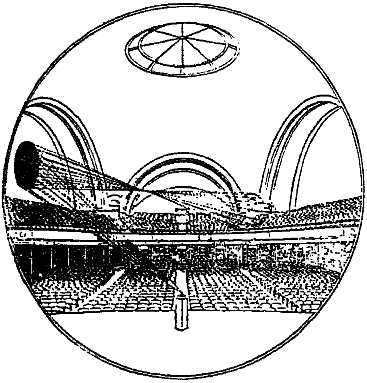


FIG. 212.—Sound reflected from a concave wall in a balcony. (*Watson, Bulletin Engineers Experiment Station, University of Illinois.*)

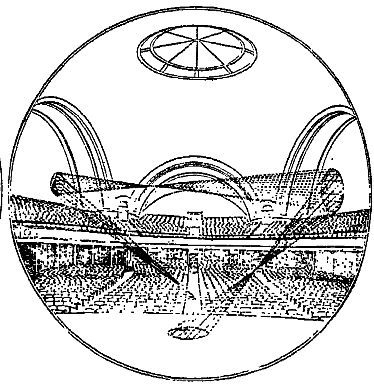


FIG. 213.—Echo formed on the [illegible] by two reflections. (*Watson, Bulletin Engineers Experiment Station, University of Illinois.*)

equilibrium is reached, and the intensity does not increase further with the time. In this state, the energy absorbed per second by the room is just as great as that supplied by the sounding body. When the source of sound is discontinued, the intensity of the

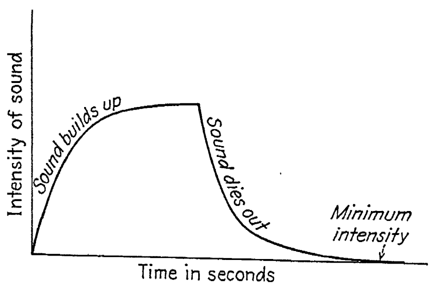


FIG. 214.—Rise and decay of sound in a room.

sound dies down as indicated in the second half of the curve in Fig. 214.

238. Absorption of Sound.—The reverberation or decay of a sound in a room is determined by the shape and size of the room and the character of its walls. For good acoustics the time of

reverberation should be short, so that when a sound has been produced, it will die away rapidly. The introduction of absorbing material increases the rate at which the sound decays. The behavior of sound in a room with little absorption and in the same room when absorbing material has been introduced is shown in Fig. 215. With small absorption the sound rises slowly to a maximum and then dies out slowly. The acoustics of such a room are not good. When the absorption is large, the maximum intensity is reached quickly, and then the sound dies out rapidly when the source of sound is removed. Such a room has good acoustics. The absorption, however, must not be made too

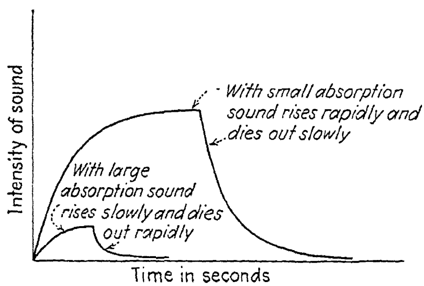


FIG. 215.—Effect of absorption on rise and decay of sound in a room.

large or the room will absorb the sound too rapidly and thus cause the acoustics to be defective.

239. Insulation of Sounds in Buildings.—The amplitude of vibration in sound waves varies approximately between 0.00000005 in. for barely audible sounds to 0.004 in. for loud sounds. A small motion of the partition between buildings will be sufficient to produce an audible sound. In making buildings sound-proof, two types of sound must be considered. One type includes sounds which originate in the air and travel through the air to the boundaries of the room. The other type of sounds comprises those sounds which are generated in the framework of the building by motors, elevators, etc. Sounds of moderate intensity, such as those generated by the human voice, are stopped with relative ease when the walls of the room are continuous and fairly rigid. All breaks in the walls for ventilators, doors, or pipes must be protected by effective insulation. Compressional waves generated in the structure of the building pass easily along the continuous solid materials.

240. Passage of Sound from One Medium to Another.—When sound waves in one medium encounter a second medium with a different elasticity or density, their regular progression is dis-

turbed. Part of the energy (Fig. 216) is thrown back in the form of reflected waves, part is absorbed in the second medium, and part is transmitted. The relative amounts in each case depend on the elasticity and density of the second medium compared with the first. In a material like hairfelt, the reflection at the surface is small, but the absorption in the porous channels may be quite large. If sound waves generated in a room fall on solid plaster, nearly all of the sound will be reflected because of the large change in the elasticity and density between air and the solid plaster. In a similar way, sounds generated in the solid structure of the building are confined to this structure by almost total reflection at the air boundaries and will pass with little diminution through the steel and concrete to distant parts of the building.

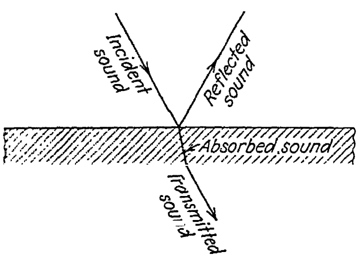


FIG. 216.—Reflection, absorption, and transmission of sound. (*Watson, Bulletin Engineers Experiment Station, University of Illinois.*)

When air passages in which sound is traveling become small in cross section, some of the energy of the sound waves is converted into heat because of friction between the sides of the passages and the oscillating air particles. The channels and interstices in carpet, hairfelt, and other porous materials absorb much of the energy in the sound waves. The amount of this absorption varies with the thickness of the absorbing material, but it is not proportional to the thickness. For example, if 1 in. of hairfelt stops 10 per cent of the incident sound, 2 in. will stop 19 per cent and 3 in. will stop 27 per cent, etc. The intensity of the transmitted sound decreases according to an exponential law,

$$I = I_o a^{-bx}$$

where I = the intensity of the transmitted sound.
I_o = the intensity of the incident sound.
x = the thickness of the substance.
a and b = constants.

Figure 217 shows the apparatus used by Watson for testing the percentage of sound transmitted, reflected, and absorbed by

different substances. The intensity of the original sound and the intensities of the transmitted and reflected parts were then determined. The amount absorbed may be found by finding the

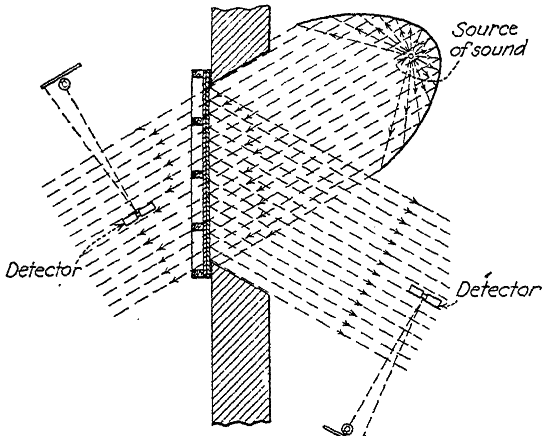


FIG. 217.—Apparatus for testing transmission and reflection of sound. (*Watson, Bulletin Engineers Experiment Station, University of Illinois.*)

difference between the intensity of the incident sound and the sum of the intensities of the reflected and the transmitted sound. Figure 218 shows the results for various thicknesses of hairfelt.

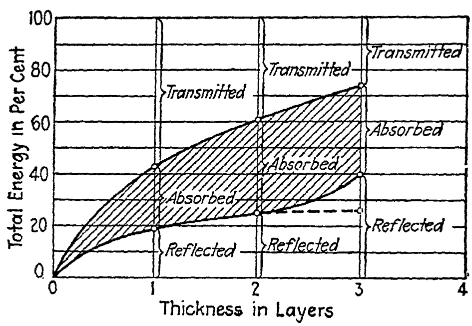


FIG. 218.—Relative amounts of sound absorbed, reflected, and transmitted by hairfelt. (*Watson, Bulletin Engineers Experiment Station, University of Illinois.*)

241. Reverberation.—Reverberation is due to the echoing and reechoing of sounds in a room because of the repeated reflection of the waves from the floor, ceiling, seats, etc. For halls of the same volume, other things being equal, the reverberation is the

same; but as the size of the hall increases, the reverberation also increases. The reverberation is greatly decreased by putting soft coverings on the floor, seats, or walls, by making the walls so they are less rigid, and by the presence of an audience in the hall. Whatever increases the absorption of the sound in the hall decreases the reverberation; for as the sound falls on the walls, seats, or audience, it is either reflected or absorbed. If a great deal of it is absorbed, the reverberation is small. If, on the other hand, most of the sound is reflected by the floors, walls, and audience, a sound in a room will continue for some time after the source of sound has been discontinued. In such a case, the reverberation in the room is great.

242. Interference of Sound Waves.—When two trains of waves of the same wave length arrive at the same point, their superposi-

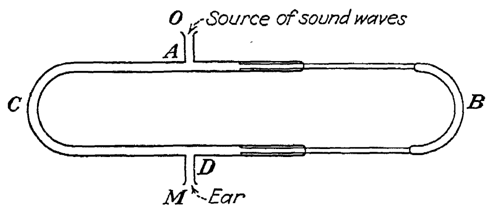


FIG. 219.—Interference of sound after traveling along paths of different lengths. When the difference in path is equal to the wave length, the sounds reinforce each other.

tion produces a disturbance which has a frequency equal to the frequency of either of the component disturbances. The amplitude of the resultant disturbance depends on the phases of the two component disturbances. If the amplitudes of the original disturbances are the same and the phases the same, the superposition of these disturbances gives a new disturbance which has double the amplitude of either of the original disturbances and four times the energy of either of them. If the phases are opposite, the union of the two trains produces no resultant vibration and there is silence. When the phases have an intermediate relation, there will be a vibration of intermediate amplitude.

The fact that sound waves interfere can be shown from the following experiment: In Fig. 219 sound waves can be carried by two paths $OACDM$ and $OABDM$ from a source at O to a detecting instrument at M. The length of the path $OABDM$ is adjustable, and when the total length of this path exceeds the total

length of the path $OACDM$ by one-half wave length or by an odd number of half wave lengths, the two trains of waves arrive at M in opposite phases and neutralize each other. With a shorter or longer difference of path, some sound can be detected at M. When the difference of path for the two trains is equal to one whole wave length or to any whole number of wave lengths the two trains of wave reinforce each other at M.

243. Beats.—Let two organ pipes which have exactly the same frequency be sounded together. At any given place the state of the disturbance emitted by the pipes remains constant. If the upper end of one of the pipes is now partially closed by sliding a brass plate over it, the pitch of this pipe is slightly lowered. When the two pipes are now sounded together, there

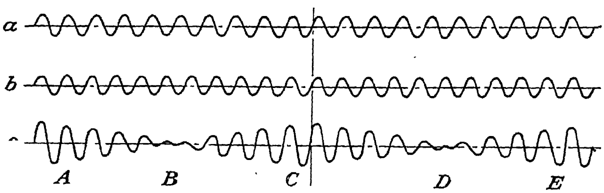


FIG. 220.—Beats. Superposition of sounds differing slightly in frequency.

is no longer unison, but bursts of sound with comparative silence between them are now noticed. These bursts of sound where the disturbance from one pipe reinforces the disturbance from the other pipe are called **beats.** The reason for these beats is that at a certain instant the compressions from both sounds arrive at a given point simultaneously. A short time later (Fig. 220), the more rapidly vibrating source is one-half vibration ahead of the other, and the compression from one sound arrives at the same time as the rarefaction from the other body. The two disturbances neutralize each other and produce a minimum of sound. One train of sound waves gradually falls still further behind the other until there is now a difference of one period, and the compressions from each sounding body arrive at the point at the same time. The disturbances are thus added together and a maximum of sound is produced. There results a pulsating effect in which soft and loud pulses alternate. These pulses are **beats.**

When the two sounding bodies are almost in unison, the number of beats per second is small. As the difference in the fre-

quencies of the sounds is increased, the number of beats is increased. If the sounds differ in frequency by unity, they will reinforce once each second and there will be one beat each second. In general, the difference in the frequency of the two sounding bodies is equal to the number of beats per second.

Number of beats = difference in frequencies.

$$N = n_1 - n_2.$$

Beats are of great service in tuning stringed instruments. As two strings are brought more and more nearly into unison, the number of beats per second become less, and, when no beats can be observed, the strings have the same pitch. In organs, two pipes having nearly the same frequency are sometimes used to produce a beating of the sounds giving a tremulous effect imitating the human voice.

244. Doppler Effect.—When a source of sound is moving toward the observer, or the observer is moving toward the source of sound, the sound appears to have a pitch which is higher than its normal pitch. If, on the other hand, the source of sound is moving away from the observer, or the observer is moving away from the source of sound, the pitch seems to be lowered. This change in pitch is often observed in listening to the whistle of an approaching or receding locomotive. The reason for this change of pitch is the fact that in one case fewer, and in the other case more, sound waves reach the ear than would reach it if the observer and the source of sound were not moving with respect to each other.

Let V = the velocity of sound.
v = the velocity with which the source of sound is approaching the observer or the observer is approaching the source of sound.
n = the true frequency of the sound waves = the number of sound waves sent out per second by the source.
n' = the apparent frequency of the sound = the number of sound waves reaching the ear when either the source of sound or the observer is in motion.

When the observer and the source of sound are both at rest, the ear receives in 1 sec. all the waves which lie in the distance V (Fig. 221), but when the observer is moving toward the source of

sound, his ear receives in 1 sec. all the waves which lie in the distance $V + v$. Hence,

$$\frac{V + v}{V} = \frac{n'}{n}.$$

When the observer is moving away from the source of sound, v is negative.

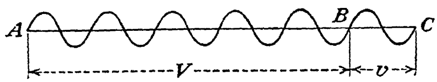


FIG. 221.—Doppler effect.

Example.—A siren is making 150 vibrations per second. An observer is moving toward it with a velocity of 40 ft. per second. The velocity of sound in air is 1,080 ft. per second. What is the apparent frequency of the siren?

$$\frac{\text{velocity of sound} + \text{velocity of observer}}{\text{velocity of sound}}.$$

$$\frac{n'}{n} = \frac{V + v}{V}.$$

$$\frac{n'}{150} = \frac{1{,}080 + 40}{1{,}080}.$$

$$n' = \text{apparent frequency} = 155.5 \text{ per second.}$$

If the source of sound is moving toward the observer, the effect on the pitch is similar to that when the observer moves toward the source. Each wave is shortened by a distance vt, where t is the time for one complete vibration of the sounding body. When the source of sound is at rest, the distance between two successive waves is $Vt = \lambda$, but when the source of sound is in motion, the distance between two successive waves is $Vt - vt = \lambda'$. Since the frequency is inversely proportional to the wave length,

$$\frac{\lambda}{\lambda'} = \frac{n'}{n} = \frac{V}{V - v}.$$

Example.—The whistle on a locomotive which is moving 80 ft. per second toward the observer is making 250 vibrations per second. What is its apparent frequency for an observer at rest?

$$\frac{\text{Apparent frequency}}{\text{True frequency}} = \frac{\text{velocity of sound}}{\text{velocity of sound} - \text{velocity of source}}.$$

$$\frac{n'}{n} = \frac{V}{V - v}.$$

$$\frac{n'}{250} = \frac{1{,}080}{1{,}080 - 80} = \frac{1{,}080}{1{,}000} = 1.08.$$

$$n' = \text{apparent frequency} = 270 \text{ per second.}$$

The positions of the wave fronts from a moving source of sound are quite different from what they would be from the same source of sound at rest. Figure 222 shows the positions with respect to each other of the wave fronts from a point source when

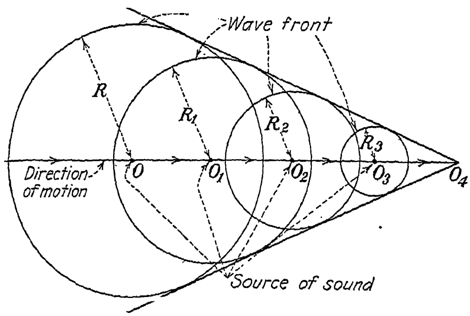


FIG. 222.—The wave fronts from a source of sound which is moving forward. The wave fronts are circles with centers displaced along the axis. The line tangent to these circles represents the resultant wave front.

the source is moving with a velocity greater than the velocity of sound.

Problems

1. A set of circular ripples is produced on the surface of a pond by throwing a stone into the water. At a certain instant, the first crest is 2 m. from the point where the stone hit the water, and the third crest is 30 cm. from the same point. What is the wave length of the disturbance?

2. An under-water signaling system at a lighthouse sends out a sound signal simultaneously with a flash of light. A vessel receives the sound signal through sea water 3 sec. after observing the flash of light. How far is the vessel from the lighthouse?

3. A sound wave travels through two branches of a tube 4 ft. long and 5 ft. 8 in. long, respectively. Name three frequencies which would suffer destructive interference on being recombined after traveling through the branches.

4. Compressional waves with a frequency of 12,000 cycles are sent down a tube, the far end of which has a movable piston. The reflected wave reinforces the source at two successive positions of the piston differing by 5.5 in. What is the velocity of the waves in the tube?

5. Two strings A and B originally had the same frequency. The tension of B is released slightly, and the strings produce five beats per second when sounded together. If the frequency of A is 515 cycles per second, what is the frequency of B?

6. At what rate must a source of sound approach an observer in order to have the pitch of each note raised by a half tone, that is, to $\frac{16}{15}$ of the original frequency?

7. What speed of a source of sound, away from an observer, will cause the sound heard to have a pitch which is $^{15}/_{16}$ of the true pitch?

8. A man on a train which is running 40 miles per hour listens to a siren which has a frequency of 360 vibrations per second. What is the apparent frequency of the sound when the train is approaching the siren?

9. Two bells, each of which has a fundamental frequency of 500 cycles, are sounded together. One of the bells is stationary and the other is moving away from the observer at the rate of 20 ft. per second. How many beats will the observer hear in 15 sec.?

10. How rapidly must a train be moving away from a stationary observer so that the pitch of its whistle will seem to drop from 1,100 to 900 vibrations per second?

11. An engine is approaching an observer at the rate of 60 miles per hour. If the pitch of the note seems to be the same as that of a tuning fork which makes 250 vibrations per second, what is the actual pitch of the whistle?

CHAPTER XX

SOUNDING BODIES

245. Vibrations of Wires and Strings.—When the center of a stretched string is displaced and then released, the disturbance is handed on from one element of the string to the next and a transverse wave is produced. These waves are reflected at the fixed ends of the string, return to the center of the string where they pass each other, and go on to the ends where they are again

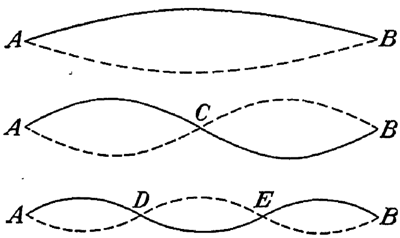


FIG. 223.—Vibrations in strings. Fundamental, first, and second overtone.

reflected. They combine to form standing waves in the string. When the string is plucked in the middle (Fig. 223), the whole string vibrates as one segment. When the string is held lightly at the middle point and plucked at a point midway between the middle and one end, the string vibrates in two segments. In a similar way, the string may be made to vibrate in three, four, etc., segments. The frequency of the sound is increased as the number of segments increases.

If plucked at random, the string may vibrate as a whole and at the same time be vibrating in segments. The note from the string then consists of the fundamental together with one or more overtones. In this case, the stationary wave in the string is the resultant of the fundamental vibration and all the overtones.

Let L = the length of the string, n = the number of vibrations per second, v = the velocity of the disturbance in the string, and λ = the wave length of the disturbance. Then,

$$v = n\lambda.$$

In the case of the fundamental, the disturbance travels twice the length of the string for each vibration. Therefore, $\lambda = 2L$ and

$$v = 2nL.$$

The velocity of a disturbance in a string (Sec. 216) is equal to the square root of the tension in the string divided by the mass of the string per unit length.

$$v = \sqrt{\frac{T}{m}},$$

where T = the tension in dynes, m = the mass in grams per unit length, and v = the velocity in centimeters per second. From these two expressions it follows that

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}}.$$

By means of this relation, the pitch or frequency of a sound emitted by a string can be calculated.

An illustration of this law is found in all stringed musical instruments like the violin or guitar. The strings are of the same length, and the pitches

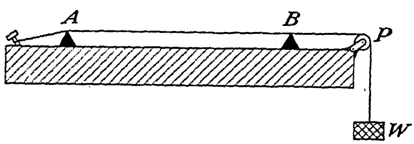


FIG. 224.—Sonometer for studying the laws of vibrations of strings.

are determined by the masses of the strings per unit length and the tensions to which they are subjected. The strings of lowest pitch have large masses per unit of length and are under small tensions. The strings of high pitch are light in weight and are under greater tensions.

These laws may be verified by a sonometer (Fig. 224). A weight W is fastened to one end of a string passing over a pulley P. By adding different weights, the tension in the string may be varied; and by moving the supports A and B, the length of the part of the string in vibration can be changed. Finally, strings of different mass per unit of length can be used. In this way, the law for the frequency of vibration of strings can be verified.

246. Vibration of Closed Pipes.—In an organ pipe, in addition to the box containing the column of air, there is a device at one end by which the column of air can be set in vibration. A compression thus set up travels to the end of the column, where in the

case of the closed pipe (Fig. 225), it is reflected as a compression and returns to the open end of the pipe. The column of air is thus set in vibration and imparts its motions to the surrounding air. The frequency of the wave emitted is determined by the length of the column.

When a steady state of vibration has been reached, the length of the pipe is equal to one-quarter of a wave length for the lowest pitch or fundamental tone of the pipe. The closed end of the pipe is a node where the movement of the air is zero, and the open end is an antinode where the movement of the air is a maximum.

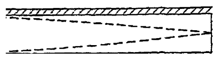


FIG. 225.—Fundamental in a closed organ pipe.

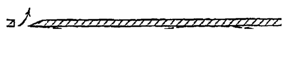


FIG. 226.—Second overtone in a closed organ pipe.

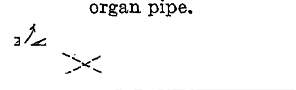


FIG. 227.—Fourth overtone on a closed organ pipe.

There are other possible modes of vibration of this column of air which will make the closed end of the pipe a node and the open end an antinode. Any frequency of vibration is possible for which the closed end is a node and the open end an antinode. The mode of vibration when the pipe emits the second overtone is shown in Fig. 226, and for the fourth overtone in Fig. 227. When the pipe is emitting its fundamental (Fig. 225) the frequency is,

$$n = \frac{v}{4l}.$$

For the second overtone (Fig. 226) the frequency is

$$n_1 = \frac{3v}{4l}$$

and for the fourth overtone it is (Fig. 227)

$$n_2 = \frac{5v}{4l}.$$

Overtones which have a frequency 2, 4, 6, etc., times the fundamental cannot be emitted by a closed organ pipe.

It, therefore, follows that the possible frequencies of vibration in a closed pipe are in the ratio of the odd numbers 1, 3, 5, 7, etc.

247. Vibrations of Open Pipes.—In an open pipe (Fig. 228) the displacement of the air must be a maximum at the open end so that the open end must be an antinode or a place of maximum vibration. In such a pipe the simplest possible mode of vibration is the case in which there is a node at the middle and an antinode at each end (Fig. 228). It follows that the length of the pipe, which in this case is the distance between two antinodes, is equal to one-half the wave length of the emitted sound. Then, since $v = n\lambda$, the frequency of the fundamental is

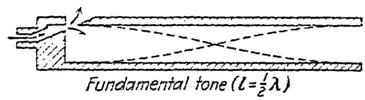


FIG. 228.—Fundamental in an open organ pipe.

$$n = \frac{v}{2l}.$$

The next possible mode of vibration (Fig. 229) is the case where the pipe is one wave length and there are two nodes in it. This is the first overtone and for it

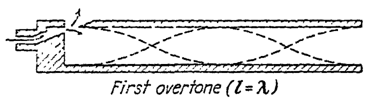


FIG. 229.—First overtone in an open organ pipe.

$$n_1 = \frac{v}{l}.$$

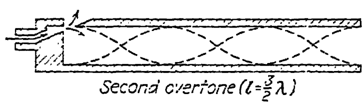


FIG. 230.—Second overtone in an open organ pipe.

It has twice the frequency of the fundamental. For the second overtone (Fig. 230),

$$n_2 = \frac{3v}{2l},$$

and this overtone has three times the frequency of the fundamental. The frequencies of the overtones in the open pipe are, therefore, in the ratio of the natural numbers 1, 2, 3, 4, etc. In the open pipe it is possible to have all the overtones, but in the closed pipe only the even-numbered overtones are possible.

Problems

1. A string with a mass of 0.02 g. per centimeter is stretched by the application of 3.2×10^7 dynes. What length of string will be required to produce a frequency of 625 cycles?

2. A steel string with a diameter of 0.5 mm. and a length of 90 cm. is stretched with a force of 10-kg. weight. What is the fundamental frequency of vibration?

3. If the fundamental of a wire 96 cm. long has a frequency of 212 vibrations per second when the wire is stretched by a load of 12 kg., what will be its frequency when the load is decreased to 8 kg.?

4. Calculate the lengths of open and closed pipes, respectively, for a fundamental frequency of 440 cycles per second.

5. The whistle of a steamer is in the form of a closed pipe 5½ ft. long. Calculate the frequency of the fundamental produced.

6. Two open organ pipes of length 30 and 29 in., respectively, are sounded simultaneously. How many beats per second will be produced if the velocity of sound in the pipes is 1,090 ft. per second?

7. Two closed organ pipes of length 40 and 41 in., respectively, are sounded together, and produce 2 beats per second. What is the velocity of sound in the medium with which the organ pipes are filled?

8. The frequency of the first overtone in a wire 200 cm. long is 64. The mass of the wire is 2.5 g. Find the tension in the wire.

9. The first overtone of an organ pipe produces 4 beats per second, when sounded with a tuning fork which has a frequency of 360 per second. What is the length of the pipe?

10. One ship sends signals to a neighboring ship. The sound waves travel by two paths, one in air and the other in sea water. The signals are heard on the neighboring ship at intervals which are 6 sec. apart. How far is it from one ship to the other?

CHAPTER XXI

AUDITION AND VOICE SOUNDS

248. Speech Sounds.—Speech sounds produced in ordinary conversation are radiated from the speaker and transmitted through the air by means of pressure waves. These pressure waves have small amplitudes, but they are exceedingly complicated. The amplitudes and frequencies of the different components which are present in different speech sounds vary from one

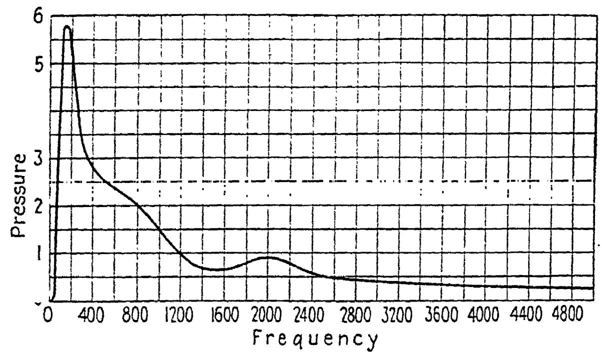


FIG. 231.—Pressure-frequency distribution for undistorted speech. (*Courtesy of Western Electric Company.*)

voice to another, but the average speech may be represented by such a curve as that shown in Fig. 231, which represents the pressure of the waves in relation to the frequency of the different components of the sound. It is seen from this curve that speech energy extends from a frequency of about 60 per second to a frequency of more than 6,000 per second. The energy is a maximum where the frequency is somewhat less than 200 per second. The vowel sounds carry most of this speech energy. The consonants are weak in energy and rather high in frequency. The speech output of the normal human voice is about 125 ergs per second. This is a very small amount of energy so that in terms of power or energy the human voice is very weak.

For comparison, a corresponding curve (Fig. 232) for pipe organ music is given. This curve shows that the pressure is a maximum when the frequency is about 64 vibrations per second.

249. Distribution of Energy in Speech Sound.—Figure 233 shows the results of the analysis of some simple vowel sounds by Fletcher. In these figures, the frequencies present in these sounds are plotted on the horizontal axis, and the amplitude of the sound of a given frequency is plotted on the vertical axis. The lengths of the vertical lines give the relative amount of the changes of pressure in the air through which the sound passed. These figures show that a simple vowel sound is really a complex sound composed of a number of frequencies, and that the amplitudes of the sounds corresponding to these characteristic frequencies vary over a wide range. This distribution of energy

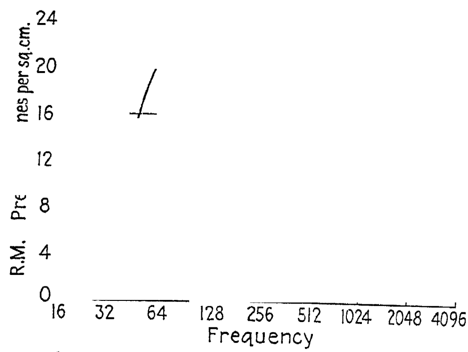


FIG. 232.—Pressure-frequency distribution for pipe-organ music. (*Courtesy of Western Electric Company.*)

among the sounds of different frequencies varies greatly from one vowel to another, or more generally from one sound to another sound.

250. Production of Speech Sounds.—The organs of speech are the lungs which by their bellows action supply the streams of air which pass in and out through the vocal passages—the vocal cords, the tongue, the lips, and the cavities of the nose and throat. These organs impress on the streams of air vibrations which are heard as speech sounds. The vibrations of the vocal cords start a train of sound waves which are conducted through the vocal passages. These passages impress on this train of waves certain resonant characteristics, and the vibrations finally emerge from the mouth as speech sounds. These so-called voiced sounds include all the vowel and consonant sounds except *p, t, ch, k, f, s, th* (thin), and *sh*. The vocal cords do not enter into the production of these last mentioned speech sounds. They are produced by certain vibrations set up in the mouth its

The voiced sounds may be divided into two classes: (1) those [illegible] a continuous flow of air, and (2) those

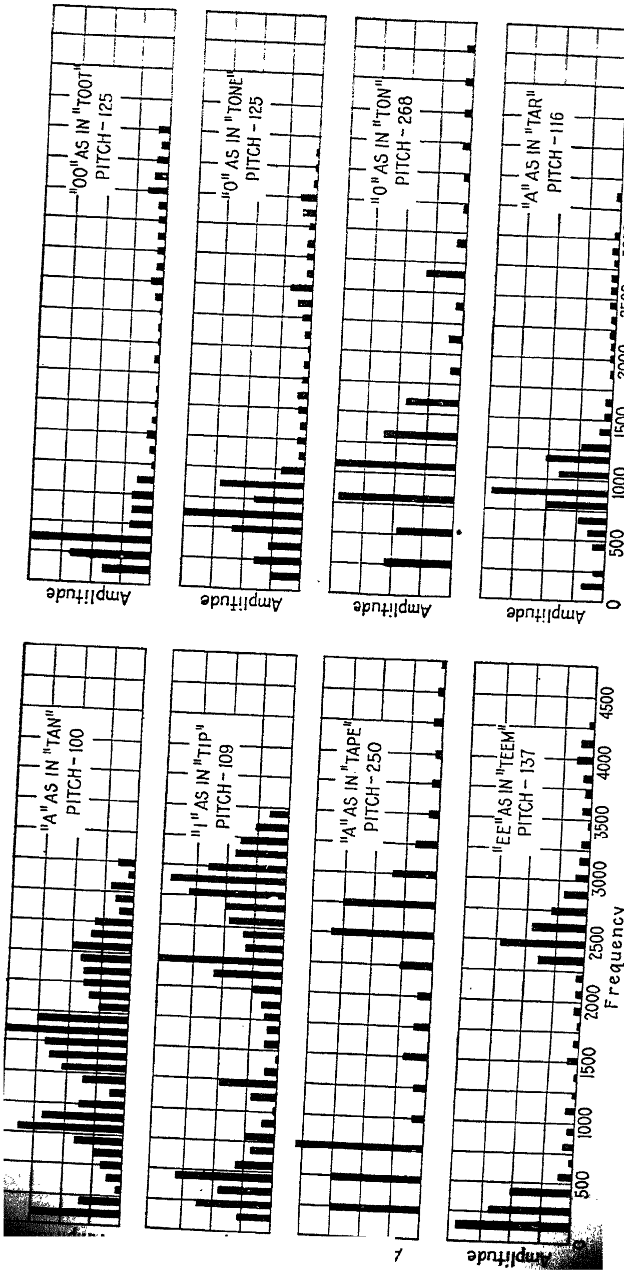


FIG. 233.—Distribution of energy in vowels and combinations of vowels. (*Courtesy of Bell Laboratories.*)

the air passages. Sounds from the vocal cords must pass through two variable, resonating cavities, namely, the throat and the mouth cavity. These resonating cavities magnify certain frequencies in speech sounds. The differentiation of speech sounds is nearly all accomplished by the mouth and positions of the lips. Figure 234 shows the changes in the shape of the mouth in sounding the vowels *a*, *u*, and *i*.

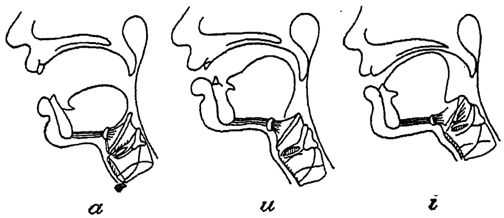


FIG. 234.—Position of vocal organs in uttering vowels.

251. Minimum and Maximum Audibility.—The least change of pressure which can produce an audible sound depends on the frequency of the sound. In like manner, the maximum change of pressure which the ear can experience and yet be able to hear the sound depends on the frequency. In Fig. 235, the change of pressure has been plotted on the vertical axis and the frequency of the sound on the horizontal axis. Because the range of pressure and the range of frequency are large, these curves have

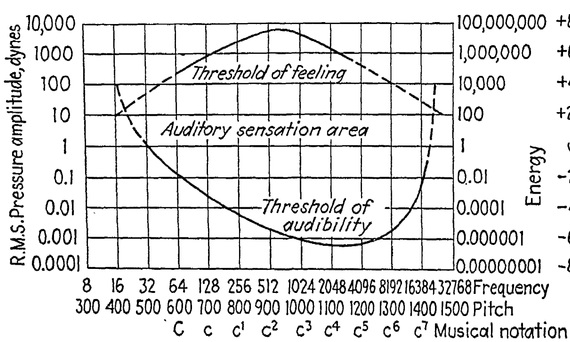


FIG. 235.—Threshold of audibility for different frequencies. (*Courtesy of Western Electric Company.*)

been plotted on logarithmic paper. From the lower one of these curves it is seen that the ear is most sensitive to sounds with a frequency of about 3,000 per second. A sound louder than that corresponding to the change of pressure indicated on the upper curve produces a painful sensation rather than hearing. This curve

has a maximum for sounds with a frequency of about 800 per second, so that when a sound has this frequency the ear can sustain the largest change of pressure without a painful sensation. These curves represent the mean of observations on a number of normal ears.

The least perceptible difference in frequency which can be detected by the ear depends on the frequency of the sound. The way in which the fractional change in frequency that can just be detected varies with the frequency is shown in Fig. 236. The ear distinguishes variations in frequency most readily when

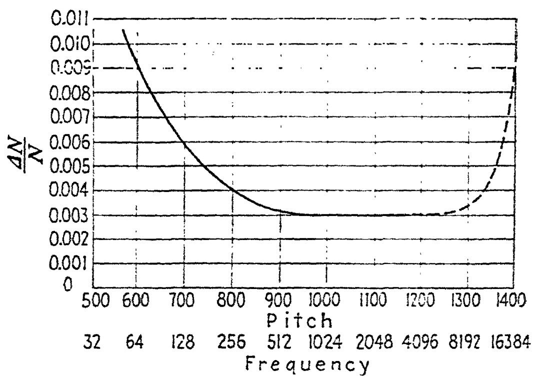


FIG. 236.—Minimum perceptible difference in frequency. (*Courtesy of Western Electric Company.*)

the frequency of the sound lies between 1,000 and 4,000 vibrations per second.

252. Heart and Lung Sounds.—The physician is often interested in a number of normal and abnormal sounds produced within the human body. The nature of organic lesions can often be determined by a careful study of these sounds. The quality of these sounds as interpreted by the ear depends on: (1) the relative intensities of the different frequency components which make up the sound, and (2) the relative sensitivity of the ear at these frequencies. These sounds can be analyzed by means of an electrical stethoscope. With this device, it is possible to eliminate all the components of a sound with frequencies above or below any desired frequency without seriously changing those components to which the observer wishes to listen. In this way, sounds of immediate interest such as a particular heart murmur can be examined. By this method, the frequencies and relative intensities of the different components of the sounds originate in the lungs, heart, etc.. have been

253. The Phonodeik.—The phonodeik is an instrument for studying the characteristics of sound waves. It was devised by Prof. D. C. Miller. The sound waves to be studied pass down the horn H (Fig. 237) to a diaphragm of glass D about 0.003 in. thick. This diaphragm is held lightly between soft-rubber rings. To the middle of the diaphragm is fastened a silk fiber or very fine platinum wire. This fiber after passing once around a tiny pulley is connected through a small spiral spring S to a fixed post P. The pulley is connected to a spindle which carries a small mirror M, 1 mm. square. The whole mass of the rotating part is about 0.002 g. Light from a lamp passes to the mirror and is focused by means of a lens on a film or screen F. The sound waves falling on the diaphragm cause it to vibrate back and forth. This motion is transmitted through the fiber and spring to the mirror which is thus set in rotation about its axis. This

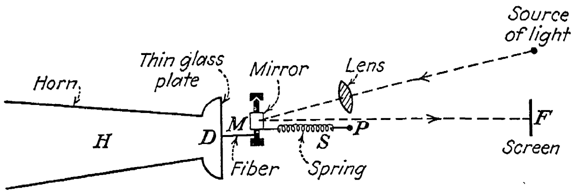


FIG. 237.—Phonodeik.

rotation causes the spot of light to move back and forth across the film. The film moves with uniform motion in a direction perpendicular to that in which the spot of light is moving. The spot of light thus traces out a path on the film in such a way as to give a record of the sound waves which were received by the instrument. The apparatus can be arranged with a rotating mirror and screen instead of the camera, and the curves can thus be projected so that they are visible to an audience. Many interesting results of the analyses of sounds have been obtained by means of this instrument.

254. Phonograph.—In one form of phonograph a wax cylinder or drum is rotated by clockwork and the sounds to be recorded are spoken into a horn. These sound waves travel down the horn and fall on a thin disk which is somewhat like the disk used in a telephone transmitter. The sound waves cause the disk to vibrate back and forth in unison with them. These vibrations are recorded on a drum or disk of hard wax. A short

needle fastened to the vibrating disk presses on the wax and ploughs a fine furrow in it.

The furrow is the record of the vibration of the disk. To reproduce the sound, the needle is allowed to travel again along the furrow. The needle moving up and down reproduces the motion by which the furrow was made. This causes the disk attached to the needle to vibrate and repeat the original sound waves which were sent down the horn. In practice it is found

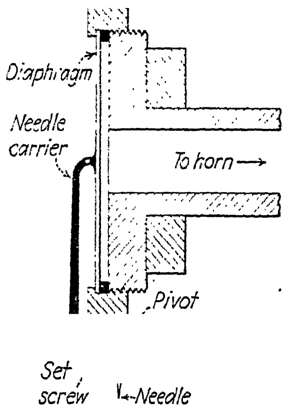


FIG. 238.—The phonograph.

better to use different needles and disks for recording and for reproducing the sounds.

In the gramophone a disk-shaped record is used instead of a cylindrical record. The stylus moves parallel to the surface in a wavy groove of uniform depth. The needle moves from side to side instead of up and down. The construction of the part of the gramophone which reproduces the sound is represented in Fig. 238. The needle carrier is attached to the center of the diaphragm which in turn is immediately in front of the end of the horn. The vibration of the diaphragm causes the needle to move to and fro, and conversely the motion of the needle to and fro causes the diaphragm to vibrate and reproduce the sounds by which the record was formed.

PART III.—HEAT

CHAPTER XXII

TEMPERATURE AND ITS MEASUREMENT

255. Temperature.—The most familiar temperature is that of the human body. Objects are said to be warm, hot, cool, or cold, compared with this temperature. This is not a very reliable or accurate standard. It is, therefore, necessary to look for some more accurate method of estimating temperatures. For this purpose it is customary at ordinary temperatures to use a thermometer which depends for its operation on the fact that a liquid like mercury expands when its temperature is increased. Any substance which expands uniformly may be used between suitable temperatures.

256. Mercury Thermometer.—The most common type of thermometer for ordinary use is the mercury-in-glass thermometer. It consists of a small glass bulb to which is sealed a glass tube with a very small bore. The bulb and part of the tube are filled with mercury. The residual air above the mercury in the tube is carefully removed so that the space above the mercury is empty. The glass tube is then sealed off. In order to use this bulb with its fine tube for a thermometer, it is necessary to have on the stem a scale divided into equal divisions called degrees.

Fixed Points.—The two fixed points which are ordinarily chosen for a thermometer are the melting point of ice and the boiling point of water under atmospheric pressure.

To determine the first of these fixed points, the bulb of the thermometer is surrounded with finely divided ice or snow. This melting ice or snow keeps the same temperature while melting. After the mercury in the bulb has reached the same temperature as the ice or snow, the height of the mercury in the stem of the thermometer does not further change. The point at which the mercury stands is now taken and used as one of the fixed points on the thermometer. On the centigrade scale, this point is called

0°, while on the ordinary Fahrenheit scale it is arbitrarily called 32°. The bulb and as much as possible of the stem of the thermometer are now placed in steam rising from water boiling at standard atmospheric pressure (Fig. 239). The mercury expands and assumes a new position in the stem. This position which does not change after the temperature of the thermometer has reached the temperature of the steam, is marked on the scale and

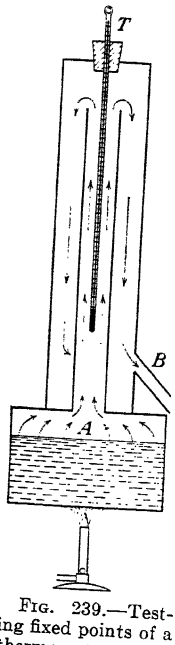


FIG. 239.—Testing fixed points of a thermometer.

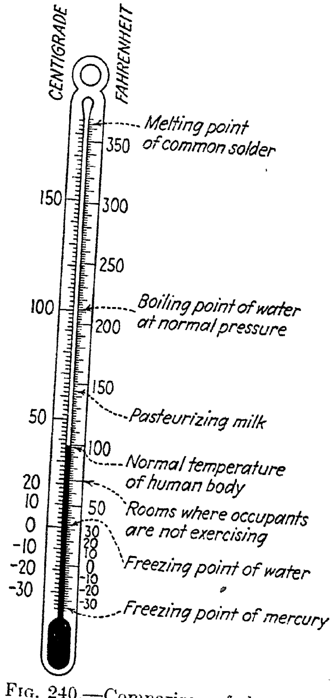


FIG. 240.—Comparison of thermometer scales.

used as a second fixed point for the thermometer. On the centigrade scale this point is called 100°, and on the Fahrenheit scale it is called 212°.

257. Comparison of Centigrade and Fahrenheit Thermometers.—Consider the thermometer shown in Fig. 240. On one side is a centigrade thermometer and on the other is a Fahrenheit thermometer. The freezing point on the centigrade scale is 0° and that on the Fahrenheit is taken as 32°. The boiling point on

the centigrade scale is 100° and that on the Fahrenheit is 212°. Hence, 100 divisions or degrees on the centigrade scale correspond to 180 on the Fahrenheit scale, and 1° on the Fahrenheit scale equals five-ninths of 1° on the centigrade.

To reduce from the Fahrenheit to the centigrade scale, first find how many degrees above or below the freezing point of water the temperature is on the Fahrenheit scale and then take five-ninths of this; the result will be the reading on the centigrade scale. In other words, subtract 32° from the reading on the Fahrenheit scale and take five-ninths of the remainder.

Reading on centigrade = $5/9$ (Reading on Fahrenheit − 32°).
Reading on Fahrenheit = $9/5$ (Reading on centigrade) + 32°.

Example.—The reading of a thermometer for the temperature of a room is 77°F. What is the reading on the centigrade scale?

$$\text{Reading on centigrade} = \frac{5}{9}\,(\text{Reading on Fahrenheit} - 32°)$$
$$= \frac{5}{9}(77 - 32) = \frac{5}{9} \times 45 = 25°\text{C.}$$

Example.—The temperature of an oven is 125°C. Find its temperature on the Fahrenheit scale.

$$\text{Reading on Fahrenheit} = \frac{9}{5}\,(\text{Reading on centigrade}) + 32°$$
$$= \frac{9}{5} \times 125 + 32$$
$$= 225 + 32 = 257°\text{F.}$$

258. Maximum and Minimum Thermometers.—In making observations on the temperature of the atmosphere, the U. S. Weather Bureau usually finds it convenient to use a maximum and minimum thermometer, which registers the highest and lowest temperatures reached since the last setting. There are several types of these thermometers in use. One of the most common forms is shown in Fig. 241. It consists of a bulb *A* filled with benzol or some other liquid having a large coefficient of expansion. A mercury column fills the lower part of the tube while the tube above *C* is also partly filled with phenol. When the temperature rises, the expansion of the liquid in the bulb *A* causes the mercury column to sink at *B* and rise at *C*, pushing upward a little index of iron in the tube above *C*, which in consequence of friction remains where pushed and marks the maximum temperature. On cooling, the contraction of the liquid in *A* causes the mercury to rise at *B*, pushing upward another little index of iron which

marks the minimum temperature. To set the instrument, the indices are drawn downward against the mercury by means of a small magnet.

259. The Clinical Thermometer.—The thermometers commonly used by physicians are maximum thermometers, having a short scale ranging from about 95 to 105°F. The tube is made very flat and narrow just above the bulb. The mercury passes readily through this constriction in rising; but as it contracts, capillarity causes the column to separate at that point, leaving the upper end of the mercury column to mark the highest temperature.

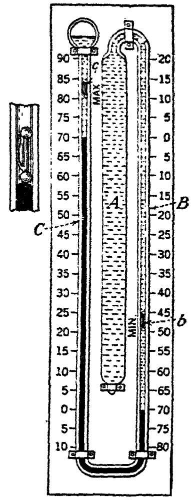


FIG. 241.—Maximum and minimum thermometer.

The usual type of this thermometer is made so that the front of the glass tube acts as a lens, magnifying the width of the mercury thread. To read such a thermometer, it should be held in the hand and turned until the mercury column suddenly appears magnified to a considerable width. This will occur when the clear corner of the triangular tube is directly in front. After a reading the mercury is shaken back into the bulb by holding the thermometer firmly between the thumb and forefinger and giving a few brisk shakes from the wrist.

	Temperature, Degrees Centigrade
Interior of hot stars	30,000,000
Surface of the sun	5,700
Carbon arc	3,500
Tungsten lamp	2,900
Boiling point of mercury	357
Boiling point of water	100
Freezing point of mercury	− 40
Liquid oxygen	−183
Liquid helium	−268

Problems

1. Fahrenheit thermometer shaded from the sun on a hot day reads 104°. What is the temperature on the centigrade scale?

2. A thermostat is set to maintain the temperature at 15°C. What is the corresponding temperature on the Fahrenheit scale?

3. What is the temperature on the centigrade scale when a Fahrenheit thermometer indicates 14°?

4. A piece of frozen carbon dioxide has a temperature of −80°C. What is the corresponding Fahrenheit temperature?

5. Find the temperature for which the centigrade and Fahrenheit readings are numerically equal but opposite in sign.

CHAPTER XXIII

MEASUREMENT OF HEAT

260. Unit of Heat.—Although heat is a form of energy and may be measured in the units in which energy is measured, it is convenient to use a unit which is based on the effect of heat in raising the temperature of a substance. In looking for a substance to use as a standard, it is natural to choose water on account of the ease with which it is obtained. Since it always

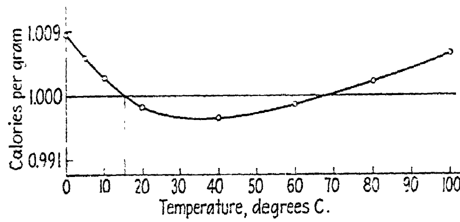


FIG. 242.—Specific heat of water in calories as a function of the temperature.

takes the same amount of energy to raise the temperature of 1 g. of water from 15° to 16°C., it is possible to define an arbitrary unit in which to measure other quantities of heat. Here the choice of the unit is largely a matter of convenience. In this respect, however, it does not differ from the unit of length or the unit of mass which is also chosen arbitrarily.

261. Calorie.—The unit of heat in the c.g.s. system is called the **calorie. It is defined as the quantity of heat or energy which is necessary to raise the temperature of 1 g. of water from 15° to 16° on the centigrade scale.** Because of the fact that the heat required to raise the temperature of 1 g. of water 1°C. is not the same at all temperatures, it is necessary to state the temperature at which the calorie is defined (Fig. 242).

262. The British Thermal Unit.—In the English system of units, the unit of heat is known as the British thermal unit, (B.t.u.). **It is defined as the quantity of heat required to raise the temperature of 1 lb. of water from 59° to 60° on the Fahrenheit scale.** This is a much larger unit of heat than the calorie. Here,

as in the case of the calorie, it is necessary to state the temperature, for the amount of heat required to raise 1 lb. of water 1°F. varies with the temperature.

263. Specific Heat.—So far we have been dealing with a single substance, namely, water. The amount of heat required to raise the temperature of 1 g. of another substance 1°C. may be compared with the amount of heat required to raise the temperature of 1 g. of water from 15° to 16°C. Such a comparison gives a definition of what is called the specific heat of the substance. **The specific heat of a substance is numerically equal to the number of calories required to raise the temperature of 1 gram of the substance 1°C.,** or, what amounts to the same thing, **the number of British thermal units required to raise the temperature of 1 lb. of the substance 1°F.**

Let Q denote the quantity of heat added to a mass of M g., let t and t' be the initial and final temperatures, and let S be the specific heat of the body. Then

$$Q = SM(t' - t).$$

Example.—Find the number of calories required to raise the temperature of 100 g. of brass from 25 to 75°C. Specific heat of brass is 0.088.

$$\begin{aligned}\text{Heat} &= \text{mass} \times \text{specific heat} \times \text{change of temperature}\\ &= M \times S \times (t' - t)\\ &= 100 \times 0.088 \times (75 - 25) = 440 \text{ cal.}\end{aligned}$$

264. Water Equivalent.—The water equivalent is defined as the heat required to raise the temperature of the entire body 1°. To find the water equivalent it is only necessary to multiply the specific heat by the mass of the body.

$$\text{Water equivalent} = MS,$$

where M is the mass of the substance and S is its specific heat.

Example.—Find the water equivalent of a clay soil which covers a field of 1 acre to a depth of 1 ft. Take 0.25 as the specific heat of the soil and 156 lb. per cubic foot as its density.

$$\begin{aligned}\text{Water equivalent} &= \text{mass} \times \text{specific heat}\\ &= \text{volume} \times \text{density} \times \text{specific heat}\\ &= 43{,}500 \times 156 \times 0.25\\ &= 17 \times 10^5 \text{ B.t.u., lb.}\end{aligned}$$

If a piece of soapstone and a piece of iron of equal weight are heated to the same temperature and each of them dropped into an equal mass of water, it will be found that the temperature of the water into which the stone was dropped is increased more than the temperature of the water

into which the iron was dropped. From this it is seen that a mass of soapstone has a greater water equivalent than an equal mass of iron.

265. Measurement of Heat by Method of Mixtures.—One of the most familiar methods of measuring a quantity of heat is by imparting this heat to a known mass of water and observing the change which it produces in the temperature of the water. If the mass of the water is known and its rise in temperature is observed, the heat imparted to the water can be found by multiplying the mass of water by its rise in temperature. Since it requires 1 cal. to raise the temperature of 1 g. of water 1°C., to raise M g. from temperature t to temperature t' would require H cal., where

$$H = M(t' - t)1.$$

Let it be assumed that it is desired to determine the specific heat of iron. Let m g. of iron be heated to a temperature T. This can be conveniently done by placing the iron for a sufficient time in a steam bath. Meanwhile M g. of cold water are weighed out in a vessel of known mass M'. The temperature of the vessel and that of the water being the same, both can be denoted by t. The piece of iron is suddenly plunged into the vessel of water, and, after stirring the water, an equalization of temperature takes place. Such a vessel (Fig. 243) is known as a **calorimeter.** The iron falls in temperature, thus giving heat to the water and its containing vessel. The water and the containing vessel, on the other hand, gain heat and rise in temperature. After mixing, the final temperature is noted and called t'. According to the law of conservation of energy, **the heat lost by the iron must be just equal to that gained by the water and its containing vessel.** The heat gained by the water is

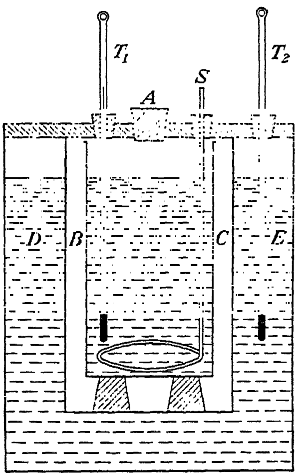


FIG. 243.—Calorimeter for measuring heat transferred from one body to another.

$$M(t' - t)1,$$

its mass times its change in temperature times its specific heat. The heat gained by the vessel is

$$M'(t' - t)S',$$

its mass times its change in temperature times its specific heat. The heat lost by the iron is

$$m(T - t')S.$$

its mass times its change in temperature times its specific heat.

Heat gained = heat lost.

Hence,

$$M'(t' - t)S' + M(t' - t) \times 1 = m(T - t')S.$$

$$S = \frac{M(t' - t) \times 1 + M'(t' - t)S'}{m(T - t')} = \text{specific heat of iron.}$$

Example.—Suppose that 450 g. of lead are heated to 100°C. and then dropped into a calorimeter containing 100 g. of water at 10°C. After stirring, the temperature of the water rose to 21.5°C. Neglecting the heat given to the calorimeter, find the specific heat of lead.

Heat lost by lead = heat gained by water.
Heat lost by lead = mass × specific heat × change in temperature
$= 450 \times S \times (100 - 21.5)$.
Heat gained by water = mass × specific heat × change in temperature

$$= 100 \times 1 \times (21.5 - 10).$$

Hence,

$$450 \times S \times (100 - 21.5) = 100 \times 1 \times (21.5 - 10).$$

$$S = \frac{100(21.5 - 10) \times 1}{450 \times (100 - 21.5)} = \frac{11.5 \times 100}{450 \times 78.5} = 0.033 \text{ cal. per gram.}$$

266. Influence of Temperature on the Specific Heat of Solids. According to the law of Dulong and Petit, the product of the specific heat of an element and its atomic weight is a constant for all elements. The following table shows the values of the atomic heats for a number of elements. It is easily seen that the atomic heat is in each case about 6 g.-cal.

Element	Specific Heat × Atomic Weight = Atomic Heat
Lead	6.35
Gold	6.15
Tin	6.4
Zinc	6.0
Iron	5.95
Chlorine	6.7
Aluminum	5.75

But this result is valid only for sufficiently high temperatures. At low temperatures, the specific heat of an element changes with the temperature. The dependence of the specific heat on the temperature for aluminum, silicon, and diamond is represented in Fig. 244. From these curves, it is evident that the specific heat approaches zero as the temperature approaches absolute zero.

Hence at low temperatures Dulong and Petit's law is no longer valid, but at sufficiently high temperatures the atomic heats of the elements are nearly the same.

The fact that the atomic heat of an element at a sufficiently high temperature is about 6 g.-cal. was satisfactorily explained on

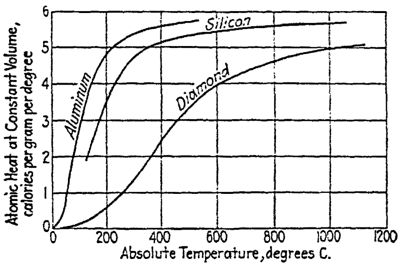


FIG. 244.—Atomic heat at constant volume becomes zero at the absolute zero.

the basis of the kinetic theory, but the kinetic theory is unable to account for the decrease of the atomic heat as the temperature is lowered to absolute zero. The quantum theory which assumes that energy exists in discrete grains or units offers a very satisfactory explanation of this phenomenon. A discussion of the essentials of the theory will be found in Part VI.

267. Heat of Combustion.—*The heat of combustion is the heat liberated by burning unit mass or unit volume of a fuel such as coal or gas.* To find it, the fuel (Fig. 245) is placed in a crucible, C, inside a bell jar which is closed so that the products of combustion cannot escape except through the openings at the base of the jar. The bell jar is placed inside a vessel, the mass of which is known, and this vessel then filled with a known weight of water. The temperature of the water is determined, and then a supply of oxygen is admitted through the opening at the top of the bell jar until all of the fuel has been burned. The products of combustion bubble up through the water. When the combustion is complete, the

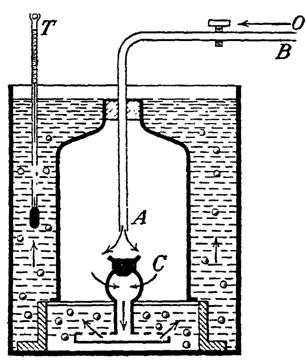


FIG. 245.—Calorimeter for heat of combustion of fuel.

temperature of the water is again observed. From these data the heat of combustion of the fuel can be found.

$$\text{Heat of combustion} = \frac{\text{mass of water} \times \text{temp. change} \times \text{sp. heat}}{\text{mass of fuel}}.$$

In this expression for the heat of combustion, it is assumed that the water equivalent of the calorimeter is so small that it can be neglected in comparison with that of the water in the calorimeter. In case this is not true, the water equivalent of the calorimeter must be added to that of the water in the calorimeter.

Example.—A sample of coal weighing 0.15 lb. was burned in the crucible in Fig. 245. The water weighed 100 lb., and its temperature at the beginning was 60°F.; the temperature at the end was 80°F. Find the heat of combustion of the coal. Neglect the heat given up to the vessel which forms the calorimeter.

$$\text{Heat of combustion per pound} \quad \frac{\text{mass of water} \times \text{temp. change} \times \text{sp. heat}}{\text{mass of coal}}$$
$$\frac{100 \times (80 - 60) \times 1}{0.15} = \frac{2{,}000}{0.15} = 13{,}300 \text{ B.t.u.}$$

268. Continuous-flow Calorimeter.—Another method of measuring the heat of combustion of a fuel which can be burned as a gas is shown in Fig. 246. The gas to be studied is burned in a burner B, which is inside the calorimeter. The products of combustion pass out through a series of pipes. Around these pipes there is a continuous flow of water. This water enters the calorimeter at a constant temperature which is determined by the thermometer T_2. The water leaves by the outlet C. Its temperature is determined just before it leaves the calorimeter. The temperature of the water as it leaves is, of course, higher than it is when it enters. By weighing the water and measuring the quantity of gas which has been burned, it is possible to cal-

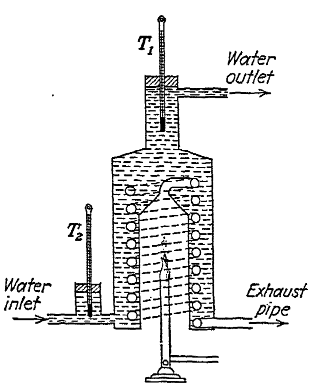


FIG. 246.—Calorimeter for heat of combustion of gases.

culate the quantity of heat generated per cubic foot, or per liter of gas. If the rate of generation of heat is constant, the difference between the temperature of the ingoing and outgoing water will remain constant.

Let Q = the number of heat units liberated.
V = the volume of the gas burned.
W = the weight of water which flowed through the calorimeter.
T_2 = the temperature of ingoing water.
T_1 = the temperature of outgoing water.
S = the specific heat of water.
$H = WS(T_1 - T_2)$.

$$\text{Heat per unit volume} = \frac{Q}{V} = \frac{WS(T_1 - T_2)}{V}.$$

Example.—In determining the heat of combustion of natural gas it was found that 2,400 g. of water flowed through the calorimeter while 3 l. of gas were being burned. The temperature of the ingoing water was 20.0°C. and that of the outgoing water was 30.0°C. Find the number of calories generated by the combustion of 1 l.

$$\text{Heat of combustion per unit volume} = \frac{\text{mass of water} \times \text{temp. change} \times \text{sp. heat}}{\text{volume of gas}}$$

$$H = \frac{2{,}400(30.0 - 20.0) \times 1}{3} = 8{,}000 \text{ cal. per liter.}$$

269. Measurement of the Energy Value of Foods.—The energy value of a food can be determined by the combustion of a known amount of the food in a bomb calorimeter (Fig. 247). The heat evolved is absorbed in a known amount of water. From the mass of the water and its rise in temperature, the heat obtained from the food can be determined.

If the law of conservation of energy holds for living as well as non-living matter, the same amount of energy should be liberated by the utilization of food inside as well as outside the body, provided the physical state and the chemical end products are the same in each case. To test this result, an animal is put inside of a calorimeter and given a definite amount of food. The heat which is evolved is measured and compared with the heat which is obtained when an equal amount of food is burned in a

bomb calorimeter. A special type of calorimeter is necessary, and there are many difficulties in obtaining accurate measurements, but the results of such observations are sufficient to show that the oxidation of assimilated foodstuffs in the living body produces the same evolution of energy as they would produce if

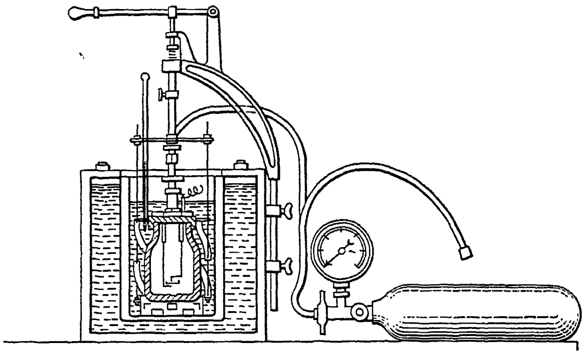


FIG. 247.—Bomb calorimeter.

burned in a bomb calorimeter, provided the end products are identical.

Problems

1. How many British thermal units are required to raise the temperature of 8 cu. ft. of water from 40 to 190°F.?

2. A piece of sheet aluminum rolled into a spiral with a mass of 62 g. is heated in a steam jacket to a temperature of 98°C. and then plunged into 120 g. of water at 9°C., causing the temperature to rise to 18°C. What is the specific heat of aluminum, assuming that no heat was lost or gained during the experiment?

3. A piece of iron weighing 130 g. was taken out of an oil bath and immediately immersed in a beaker containing 160 g. of water at 14°C. The temperature of the water rose to 29°C. What was the temperature of the oil bath? Specific heat of iron = 0.11.

4. A quantity of liquid having a specific heat of 0.45 and a temperature of 26°C. was mixed with another liquid with a specific heat of 0.26 and a temperature of 8°C. The resulting temperature was 20°C. In what proportions were the liquids mixed?

5. Into a copper calorimeter weighing 120 g. and containing 250 g. of water at 12°C., there are dropped simultaneously 55 g. of silver at 110°C., 40 g. of iron at 60°C., and 20 g. of platinum at 72°C. What is the resultant temperature?

6. A copper calorimeter contains 200 g. of water at 0°C. and 216 g. of silver. The calorimeter weighs 150 g. How much will the temperature be increased by adding 750 cal.?

7. In finding the temperature of a furnace, a piece of platinum was inserted in the furnace. The platinum was then dropped into a copper calorimeter which had a mass of 250 g. and contained 1,200 g. of water. The temperature of the water rose from 12 to 18°C. If the mass of the platinum was 200 g. what was the temperature of the furnace?

8. How many cubic feet of gas is burned in heating the water in a bath tub containing 6 cu. ft., if the temperature is raised from 60 to 105°F.? Assume that all the heat goes to heat the water and take 21,200 B.t.u. per pound as the heat of combustion of the gas. Density of the gas was 0.045 lb. per cu. ft. under standard conditions.

9. A specimen of gasoline was tested by burning a sample of 6 g. in a fuel calorimeter containing 10 lb. of water, causing a rise of temperature from 62 to 81°F. If the water equivalent of the calorimeter was 0.96 lb., how many British thermal units could be obtained from each pound of gasoline?

10. Natural gas from the mains with a heat of combustion of 1,050 B.t.u. per cubic foot is used to heat water. Assuming that half of the available heat is wasted, how much gas will be required to heat 2 lb. of water from 50°F. to the boiling point?

11. A quantity of gas which measured 1.75 cu. ft. as obtained from the mains was burned in a continuous-flow calorimeter. During the test 60 lb. of water flowed through the calorimeter, with a temperature of 58°F. when entering, and 85°F. when leaving. The products of combustion carried away 7.2 B.t.u. Find the heat of combustion per cubic foot.

12. A specimen of coal with a mass of 2.5 g. was burned in a copper calorimeter having a mass of 1,200 g. The mass of the water in the calorimeter was 1,800 g., and the initial temperature was 14°C. The final temperature of the water was 21°C. Find the heat of combustion in calories per gram.

13. A sample of methyl alcohol weighing 15 g. was burned in a fuel calorimeter containing 9 kg. of water. The water equivalent of the calorimeter was 600 cal. The initial temperature was 12.2°C., and the final temperature was 20.5°C. Calculate the heat of combustion of the methyl alcohol.

14. A sample of coal weighing 2 oz. with a heat of combustion of 12,500 B.t.u. per pound was burned in the crucible of a calorimeter (Fig. 221). The calorimeter contained 12 lb. of water, and it had an initial temperature of 50°F. The glass parts of the calorimeter weighed 6.2 lb. and had a specific heat of 0.19. To what temperature was the water raised?

CHAPTER XXIV

EXPANSION BY HEAT

270. Expansion of Solids.—When a railroad track is laid, it is customary to leave a gap between the rails to allow for the expansion of the rails in the summer. The expansion of solids may be demonstrated in the following manner. A brass ball is made so that it will just pass through a ring when both ring and ball are cold. When the ball is heated, it will not pass through the ring; but when it is allowed to cool, it passes through as before. From this experiment, it is seen that solids expand when heated and contract when cooled.

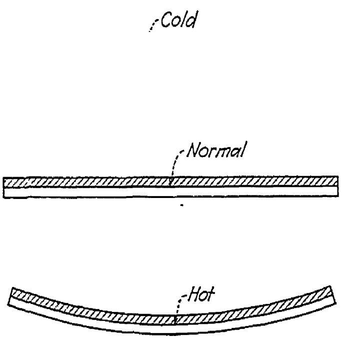


FIG. 248.—Unequal expansion of metals. At high temperatures the metal with the greater expansion is on the outside.

271. Unequal Expansion of Solids.—Different substances expand at different rates. The unequal expansion of two metals such as iron and brass is shown in the following way. Consider two strips of these metals riveted together (Fig. 248) so as to form a composite bar. If this bar is heated, it will be found to bend because of the unequal expansion of the metals. The brass expands more rapidly than the iron, so that the brass will be on the outside of the curve which the bar makes. The iron, expanding less rapidly than the brass, will be on the inside.

The bending of a bar which arises out of this unequal expansion is of importance in the regulation of temperatures. When the temperature changes, one of these compound bars will bend and may (Fig. 249), on account of the motion thus resulting, be made to control the supply of heat.

272. Measurement of Expansion.—For many purposes it is necessary to know how much a substance expands when the temperature is raised 1°.

To determine the expansion of a rod, insert the rod (Fig. 250) into a jacket which can be filled with steam or water as desired. The rod is fixed between two screws. One of these screws, S, is an adjusting screw; the other, R, is a micrometer screw. Adjust the micrometer screw S to read zero and then tighten up the screw R when water is flowing through the jacket.

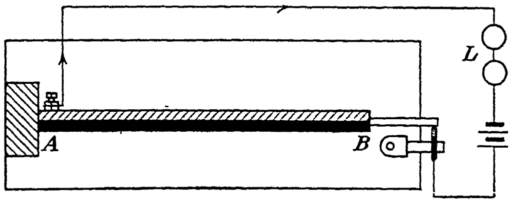


FIG. 249.—Temperature regulator for a thermostat. Change in temperature bends the composite rod AB and opens or closes an electric circuit.

Read the temperature indicated by the thermometer T, and replace the water in the jacket by steam from a boiler, at the same time releasing the micrometer screw to allow for expansion of the rod. After the rod has been heated, turn the micrometer screw until it is again tight. The amount it has been advanced gives the increase in the length of the rod. Note the temperature

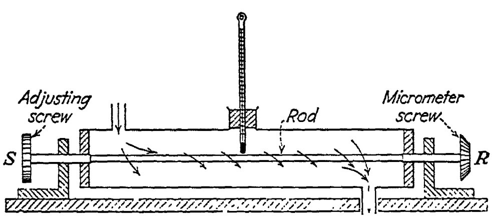


FIG. 250.—Measurement of coefficient of linear expansion, that is, change in length per unit length per degree change of temperature.

of the water, and measure the original length of the rod. Divide the increase in length by the original length. This gives the increase in length per unit length. Now, divide the increase in length per unit length by the change in temperature. This gives the increase in length per unit length per degree change in temperature.

$$e = kl(t_1 - t_2),$$

where e = the change in length.
k = the coefficient of expansion.
l = the original length.
t_1 = the highest temperature.
t_2 = the lowest temperature.

Or

$$k = \frac{\quad}{l(t_1 - t_2)}$$

The coefficient of expansion, k, is the increase in length per unit length of the bar for 1° rise in temperature. This quantity is a small fraction which varies from substance to substance. It is independent of the units in which the length is measured but depends on the units in which the temperature is measured. For the centigrade scale, it will have one value, and for the Fahrenheit scale, the value will be only five-ninths as large. (Appendix D-10.)

Example.—A telephone wire is 1 mile long when the temperature is 0°C. How much would its length increase by raising the temperature to 35°C.? Assume the coefficient of linear expansion of iron to be 0.000012 per degree centigrade.

Change in length = original length × temp. change × coef. of expansion.

Change in length in feet = 5,280 × 35 × 0.000012 = 2.22 ft.

273. Illustrations of Expansion.—In the construction of an iron bridge (Fig. 251) provision is made for the expansion of the iron. To make it possible for the bridge to expand, one end of the bridge is mounted on a roller. As the length of the

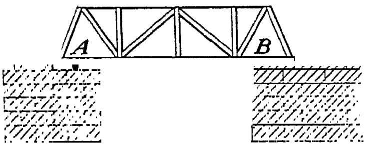


FIG. 251.—Expansion of a bridge. Change of temperature causes A to move along the pier.

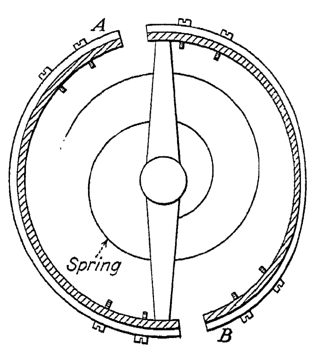


FIG. 252.—Balance wheel of a watch. Change of temperature changes the rotary inertia of the wheel.

bridge increases or decreases, the end of the bridge moves forward or back without injuring the piers.

A wire fence is made of wires which are not straight but bent back and forth in kinks so that, as the temperature of the fence increases, the wire is

not stretched, as these bends in the wire take care of the increase or decrease in length.

In a watch, the balance wheel (Fig. 252) is made of two metals which are fastened together. As the temperature of the wheel increases, the end *B* and the end *A* are carried in, making the effective radius of the wheel less. The expansion of the radius tends to carry the rim of the wheel farther away from the center. At higher temperatures, the elasticity of the hair-spring is less, and this makes the wheel tend to move more slowly. As the free ends of the wheel move in, the wheel tends to vibrate more rapidly. These two tendencies, one making the wheel go more slowly and the other making it go more rapidly, are made to compensate each other, so that the period of vibration remains the same at all temperatures.

274. Unequal Expansion of Substances.—The unequal expansion of various substances when heated is a matter of much importance in the formation of soils. If the sun shines on rocks made up of minerals containing different kinds of crystals which do not expand at the same rate, the unequal expansion or contraction of these crystals tends to loosen them. In this way, the rock breaks up into small fragments which gradually form soil.

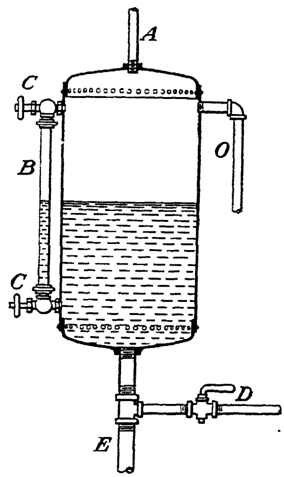


FIG. 253.—Expansion tank for a hot-water system. It takes care of the increase of volume with rise of temperature.

275. Expansion of Liquids.—To show that liquids expand when heated, fill a flask with kerosene or with water. Insert in it a stopper through which passes a glass tube. Allow the liquid to rise a small distance in the tube. Now insert the flask in a vessel filled with hot water. Soon the liquid begins to expand, and the surface of the liquid in the tube rises. If two such flasks are put in the vessel side by side, and if one of these flasks is filled with water and the other filled with alcohol, it will be found that the alcohol expands much more than the water for the same rise in temperature. Kerosene and most oils expand more rapidly than water. In a hot-water heating system, an expansion tank (Fig. 253) is provided to take care of the increase in the volume of the water when the furnace is heated.

In the case of solids, we are usually concerned with the change in length with change of temperature. In liquids and gases, on the other hand, we are concerned with the change in volume with

change of temperature. **The coefficient of cubical or volume expansion may be defined as the change in volume per unit volume per degree change in temperature.** It may also be expressed as follows:

Let v = the change in volume.
V = the original volume.
t_1 = the initial temperature.
t_2 = the final temperature.
a = the coefficient of volume expansion.

Then

$$a = \frac{\quad}{V(t_2 - t_1)} \qquad \text{(Appendix, D-10.)}$$

Example.—A barrel which holds 30 gal. is completely filled with kerosene when the temperature is 0°C. How much kerosene will overflow when the temperature is raised to 75°C.?

Coefficient of expansion of kerosene = 0.00085.
Change in volume = original volume × change in temperature × coefficient of expansion.
= 30 gal. × 75 × 0.00085.
= 1.9 gal.

276. Maximum Density of Water.—It has been seen that liquids like alcohol and kerosene expand whenever heated. The expansion of water at temperatures in the neighborhood of 4°C. is very peculiar. Fill a bulb to which is attached a capillary tube filled with water, as in the preceding experiment, and then place it in a vessel filled with cracked ice. When the temperature has become constant, note the position of the liquid in the capillary tube. On removing the flask from the vessel of ice water and allowing it to warm gradually, it will be noticed that the liquid in the capillary tube first descends and then rises on further heating. This means that the water on being heated above 0°C. at first contracts and then expands. Careful experiments show that water has its least volume at 4°C. When it is warmed above this temperature or cooled below this temperature, it increases in volume. As the volume of the liquid increases, its density decreases; and as the volume of the liquid decreases, its density increases. Hence, where the volume of the liquid has its least

value, the density has its greatest value. The facts with respect to water may, therefore, be expressed by saying that the volume of the liquid is a minimum at 4°C. (Fig. 254) and the density is a maximum.

277. Expansion of Gases.—Gases as well as liquids and solids expand when heated and contract when cooled. If a spherical bulb (Fig. 255) has a small glass tube with its end immersed in a beaker of water, it will be found that on heating the bulb, air bubbles out of the beaker; and as the bulb is allowed to cool, the liquid rises in the tube. From this experiment it is seen that the volume of the gas increases as the temperature rises and decreases as the temperature falls. If the pressure is kept constant, the whole volume of the air occupies more space than it did when it was cold. Consequently, hot air weighs less per cubic inch or per cubic centimeter than cold air; that is, the density of air or any other gas decreases as the temperature rises.

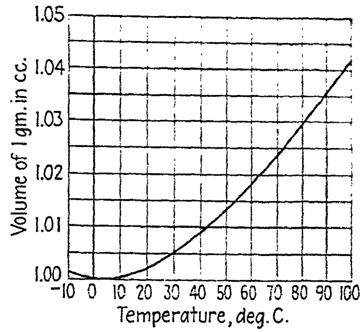


FIG. 254.—Specific volume of water. The volume is a minimum at 4°C.

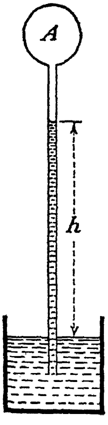


FIG. 255.—Expansion of gases with rise of temperature.

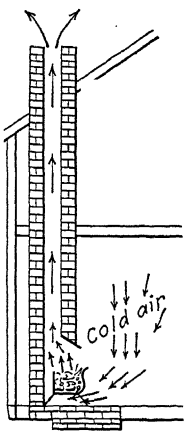


FIG. 256.—Rise of smoke in a chimney. Rising warm air is replaced by cold air.

278. Rise of Smoke in a Chimney.—When the fire is lighted in a fireplace, the air in the neighborhood of the fire is immediately warmed (Fig. 256). This causes its density to decrease due to the expansion of the air. Since the density of the warm air is less than that of the cold air nearer the top of the chimney, the heated air rises and the cold air descends to take its place. The heated air in this case behaves like a light gas in a balloon. It weighs less than the cold air which it displaces. The hot air is consequently pushed up by the cold air, and an upward circulation of the air takes place. After a short time the entire chimney is filled with hot air,

and this entire volume of hot air is pushed up just as a body immersed in a liquid is lifted by the liquid which it displaces. Since a tall chimney has a larger volume of hot air than a short chimney has, the lifting effect in the tall chimney is greater than in the short chimney. Hence, tall chimneys draw better than low ones.

279. Expansion of Gases in Cooking.—In making bread, the yeast gives off carbon dioxide. This gas is liberated all through the dough, and there are formed in the dough thousands of small closed pockets filled with gas. As more gas is liberated, these pockets grow. This process causes the bread to rise. When the dough is placed in the oven, the yeast ceases to produce the carbon dioxide, but the dough continues to rise for the reason that the heat expands the carbon dioxide.

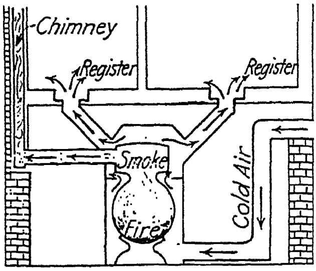


FIG. 257.—Hot-air heating system. It depends for its action on change of density of the air with the temperature.

When eggs are beaten, small bubbles of air are included. As the eggs are being heated, these bubbles of air expand, and the volume of the egg increases, thus making the omelet rise. The eggs should be beaten in a cool place for the reason that they are more rigid and thus able to include more air, and for the further reason that the colder the air, the greater will be the change in temperature on heating and the greater the increase in volume.

280. Hot-air Heating System.—A hot-air heating system (Fig. 257) depends for its operation on the unequal heating of the air in the furnace and the expansion of the air resulting from this heating. The air directly over the fire box becomes heated and expands. On account of the decrease of density due to this cause, it rises and passes out through the registers into the rooms. Cold air from the outside enters through the cold-air duct and is in its turn heated by the furnace. The process thus continues, giving a continuous supply of hot air to the rooms.

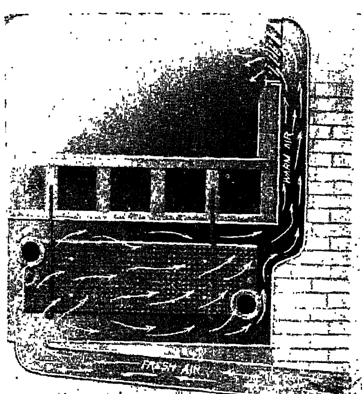


FIG. 258.—Ventilation by indirect heating. Rising warm air is replaced by incoming cold air.

281. Ventilation.—Except where forced circulation is provided, most buildings depend for ventilation on the unequal heating of the air. The open fireplace in Fig. 256 calls attention to the circulation of air resulting from the warming of the air in the chimney. The air to replace that which rises up the chimney comes from the room, and

air from outdoors comes in to take the place of that which has gone up the chimney.

Figure 258 shows a case of ventilation by indirect heating and Fig. 259 a case by forced draft.

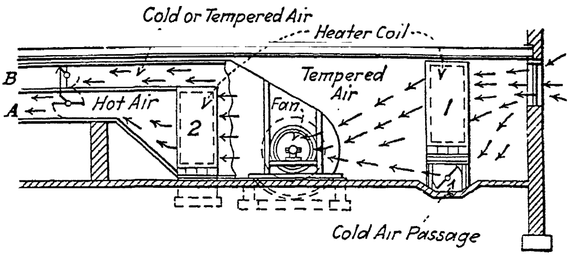


FIG. 259.—Ventilation by forced draft.

282. Charles's Law.—It has been seen that gases, like solids and liquids, expand when the temperature is increased. In order to determine the rate of this expansion with accuracy, it is necessary to specify the way in which the gas is allowed to expand. Suppose that it is agreed that during the expansion the pressure on the gas will be kept

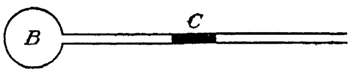


FIG. 260.—Expansion of gases at constant pressure.

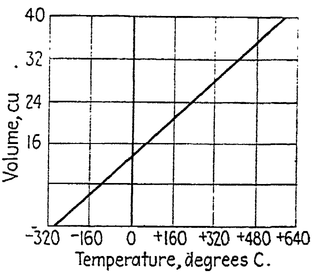


FIG. 261.—Relation between volume and temperature of a

unchanged. It is then said to expand at constant pressure (Fig. 260). The coefficient of expansion can now be defined as it was defined for liquids.

Let V_0 = the original volume at 0°C.
V = the volume at t°C.
t = the temperature on centigrade scale.
b = the coefficient of cubical expansion.

Then

$$b = \frac{V - V_0}{V_0 t},$$

or

$$V = V_0(1 + bt).$$

The relation between the volume of a gas at constant pressure and its temperature is shown in Fig. 261.

By careful experiments Charles found that the coefficient of cubical expansion of all gases at constant pressure is the same and equal to 0.00367 or 1/273. This is known as Charles's law. It may be expressed as

$$V = V_0\left(1 + \frac{t}{273}\right)$$

where t is measured on centigrade scale.

Example.—The volume of a certain mass of gas at 0°C. is 350 c.c. Its temperature is raised to 175°C. What is the new volume, and what is the change in volume?

$$= V_0\left(1 + \frac{t}{273}\right).$$

$$V = 350\left(1 + \frac{175}{273}\right) = 574.3 \text{ c.c.}$$

Change in volume = 574.3 c.c. − 350 c.c. = 224.3 c.c.

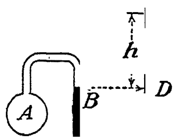


FIG. 262.—Heating a gas at constant volume. The pressure is proportional to the temperature.

283. Heating a Gas at Constant Volume.—A gas may also be heated in such a way that its volume remains constant. In this case, the pressure exerted by the gas increases with rise of temperature. In order to show this, a bulb connected (Fig. 262) to a tube bent in the form of a U is immersed in a bath of water at room temperature. The mercury in the U-tube is brought to a mark on the glass tube at B. The bulb is now heated, and the mercury falls at B, showing that the pressure is increasing. If enough mercury is poured into the tube, or the right-hand side of the tube is elevated so that the mercury level remains at B, the volume of the gas remains unchanged. The pressure increases, and this rate of increase has been found to be constant for all gases. This is another form of Charles's law. In this form, Charles's law states that **if the volume of a gas is kept constant, the pressure increases 1/273 of its value at 0°C. for every rise of 1°C. in temperature.**

Let P_0 = the pressure at 0°C.
P = the pressure at t°C.

Then

$$= P_0\left(1 + \frac{t}{273}\right).$$

Figure 263 shows the relation between pressure and temperature of a gas at constant volume.

Example.—The pressure of a gas in a vessel was 75 cm. of mercury when the temperature was 0°C. After the gas was heated, the new pressure was found to be 125 cm. of mercury. If the volume remained constant during the heating, what was the new temperature?

$$_0\left(1 + \frac{t}{273}\right).$$

$$125 = 75\left(1 + \frac{t}{273}\right)$$

$$\frac{125 - 75}{75} \times 273 = \frac{50}{75} \times 273 = 182°\text{C}.$$

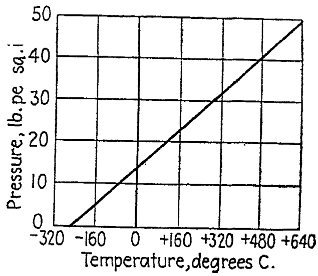


FIG. 263.—Relation between pressure and temperature of a gas.

284. Absolute Scale of Temperature.—In the study of the heating of a gas at constant volume, it was found that the pressure of a gas always increased $1/273$ of its value at 0°C. for each degree rise in temperature above 0°C. In like manner, when the temperature of the gas is lowered, the pressure decreases $1/273$ of its value at 0°C. for each degree the temperature is lowered. If this rate of decrease in pressure continue as the temperature is gradually lowered, it is clear that when the temperature has become 273° below 0°C., the gas will no longer exert a pressure. From the standpoint of the kinetic theory of gases, a gas must always exert a pressure as long as the molecules are in motion. Hence, the temperature at which the molecules cease to exert a pressure would be the temperature at which these molecules cease to move. This temperature is known as the **absolute zero.**

It is often convenient to measure temperatures from this zero instead of from the zero on the centigrade scale. Temperatures on this scale are called **absolute temperatures** to distinguish them from those measured on the centigrade scale. To find the absolute temperature in centigrade degrees, 273° is added to the reading on the centigrade scale. If we indicate temperatures on the centigrade scale by t and those on the absolute scale by T,

$$T = t + 273.$$

Example.—The reading on a centigrade thermometer is 57°. What is the absolute temperature?

$$\begin{aligned}\text{Absolute temperature} &= \text{reading on centigrade scale} + 273°.\\ T &= t + 273.\\ &= 57° + 273° = 330° \text{ absolute.}\end{aligned}$$

285. The General Gas Law.—In Boyle's law it is assumed that the temperature of a gas remains unchanged while the pressure and the volume are varied. This law can, therefore, be applied only when the temperature is fixed. In that case,

$$PV = \text{constant}.$$

On the other hand, Charles's law assumes that the pressure of the gas does not change while the temperature and volume of the gas vary. In this case,

$$V = V_0\left(1 + \frac{t}{273}\right) = \frac{V_0(273 + t)}{273} = \frac{V_0T}{273}.$$

At another temperature,

$$V' = V_0\left(1 + \frac{t'}{273}\right) = \frac{V_0T'}{273}.$$

Hence, $V'/V = T'/T$; *i.e.*, **the volumes at constant pressure are proportional to the absolute temperatures.** If the volume remains constant, Charles's law states that

$$P = P_0\left(1 + \frac{t}{273}\right) = \frac{P_0T}{273}$$

and

$$P' = P_0\left(1 + \frac{t'}{273}\right) = \frac{P_0T'}{273}.$$

Hence, $P'/P = T'/T$; *i.e.*, **the pressures are proportional to the absolute temperatures when the volume remains constant.**

It would often be more convenient to have a law in which these restrictions with respect to temperature, pressure, and volume were removed. By combining Boyle's law and Charles's law these restrictions can be removed and the **general gas law** obtained. The general gas law expresses the relation between the volume of the gas, its temperature, and its pressure.

Let a given mass of gas having a volume V_0 and a pressure P_0 at 0°C. (Fig. 264) be heated at constant pressure until the temperature is t°C. Its new volume is

$$V_1 = V_0\left(1 + \frac{t}{273}\right) = \frac{V_0T}{273}.$$

Keeping the temperature constant, compress this gas until its volume is V and its pressure is P. Then by Boyle's law,

$$PV = V_1P_0.$$

Since

$$V_1 = \frac{V_0T}{273},$$

$$PV = \frac{V_0P_0}{273}T.$$

Let

$$\frac{P_0V_0}{273} = R = \text{constant}$$

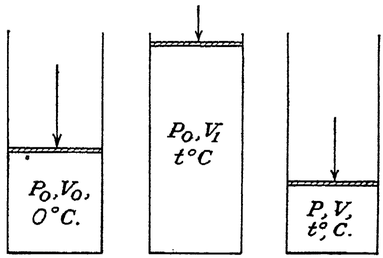


FIG. 264.—Combination of Boyle's and Charles's laws.

for a particular mass of a given gas. Then,

$$PV = RT,$$

$$\frac{PV}{T} = R = \text{constant},$$

and

$$\frac{P'V'}{T'} = \frac{PV}{T}.$$

Example.—The volume of a gas at 25°C. and atmospheric pressure is 1 l. What is its volume when the pressure is 2 atmospheres and the temperature is 300°C.?

$$\frac{\text{Original pressure} \times \text{original volume}}{\text{Original absolute temperature}} = \frac{\text{final pressure} \times \text{final volume}}{\text{final absolute temperature}}$$

$$\frac{PV}{T} = \frac{P'V'}{T'}.$$

$$\frac{1 \times 1}{298} = \frac{2 \times V'}{573}.$$

$$V' = \frac{573}{298 \times 2} = \frac{573}{596} = 0.963 \text{ l.}$$

286. Application to Different Masses of Gases.—In case the general gas law is to be applied to different masses of the same gas, the law should be written in terms of the volume per unit mass of the gas.

Let V_1 = the total volume of m_1 units of mass of the gas at pressure P_1 and temperature T_1.
V_2 = the total volume of m_2 units of mass of the gas at pressure P_2 and temperature T_2.
$v_1 = V_1/m_1$ = the volume of unit mass of the gas at pressure P_1 and temperature T_1.
$v_2 = V_2/m_2$ = the volume of unit mass of the gas at pressure P_2 and temperature T_2.

Applying the general gas law to unit mass of the gas,

$$\frac{P_1v_1}{T_1} = \frac{P_2v_2}{T_2}.$$

Since

$$v_1 = \frac{V_1}{m_1} \text{ and } v_2 = \frac{V_2}{m_2},$$

$$\frac{P_1V_1}{T_1m_1} = \frac{P_2V_2}{T_2m_2}.$$

Example.—A quantity of air having a mass of 100 g. is enclosed in a vessel having a capacity of V. The pressure of the gas is 75 cm. of mercury and its temperature is 27°C. Find the pressure exerted by the gas when 60 g. of it have been allowed to escape, and the temperature has been raised to 227°C.

Let m_1 = 100 g.
m_2 = 40 g.
T_1 = 300° absolute.
T_2 = 500° absolute.
P_1 = 75 cm. of mercury.

$$\frac{V \times P_1}{m_1T_1} = \frac{V \times P_2}{m_2T_2}.$$

$$\frac{V \times 75}{100 \times 300} = \frac{V \times P_2}{40 \times 500}.$$

$$P_2 = 50 \text{ cm. of mercury}$$

Problems

1. A bar of copper with a coefficient of linear expansion of 0.0000167 is measured at 15°C. and found to be 65 cm. long. At what temperature will it be 1 mm. shorter?

2. Steel rails in 40-ft. lengths are laid in water at 0°C. How much space between consecutive rails must be allowed to permit expansion to a summer temperature of 45°C.?

3. A sheet of brass has an area of 112.8 sq. cm. at 18°C. What will be the area at a temperature of 100°C.?

4. A steel rod 24 cm. long has a cross section of 0.8 sq. cm. What force would be required to extend the bar by the same amount as the expansion produced by heating it through 10°C.?

5. A rod of steel and one of brass have exactly the same length, 120 cm., at 0°C. The rods are heated until they differ in length by 0.08 mm. What is the temperature?

6. A brass pendulum is adjusted to beat seconds at 15°C. What will be the gain or loss per day if the temperature of the clock drops to 0°C.?

7. An iron ring which is 1 ft. in diameter is to be shrunk on a pulley which is 1.005 ft. in diameter. If the temperature of the ring is 10°C., find the temperature to which it must be raised so that it will just slip on the circumference of the pulley.

8. A glass flask with a very small coefficient of expansion holds exactly 1 l. It is filled with mercury at 0°C. and then heated to 80°C. How much mercury runs out?

9. A copper sphere 12 cm. in diameter is immersed in water at 10°C. How much will the buoyancy of the water on it be changed when the water is heated to 75°C.?

10. A long U-tube is filled with alcohol. One arm of the tube is kept at 10°C. and the other arm at 30°C. Find the length of the column in the tube at the higher temperature, if the length of the column in the tube at the lower temperature is 60 cm.

11. Air pumped into a tank has a temperature of 90°F., and the pump stops when a pressure of 150 lb. per square inch, in excess of atmospheric pressure, is reached. What will be the pressure in the tank after the air has cooled down to 68°F.?

12. What volume would be occupied by a sample of gas which occupies 400 c.c. at 0°C. and 76 cm. pressure if it were cooled to −30° and the pressure reduced to 73.5 cm.?

13. A flask contains air at room temperature, 17°C., and a pressure of 74 cm. of mercury. Find the pressure in the flask after it is sealed and cooled to −80°C.

14. Illuminating gas is stored in a tank designed so that the volume may change but the pressure remains constant. If the tank contains 30,000 cu. ft. under standard conditions, how much does the volume change when the temperature rises from 7 to 35°C.?

15. A clock which has a brass pendulum beats seconds correctly when the temperature of the room is 22°C. How many seconds per day will it gain or lose when the temperature of the room is 16°C.?

16. At the beginning of the compression stroke of an automobile engine, the gas occupies a volume of 12 cu. in. at atmospheric pressure. At the end of the compression, the pressure is 6.6 atmospheres and the volume 2.1 cu. in. What is the temperature according to the general gas law, if the original temperature was 37°C.?

17. Instruments for the study of the upper atmosphere are sent up in a free balloon made of rubber which stretches as the gas inside expands. What volume would be assumed by 120 cu. m. of gas admitted at 2°C. and 75 cm. pressure in a region where the pressure is 40 cm. and the temperature 33°C.?

18. An air bubble has a volume of 10 cu. cm. at the surface of a lake where the temperature is 27°C. What is its volume at a depth of 300 m. where the temperature is 5°C.?

19. A stratosphere balloon has a gas bag which has volume of 500,000 cu. ft. when the barometer reads 75 cm. of mercury and the temperature is 20°C. Find its volume when the balloon has risen to such a height that the atmospheric pressure is 12 cm. of mercury and the temperature −32°C.

20. A tank containing 6 cu. ft. of oxygen at a pressure of 250 lb. per square inch is opened and the oxygen allowed to escape into a sealed chamber which has a volume of 60 cu. ft. The chamber originally contained air at atmospheric pressure. What is the resulting pressure if there is no change in temperature?

CHAPTER XXV

KINETIC THEORY OF GASES

287. Brownian Motions.—The simplest and most direct evidence for the existence of molecules was first noted by an English botanist by the name of Brown. With a microscope he observed the motion of very fine particles held in suspension in water and noted that these fine particles are constantly in motion. The smaller the particles the more freely do they move. The motion of these particles is caused by the incessant bombardment of the molecules of the water or other liquid in which they are suspended.

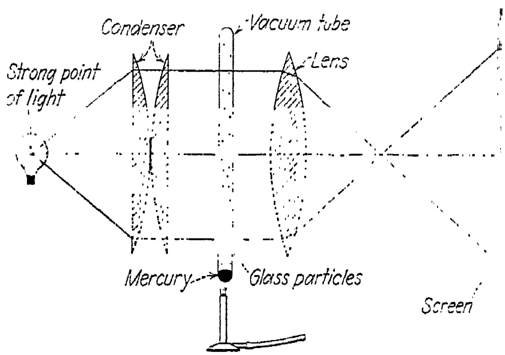


FIG. 265.—Motion of shadows cast by glass particles illustrate behavior of molecules.

This bombardment of the water molecules is not the same on the different sides of the fine particles. Hence, they are driven hither and thither. An approximate picture of the behavior of such small particles is obtained by projecting on a screen (Fig. 265) the shadows cast by finely divided glass particles which are set in motion by rapidly boiling mercury.

Perrin and others who have made careful studies of these motions have found that the distribution of these particles, their velocities, and their mean free paths are precisely what should be expected from the kinetic theory of gases. From these observations it is possible to determine the number of molecules in

1 c.c. of a gas under standard conditions. The fact that the number of molecules per cubic centimeter in a gas as determined in this way is in good agreement with the number derived from the methods involving the kinetic theory of gases shows that the motion of these fine particles obeys the same general laws as the motion of molecules.

288. Basic Assumptions.—The physical properties of a gas are well explained on the molecular theory. Three basic assumptions are, however, necessary.

1. The molecules of a gas are extremely small, perfectly elastic spheres. This assumption implies that when molecules of gas collide with other molecules or with the walls of the containing vessel, the total kinetic energy of the molecules is not diminished in any way.

2. The molecules move with high velocities through the space occupied by the gas. Between collisions, their paths are straight lines. This assumption implies that the forces acting on the molecules are negligible except at collision.

3. The time occupied in a collision between two molecules or in a collision of a molecule with the wall is small compared to the time between collisions. This assumption implies that a collision is nearly instantaneous.

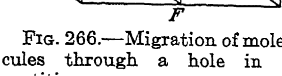


FIG. 266.—Migration of molecules through a hole in a partition.

289. Molecular Velocities in Gases.—Consider a large vessel (Fig. 266) containing two compartments. Let one of these, A, be filled with a gas and the other compartment B be empty, and suppose there is a fine circular opening O in the partition $DCEF$ between the two compartments. The gas molecules which fall upon the opening O will leave the compartment A and enter the compartment B. If by some means the pressure of the gas is maintained constant in the compartment A, the number of molecules which pass through the opening O in a given time is just equal to the number of molecules which would have collided with an area of the wall equal to the area of the opening. In this way, there is formed a stream of molecules from the compartment A to the compartment B. The velocity of this stream is equal to the mean velocity of the molecules.

If the velocities of the molecules escaping through the opening could be measured directly, the molecular motions of the molecules would be established directly and the velocities of the molecules determined directly by experiment. In general, these velocities cannot be measured since the

molecules collide with other molecules and are deflected from their paths. The collision of the molecules with their neighbors can be prevented by allowing the molecules to escape into a vacuum. The paths of the molecules can be traced in a number of ways. The velocities of molecules differ from molecule to molecule. A few have very large velocities and a few very small velocities. This variation of the velocities of the molecules is shown in Fig. 267 which gives the relation between the number of molecules and their velocities.

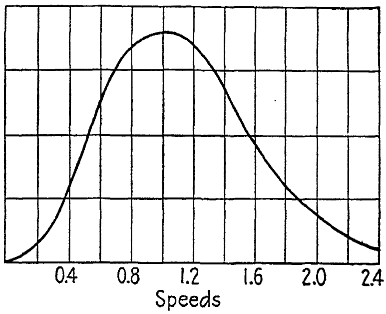


FIG. 267.—Distribution of velocities in a large group of molecules.

290. Monomolecular Films.—A maximum value of the size of a molecule has been obtained by measuring the thickness of the thinnest layer of a substance which will retain the properties of that substance. Such a layer of the substance must have a thickness at least equal to the diameter of the molecule, but the thickness may be several times the diameter of the molecule. To get a maximum value for the diameter of a molecule by this method, a known mass of oil is allowed to spread over the surface of water. Care is taken that the film of oil on the surface of the water be as thin as possible. From the area of the film of oil, the mass of the oil, and its density, the thickness of the film can be calculated from the equation,

$$\text{Thickness} \quad \frac{\text{mass}}{\text{area} \times \text{density}}.$$

The thickness obtained in this way is found to be about 0.000001 mm. Hence, the maximum diameter of a molecule of oil is less than 10^{-6} mm. Since the layer of oil is probably several molecules of oil in thickness, the diameter of the molecule is much less than 0.000001 mm.

291. The Mean Free Path and the Number of Collisions per Second.—The distance the molecules travel between collisions differs from time to time. Figure 268 represents diagrammatically the paths of a molecule between different collisions. The average of the lengths of these paths over a large number of collisions is called the *mean free path of the molecule.* In a similar way, the velocities of the molecules vary between collisions, and the mean velocity is defined as the average of the velocities between a large number of collisions. From the mean free path and the mean velocity of the molecules, it is possible to calculate the number of collisions the molecules make per second.

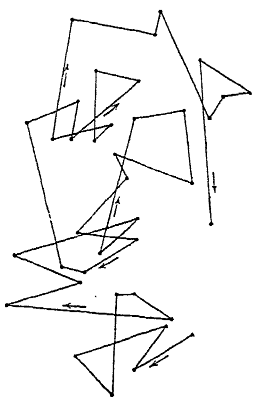


FIG. 268.—Diagrammatic representation of behavior of molecules between collisions.

Let L = the average distance the molecules move between collisions.

v = the mean velocity of the molecules.

N = the number of collisions per second.

Then,

$$N = \frac{v}{L}.$$

292. Molecular Magnitudes.—There are a number of molecular magnitudes characteristic of gases under different conditions. Some of these magnitudes are known with great accuracy. Others are only approximately known.

1. *The mean velocity* for a particular temperature differs from one kind of gas to another kind.

2. *The number of molecules* per cubic centimeter is the same for all gases at the same temperature and pressure.

3. *The mean free path* depends on the pressure and temperature of the gas.

4. *The effective diameter* of molecules is a useful quantity because there are many phenomena which can be satisfactorily described, as if the molecules were solid spheres with definite diameters. Later considerations will show, however, that molecules are not solid spheres to which a definite diameter can be assigned.

MOLECULAR QUANTITIES OF 0°C. AT PRESSURE OF 76 CM. OF MERCURY

	Hydrogen	Oxygen
Number of particles per cubic centimeter....	2.70×10^{19}	2.70×10^{19}
Diameter of each molecule	2.4×10^{-8} cm.	3.2×10^{-8} cm.
Mass of each molecule......	3.33×10^{-24} g.	53×10^{-24} g.
Mean free path..	1.83×10^{-5} cm.	1.0×10^{-5} cm.
Number of collisions per second..........	1.00×10^{10}	4.6×10^{9}
Average velocity	18.4×10^{4} cm. per second	4.6×10^{4} cm. per second
Mass per cubic centimeter....	8.99×10^{-5} g.	1.43×10^{-3} g.
Volume per gram	1.11×10^{4} c.c.	699 c.c.
Number of molecules in 1 gr.-mol..........	6.06×10^{23}	6.06×10^{23}

293. Pressure of a Gas from the Kinetic Theory.—The pressure exerted by a gas is due to the bombardment of the walls as the molecules strike against them (Fig. 269). An expression for the

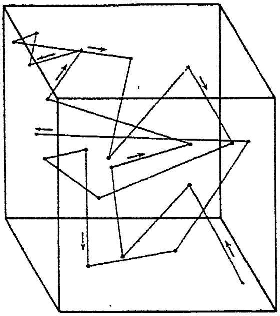


FIG. 269.—Collision of molecules against the walls of the containing vessel.

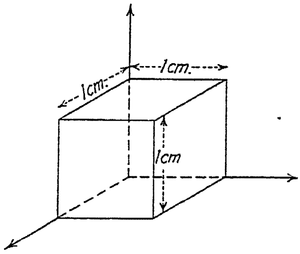


FIG. 270.—Collision of a molecule against the face of unit cube.

pressure may be derived by neglecting the size of the molecules and their impacts against each other, and considering each molecule as colliding only with the wall of the vessel. Suppose that a single molecule is enclosed in a cube (Fig. 270) with an edge

1 cm. long, and that this molecule moves to and fro across the cube at right angles to two of its opposite faces. Let v denote the velocity of the molecule and m its mass. Its momentum is mv as it approaches one of the faces. At collision, the direction of its velocity is reversed. The molecule is stopped and then started in the opposite direction, so that the change of momentum produced by the collision is $2mv$. Between two consecutive collisions with the same face, the molecule travels a distance of 2 cm. The time between collisions at the same face is therefore $2/v$. The number of collisions per second is $v/2$. Since at each collision the change of momentum is $2mv$, the change of momentum per second for each molecule is

$$2mv \times \frac{v}{2} = mv^2,$$

i.e.,

$$\text{Change of momentum per second at single face} = mv^2.$$

If there are N molecules in each cubic centimeter, they will be moving in all directions. On the average, however, it may be assumed that $N/3$ are traveling parallel to any edge of the cube. Of these $N/3$ molecules, one half is moving in one direction, and the other half in the opposite direction. Each of these molecules will strike the opposite face of the cube alternately. At each of these collisions the momentum of the molecule will be reversed in the same way as that of the single molecule considered above. The change of momentum at one face per second for all the molecules is

$$\begin{aligned}&\text{Change of momentum per second per square centimeter}\\ &\qquad\qquad\qquad\qquad\text{of face of cube} = \frac{N}{3}mv^2.\end{aligned}$$

By Newton's second law of motion, the change of momentum per second is equal to the force. In this case, the force per unit area is called the pressure. Hence, the pressure exerted on 1 sq. cm. of the cube is

$$p = \frac{N}{3}mv^2 \text{ dynes per square centimeter.}$$

The density of a gas is the mass per cubic centimeter = Nm.

$$\rho = Nm.$$

$$p = \frac{\rho v^2}{3}.$$

According to Boyle's law, p/ρ is constant as long as the temperature is unchanged. Hence, V^2, which stands for the mean square velocity of the molecules, is constant for a given temperature. The molecules have different velocities, but the mean of the square of the velocities of the individual molecules remains constant so long as the temperature is not changed.

The characteristic equation of a perfect gas states that

$$\frac{p}{} = RT.$$

Hence,

$$\frac{v}{3} = RT;$$

that is, the mean square of the velocities of the molecules is proportional to the absolute temperature of the gas. According to the kinetic theory the molecules will have lost all their motion at the absolute zero.

294. Specific Heats of Gases.—The specific heat of a gas depends on whether the gas is heated at constant volume or at constant pressure. These two specific heats are known as *specific heat at constant pressure* and *specific heat at constant volume.*

1. *Specific Heat at Constant Volume.*—When heat is supplied to a gas in which the volume is kept constant, the pressure increases, and all the energy which is supplied to the gas is used to increase the kinetic energy of the molecules. There is no external work done by the gas. When the temperature of 1 g. of the gas is raised through 1°C., the gas will absorb C_v units of heat, and this quantity of heat is its *specific heat at constant volume.*

2. *Specific Heat at Constant Pressure.*—In heating a gas 1°C., at constant pressure the heat required to increase the speed of the molecules will be the same as it was in case the gas was heated an equal amount at constant volume. In addition to this heat, it is necessary to supply a certain amount of heat to do external work while the gas is expanding. For example, if the gas is expanding in a cylinder closed by a moving piston, the molecules

after colliding with the piston will rebound with less energy than that with which they reached the piston. Additional energy must be supplied to make up this decrease. Consequently, the specific heat at constant pressure must exceed the specific heat at constant volume by an amount which is just equal to the thermal equivalent of the work which is done when unit mass of gas is heated through 1°C. at constant pressure. In a simple case this work may be done on a piston which moves as the gas expands. If A is the cross section of the piston in square centimeters, p the pressure of the gas in dynes per square centimeter, and d the distance in centimeters through which the piston moves, the work done by the gas is

$$\text{External work} = \text{force} \times \text{distance} = pAd \text{ ergs.}$$

Now $Ad = v$ is the increase in volume of unit mass of the gas for 1°C. rise in temperature. Since the coefficient of expansion of a gas at constant pressure is $1/273$,

$$v = \frac{V}{273},$$

where V is the original volume of unit mass of the gas at 0°C. Hence,

$$\text{External work for 1°C. rise in temperature} = pv = \frac{pV}{273} \text{ ergs.}$$

From the characteristic equation of a gas

$$pV = RT$$
$$\frac{pV}{T} = R = \frac{pV}{273}.$$

Therefore, the work to raise the temperature 1°C. $= R$ ergs.

$$\text{Hence } C_p - C_v = R,$$

if C_p and C_v are measured in work units.

The ratio of the specific heat of a gas at constant pressure to the specific heat at constant volume is

$$k = \frac{C_p}{C_v} = 1.41 \text{ for air,}$$
$$= 1.66 \text{ for mercury vapor.}$$

295. Relation of Temperature to the Kinetic Energy of the Molecules.—According to the kinetic theory, the temperature of a gas corresponds to the mean kinetic energy of its molecules; that is, to $\frac{1}{2} MV^2$, where M is the mass of the molecule, and V^2 is the mean square of the velocities at a given temperature. If the temperature of the gas is kept constant, the mean kinetic energy of the molecules is also constant. On this theory, the cooling of a gas is equivalent to decreasing the kinetic energy of the molecules, and heating a gas is equivalent to increasing the kinetic energy of its molecules. If the cooling of a gas were continued until its molecules no longer had any mean kinetic

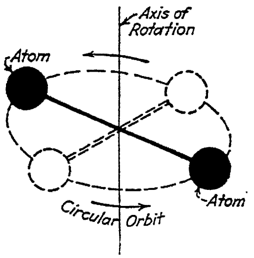


FIG. 271.—The rotation of a diatomic molecule about an axis. The molecule has energy of rotation.

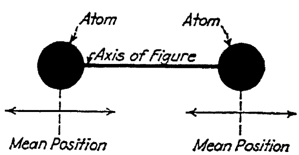


FIG. 272.—The vibration of a diatomic molecule. The molecule has energy of vibration.

energy, the translational motion of the molecules would cease. The molecules would have no kinetic energy of translation, and the temperature of the gas would be said to be *absolute zero.* From the kinetic theory (Sec. 293) and Charles's law (Sec. 283), this temperature comes out to be 273° below zero on the centigrade scale.

The molecules of a gas may have energy of rotation and energy of vibration as well as energy of translation. For example, if the molecule consists of two atoms, these may rotate around their center of gravity and thus have energy of rotation (Fig. 271); or the two atoms may vibrate with respect to each other (Fig. 272), in which case the molecule has a certain amount of energy of vibration.

296. Elasticity of Gases.—In the case of gases there will ordinarily be a change of temperature when the gas is compressed or expanded. In such cases, the coefficient of elasticity depends on the way in which the gas is compressed. It is customary to distinguish two cases: **(1) When the volume change takes place**

at constant temperature; (2) when the volume change takes place so suddenly that there is no time for the heat generated by a compression to flow out of the gas or for heat to flow into the gas during an expansion. The former is called **isothermal elasticity** and the latter **adiabatic elasticity.** The coefficient of adiabatic elasticity is always greater than the coefficient of isothermal elasticity. In the case of air, the coefficient of adiabatic elasticity is about 1.4 as great as the coefficient of isothermal elasticity. Hence, it is more difficult to compress a gas adiabatically than it is to compress it isothermally.

The coefficient of isothermal elasticity may be calculated as follows:

Let P = the pressure of the gas before compression.
P' = the pressure of the gas after compression.
$(P' \quad P)$ = the change in the pressure of the gas.
= the change in force per unit area.
= the stress.

Let V = the volume of the gas before compression.
V' = the volume of the gas after compression.
$(V - V')$ = the change in volume.
$\left(\frac{V - V'}{V}\right)$ = the change in volume per unit volume.
= the strain.

$$\text{Coefficient of elasticity} = E = \left(\frac{\text{stress}}{\text{strain}}\right) = \frac{(P' - P)}{\left(\frac{V - V'}{V}\right)}.$$

$$E_i = V\left(\frac{P' - P}{V - V'}\right).$$

From Boyle's law, $P/P' = V'/V$,

whence

$$\frac{P' - P}{P'} \quad V - V'.$$

and

$$\frac{P' - P}{V - V'} = \frac{P'}{V}$$

$$V\frac{P' - P}{V - V'} = V\left(\frac{P'}{V}\right) = P';$$

i.e.,

$$E_i = P \text{ (approx.) for small changes of pressure.}$$

The coefficient of adiabatic elasticity $= E_a = kP$ (see Appendix E-2),

where
$$k = \frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}} = \frac{C_p}{C_v}.$$

297. Molecular Attractions in Gases.—The first attempt to detect the existence of forces acting between gaseous molecules was made by Joule who connected two flasks (Fig. 273) together by means of a tube containing a stopcock. One of the flasks was exhausted and the other was filled with air, and the flasks were immersed in a vessel filled with water. When the stopcock was opened, the gas distributed itself through both flasks, and the pressure became equal in the two flasks. The average distance between the molecules was greater after this equalization. If the molecules attract each other, work must have been done in pulling them apart. The work to pull the molecules apart comes at the expense of the heat energy in the gas itself and should cause a fall in the temperature of the gas. But this method was not sufficiently sensitive to detect small changes in temperature, and consequently the experiment led to negative results.

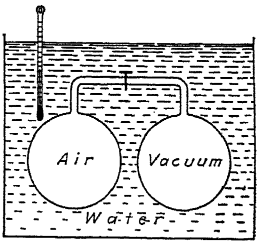


FIG. 273.—Free expansion of a gas from one vessel to another. A test for molecular attraction.

Another form of the experiment was devised. It is a less direct but more sensitive method of observing any change in temperature in a gas when it expands from a high to a low pressure.

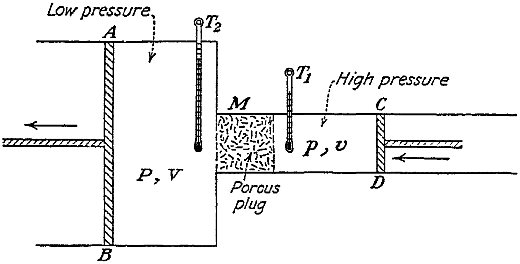


FIG. 274.—Cooling effect produced by a gas expanding from a higher to a lower pressure. A more accurate test for molecular attraction.

A diagrammatic representation of the essential parts of the apparatus used in this experiment is shown in Fig. 274. By means of a compressor, a high pressure is maintained on the right-hand side of a porous plug *M* consisting of a mass of tightly packed cotton. Through this plug the air is forced slowly and emerges on the left-hand side of the plug with an inappreciable

velocity but at a very much lower pressure. With the pressure on the right-hand side about 200 atmospheres, that on the left-hand side is scarcely more than 1 atmosphere. Thermometers T_1 and T_2 give the temperature of the gas just before and just after it passed through the plug. Precautions are taken to prevent heat from flowing into the gas from the outside or from escaping from the gas to the outside air.

The results of experiments of this type show that when proper care is taken to prevent heat from flowing out of the gas or into it, there is, in general, a decrease in temperature in passing through the plug. This dècrease in temperature is nearly proportional to the difference in pressure on the two sides of the porous plug P. The magnitude of this cooling effect gives a measure of the attraction between the individual molecules for each other. The greater the pressure, the greater is this attraction.

298. Deviations from Boyle's Law.—If Boyle's law held rigidly for actual gases, the product pv would be constant for all pressures and volumes of the gas so long as the temperature were kept constant. But accurate observations show that the product pv is not constant over any wide range of pressures. At first the product pv dècreases with increasing pressure. After passing through a minimum value, it increases with further increase of pressure. These divergences from Boyle's law are due to two causes.

1. The volume of the molecules is not negligible in comparison with the total space occupied by the gas. At low pressures, the space actually occupied by the molecules of the gas is much less in comparison with the total space occupied by the gas than it is at high pressures.

2. In ideal gases, it is assumed that the forces of attraction between the individual molecules can be neglected. In actual gases at high pressures, these forces become appreciable and cause deviations from Boyle's law which is only valid for ideal gases.

These two causes oppose each other. The temperature and the pressure of the gas determine in a given case which of the effects predominate. To take account of these two effects, corrections must be made in the general gas law. A number of equations have been suggested to provide for these corrections. The most familiar of these equations is the one proposed by Van der Waals. This equation is as follows:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where a and b are constants which are characteristic of a given gas but independent of temperature and volume. This equation differs from the general gas law by the addition of two terms: (1) (a/v^2), which is introduced to correct for the attractive force between the molecules; and (2) b, which corrects for the volume occupied by the molecules themselves in comparison with the total space filled by the gas.

Problems

1. Find the number of gas particles each with a mass of 3.3×10^{-23} g. and an average speed of 560 m. per second which would maintain normal atmospheric pressure against the walls of a containing vessel with a volume of 5 c.c.

2. A vessel containing hydrogen is attached to a pump, and the pressure is reduced to 10^{-5} mm. of mercury at a temperature of 17°C. How many molecules per cubic centimeter remain in the vessel?

3. How long would it take for a toy balloon to deflate to half its original volume at atmospheric pressure, if it were losing hydrogen molecules at the rate of a million per second? (Assume an original volume of 1 l. and 100 cm. of mercury pressure.)

4. Calculate the average velocity of particles of carbon monoxide under standard conditions if the density of the gas is 1.25 g. per liter.

5. A McLeod gauge can indicate pressures as low as 0.000001 mm. How many molecules per cubic centimeter of hydrogen would be required to produce this pressure at 0°C.?

6. How many molecules per cubic centimeter are there in a vessel in which the temperature is 27°C. and the pressure 10^{-6} mm. of mercury?

7. A carefully evacuated vessel has a volume of 2 l. It develops a small leak through which two million molecules of air pass each second. How long before the pressure of the air in the vessel will be 0.25 atmosphere?

CHAPTER XXVI

FUSION

299. The Melting Point.—If a vessel of ice or snow is heated, the temperature at first rises until it is 0°C. and then remains stationary until all the ice is melted. After all the ice has been melted, the temperature of the water begins to rise. That temperature at which the solid changes into a liquid without a change of temperature is called the **melting point.** For ice, this temperature is 0°C. or 32°F. At the melting point, the addition of heat simply serves to hasten the melting process without any change of temperature.

If a pail of water is placed in a freezing mixture of ice and snow, the temperature of the water decreases until ice begins to be formed in the pail. After this temperature has been reached, the temperature of the water in the pail remains the same until all the water has become ice. That temperature at which the liquid changes into the solid state is its **freezing point.** This temperature is ordinarily the same as the temperature at which the solid melts. For crystalline substances, like ice or copper, the freezing point or the melting point is sharply defined. For substances which are not crystalline, like wax or glass, the substance gradually softens in passing from the solid to the liquid state. Such substances do not have a definite melting point. In the cases of certain fats, the melting point is not the same as the freezing point. For example, butter melts between 28 and 32°C. and solidifies between 20 and 23°C. For other data on melting points see table, page 770.

300. Heat of Fusion.—In order to cause a solid like ice to change into a liquid, it is necessary to supply a given quantity of heat to each gram or each pound of it. This is true, though the temperature of the ice at the beginning is the same as the temperature of the water at the end of the process. The ice has a crystalline structure, and the heat which is supplied is necessary to tear down this structure. Evidently, the water in changing

back to ice or, in general, the liquid in changing back to a solid will give up the heat which it absorbed in melting. **The heat of fusion of a substance is defined to be the number of calories necessary to convert 1 g. at the melting point into liquid at the same temperature.** It may also be defined as the number of British thermal units which must be supplied to change 1 lb. of the solid to liquid without a change of temperature. To change 1 g. of ice to water at 0°C. requires 80 cal., and to convert 1 lb. of ice to water at 32°F. requires 144 B.t.u.

The heat of fusion of a substance can easily be found by the method of mixtures.

Let M = the number of grams of substance melted.
t = the melting point of the substance.
t' = the temperature of the calorimeter at the beginning.
T = the temperature of the calorimeter at the end.
m' = the initial mass of water in the calorimeter.
m = the mass of the calorimeter.
s' = the specific heat of the calorimeter.
s'' = specific heat of water.
s = the specific heat of the substance in the liquid state.
L = the latent heat of fusion.

Heat lost by water $= m's''(t' - T)$.
Heat lost by the calorimeter $= ms'(t' - T)$.
Heat required to melt the substance $= ML$.
Heat required to change the substance from the melting point to the final temperature $= Ms(T - t)$.
Since,

$$\text{Heat gained} = \text{heat lost:}$$
$$ML + Ms(T - t) = m's''(t' - T) + ms'(t' - T).$$

In the case of ice, this reduces to

$$ML + Ms''(T - 0) = m's''(t' - T) + ms'(t' - T).$$

From this equation L can be found.

Example.—In making observations on the heat of fusion of ice, the following data were taken:

Mass of ice melted = 25 g.
Initial temperature of the water in the calorimeter = 30.0°C.
Final temperature of water in calorimeter = 18.0°C.
Initial mass of water in calorimeter = 200 g.

Neglecting the heat lost by the calorimeter, find the heat of fusion of the ice.

Heat lost by the water = mass of water × change in temperature of water × specific heat of water

$$= 200 \times (30 - 18)s'' = 2{,}400.$$

Heat necessary to melt the ice = mass of ice $\times$ heat of fusion.
= $25 \times L$.

Heat necessary to raise temperature of water from 0 to final temperature = mass of melted ice $\times$ change in temperature $\times$ specific heat of water = $25(18 - 0) \times 1$.

Heat lost = heat gained.

Hence

$$200(30 - 18) \times 1 = 25L + 25(18 - 0) \times 1.$$

$$L = \frac{(200 \times 12) \times 1 - (25 \times 18) \times 1}{25} \quad 78 \text{ cal. per gram.}$$

301. Supercooling.—If a pure liquid is carefully protected from mechanical disturbances, it may be cooled below the temperature at which it normally solidifies. Thus, water may be cooled to $-10°$C. or lower without becoming ice. The liquid at such a temperature is in a state of unstable equilibrium and will immediately solidify if it is disturbed or if a crystal of the solid is dropped into it. This phenomenon is known as **supercooling.**

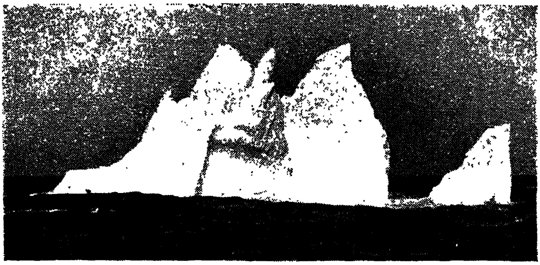


FIG. 275.—An iceberg with about nine-tenths submerged. (*Underwood and Underwood.*)

302. Change of Volume during Freezing.—It is a familiar fact that ice floats (Fig. 275) and that pipes or bottles filled with water burst when they freeze. This shows that water expands when it freezes. Thus, 1 cu. ft. of water makes about 1.09 cu. ft. of ice when it is frozen. This expansion on solidification is due to the fact that the molecules of water form crystals leaving larger spaces between them than were present when the water was in the liquid condition. Cast iron behaves like water in this respect. It also expands when it solidifies, and for this reason is suitable for making castings in which it is desired to reproduce the detail of the mold. Most metals and other substances contract on solidification. This is one reason why gold and silver coins are stamped instead of being cast.

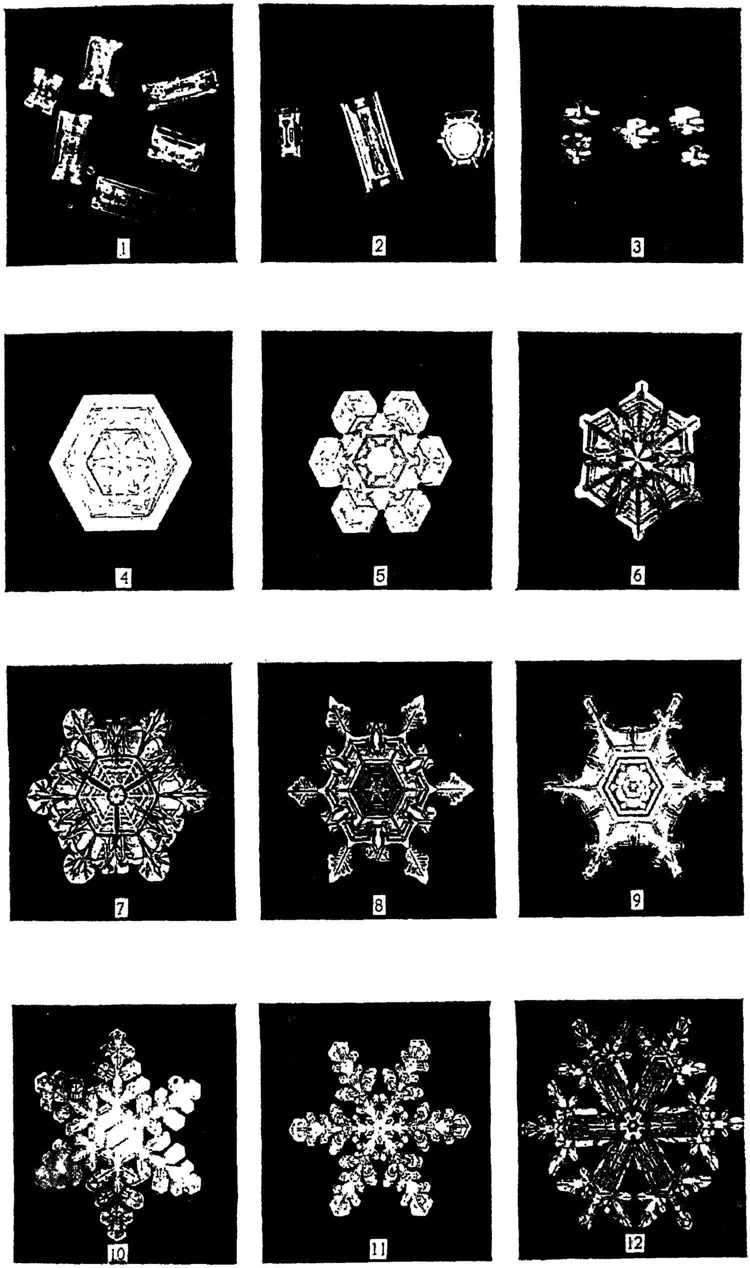


PLATE II. Snow crystals. (*Bentley.*)

303. Forces Exerted by Freezing Water.—That the forces exerted by freezing water are very large may be seen from the fact that they are sufficient to burst the strongest water pipes on a cold night. Some idea of the magnitude of these forces may be obtained by filling a cast-iron bomb with water and then putting it in a pail of ice and salt. If a screw plug has been placed in the bomb, and care is taken to fill the bomb completely with water, the solidification of the water bursts the bomb.

The forces exerted by water on freezing are of much importance in the formation of soils from rocks. The water percolates into the crevices in the rocks and then freezes. The expansion which occurs during freezing breaks off small or large fragments, which by this process after a time are made into soil. The alternate freezing and thawing of soils tend to pulverize them. Ice forming in the interstices of the soil serves to loosen compact land and give it better tilth.

304. Effect of Pressure on the Melting Point.—Since the application of pressure tends to keep a body from expanding, the

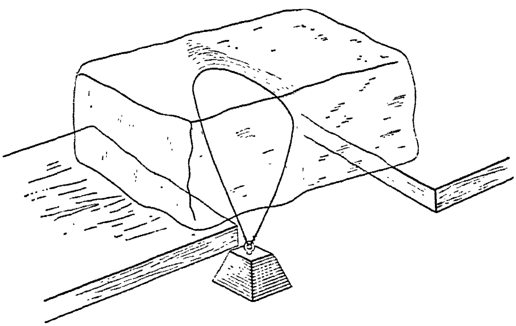


FIG. 276.—Effect of pressure on the melting point. Ice melts at a lower temperature under pressure.

expansion which takes place during the solidification of water is partially prevented by pressure. It, therefore, becomes necessary to lower the temperature below the normal freezing point before solidification takes place. Careful experiments show that the melting point of ice is lowered 0.0075°C. for an increase of 1 atmosphere in the outside pressure.

If, on the other hand, a substance expands upon melting, its melting point will be raised by the application of pressure.

The effect of pressure on the melting point of ice may be shown by taking a piece of ice (Fig. 276) which is about 1.5 ft. long and 6 in. square and hanging over it a loop of wire from which a weight of 35 or 40 lb. is supported. The pressure of the wire on the ice lowers the melting point of the ice, so that it is in a condi-

tion to melt as soon as the necessary heat is supplied. In order to melt each gram of the ice beneath the wire, it is necessary to supply 80 cal. to it. This heat is taken from the water above the wire causing the water above the wire to freeze again. The water which comes from the melting of the ice below the wire is at a temperature slightly below 0°C., at the instant melting occurs. But this water immediately flows up past the wire where the pressure on it is atmospheric. At this pressure, the water freezes again, giving up heat which is conducted through the wire to the ice below. Thus the process continues, until the wire cuts its way through the block of ice, leaving the block as solid as it was at the beginning of the experiment.

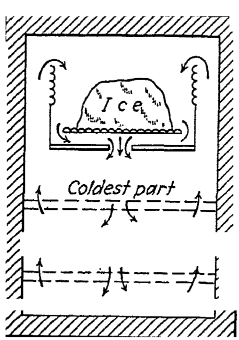


FIG. 277.—A refrigerator. Convection currents control the temperature.

305. The Refrigerator.—An important illustration of the absorption of heat in the melting of ice is found in the ordinary refrigerator (Fig. 277). The ice which is placed in the upper part of the refrigerator melts and thus absorbs heat from the air in contact with it. This air, being now heavier than the remainder of the air in the refrigerator, sinks through the openings in the bottom of the ice chamber and forces the warmer air upward. This warmer air is in turn cooled by the ice, and the process of circulation thus continues. The cool air in passing downward comes in contact with the food and takes heat from it. The temperature to which the air and the food in the refrigerator can be cooled in this way depends on the temperature of the outside air and the manner in which the inside of the refrigerator is insulated from the outside air. If the walls of the refrigerator were made of materials which would allow no heat to flow into the refrigerator, the temperature of the inside of the refrigerator might be reduced to 0°C. or 32°F. Some heat, however, always flows into the refrigerator, and this amount is greater when the temperature of the outside air is high, so that the temperature in the refrigerator is never as low as 0°C.

306. Freezing Point of Solutions.—When a solid like salt or sugar is dissolved in water, the freezing point of the solution is lower than the freezing point of water. This is true of any liquid in which some substance has been dissolved. The freezing point of the solution is always lower than that of the pure solvent. When such a solution begins to freeze, it is only the solvent which freezes out. This makes the remainder of the solution more concentrated and lowers its freezing point still farther. This process continues until the solution becomes saturated. Then

both the dissolved substance and the solvent freeze out in such a way that the concentration of the solution remains unchanged during solidification. For solutions which are not too concentrated, the lowering of the freezing point is proportional to the concentration of the dissolved substance.

Figure 278 represents the relation between the freezing point and the percentage of salt in a solution. Solidification begins at a lower temperature than the freezing point of the pure solvent. The freezing point decreases as the pure solvent freezes out, and the concentration of the solution thus increases. Curve AB shows the relation between the temperature and the concentration of the salt in the solution. If this process is continued long enough, the temperature at which the solvent freezes out finally becomes constant. At this temperature, the solvent and dissolved substance freeze out simultaneously as a mixture of two solids. The temperature at which the solvent and the dissolved substance crystallize out as a mixture is called the **eutectic temperature.**

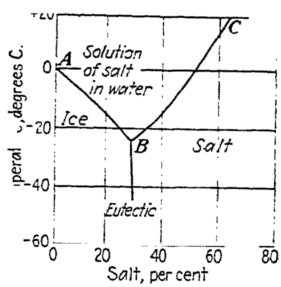


Fig. 278.—Freezing points of solutions of different concentrations.

If the temperature of the solution is higher than the eutectic temperature and the solution is saturated, the dissolved substance will crystallize out as the temperature is lowered, and the line BC will represent the relation between the temperature and the concentration. Along this curve, pure salt separates out from the solution. When the eutectic temperature is reached, both salt and solvent will crystallize out together as a mixture, and the freezing point will remain constant until the whole mass has solidified.

307. Freezing Mixtures.—In making ice cream it is necessary to lower the temperature of the cream in the freezer somewhat below the temperature of melting ice. In order to produce such a temperature, it is customary to prepare a mixture of ice and salt. The usual proportion of salt to ice is 1 part of salt to 3 parts of ice or snow. This mixture has a lower freezing point than pure ice has. With this mixture it is possible to reach a temperature of $-6°F$. Other substances besides salt may be used for this purpose, but salt is the most convenient.

Problems

1. A specimen of copper with a mass of 400 g. is heated to 220°C. and placed on a mass of ice. If 100 g. of ice is melted, what is the specific heat of copper?

2. A sphere of iron weighing 180 g. is heated in an oil bath and then placed on a block of ice, causing 48 g. of ice to be melted. What was the temperature of the iron?

3. An experiment on the heat of fusion of ice is to be performed in a room at 18°C. The mass of water in the calorimeter is to be 250 g., and the quantity of ice to be melted is 40 g. It is desired to have the initial temperature of the water as much above room temperature as the final temperature will be below. What must be the initial temperature? (Neglect the heat capacity of the calorimeter.)

4. Find the number of British thermal units required to heat 6 lb. of ice from a temperature of 12°F. to the melting point, melt it, and then heat the water to the boiling point. Specific heat of ice = 0.5.

5. Water produced by melting ice comes from a refrigerator at a temperature of 45°F. and the quantity obtained in 2 hr. is 7 lb. How many British thermal units of heat penetrate the walls of the refrigerator per hour?

CHAPTER XXVII

VAPORIZATION

308. Evaporation.—The molecules of a liquid are in constant motion with varying velocities. When the liquid is heated, the velocities of the molecules are increased and the spaces between the molecules are increased. Sooner or later, some of these molecules which are near the surface of a liquid move with sufficient velocity to escape from the surface (Fig. 279) and leave the neighboring molecules. Only the more rapidly moving molecules will escape, as the forces of attraction due to the other molecules will be sufficient to hold the more slowly moving molecules in the liquid. Those molecules which escape from the surface of the liquid move about in the space above the liquid. Since it is only the more rapidly moving molecules which escape from the surface, the average speed of the molecules in the liquid will decrease and the temperature of the liquid will drop during the evaporation.

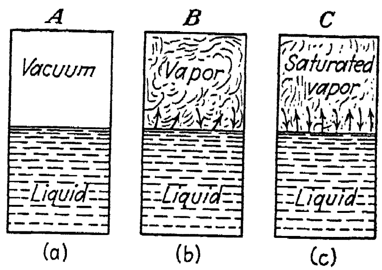


FIG. 279.—Evaporation of liquids. For saturated vapor the number of escaping molecules per unit of time is just equal to the number returning in unit time.

309. Saturated Vapor.—If the space above the surface of the liquid is enclosed, some of these particles will again return to the surface of the liquid and be held fast. As more and more molecules escape from the surface of the liquid, the number returning to the surface will also be increased. When the number of molecules in the space above the liquid has become so great that the number of molecules escaping from the surface of the liquid is just equal to the number returning to it, the space above the liquid will not gain more molecules at the expense of the liquid. The space above the liquid is then said to be saturated. The vapor which is above the liquid is called a **saturated vapor.**

If the temperature of this vapor remains unchanged, it is not possible to increase the pressure on it. When an attempt is made to increase the pressure on a saturated vapor, the vapor condenses and changes into liquid. The vapor pressures of different liquids (Fig. 280) are not the same at a given temperature.

310. Vapor-pressure Curve.—If a curve is plotted between temperatures as abscissae and the corresponding pressures of a saturated vapor, Fig. 281 is obtained. This curve shows the pressure of the vapor at each temperature. As the temperature of the liquid increases,

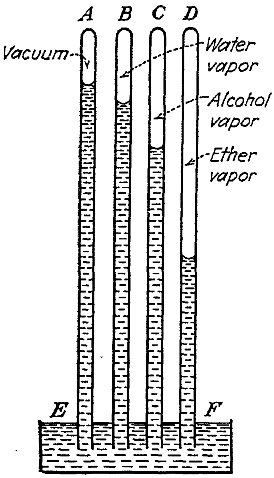


FIG. 280.—Vapor pressure of liquids. The lower the boiling point, the greater the vapor pressure at a given temperature.

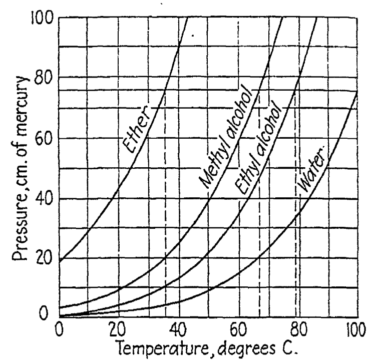


FIG. 281.—Vapor-pressure curves for water, ether, ethyl alcohol, and methyl alcohol. At the boiling point the vapor pressure is 76 cm. of mercury. Boiling points are indicated by dotted lines.

the pressure of the saturated vapor also increases. In the case of water, when the temperature has become 100°, the vapor pressure is 76 cm. of mercury. For any lower temperature the pressure is less. Whenever the temperature of the liquid is known, the vapor pressure can be found from this curve. On the other hand, when the vapor pressure is known, the temperature at which the liquid has this pressure is obtained from this curve. Such a vapor-pressure curve is obtained by observing the difference in level between the surfaces of mercury (Fig. 282) as the temperature is increased.

311. Heat of Vaporization.—Just as a certain amount of heat is required to convert a gram or a pound of ice into water without

changing its temperature, so a certain amount of heat is required to change a gram or a pound of water into steam without changing the temperature. When the steam condenses, this heat is again liberated and becomes available for heating other bodies. The quantity of heat required to cause the liquid to become a vapor depends on the temperature at which the change takes place. The heat of vaporization is, therefore, said to vary with the temperature. The higher the temperature, the easier it is to cause the liquid to become a vapor; that is, the higher the temperature, the less the heat of vaporization.

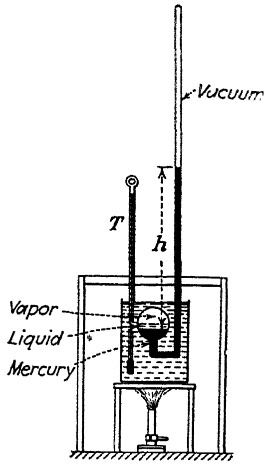


FIG. 282.—Measurement of vapor pressure. As the temperature increases, the height of the mercury in the tube increases.

The heat of vaporization is defined to be the heat necessary to change 1 g. of the substance from the liquid to the vapor state without any change in the temperature. For water at 100°C., it is found to be 538.0 cal.

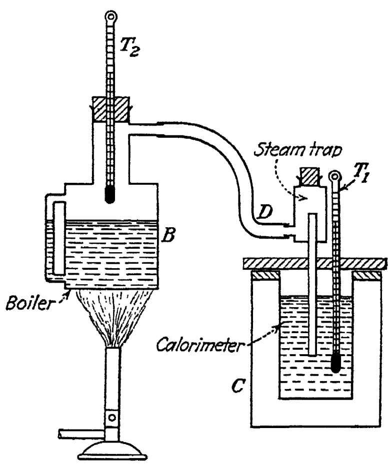


FIG. 283.—Heat of vaporization. The rise of the temperature of the liquid in the calorimeter measures the heat of vaporization.

That is to say, it takes more than five times as much heat to change 1 g. of water at 100°C. into steam at 100°C. as it takes to

heat that same gram of water from 0 to 100°C. For other temperatures the heat of vaporization of water is given in Table VIII page 770.

To determine the heat of vaporization of water, take a calorimeter C (Fig. 283) filled with a known weight of water, and a boiler B partly filled with water. Pass dry steam from the boiler into the calorimeter. In order to prevent water from being carried over, the steam passes through the trap D which catches the condensed steam. Note the temperature of the water in the calorimeter at the beginning and its temperature after a known quantity of steam has been condensed in it. The amount of condensed steam is found by weighing the calorimeter after the steam has been condensing in it for a sufficient time to produce the desired rise in temperature. The latent heat of vaporization of steam can now be calculated.

Let L = the heat of vaporization.
t = the temperature of the steam.
t' = the initial temperature of the water in the calorimeter.
T = the final temperature of the water in the calorimeter.
m = the mass of the steam condensed.
M = the mass of the water in the calorimeter.
C = the water equivalent of the calorimeter.

Heat liberated by condensation of steam $= mL$.

Heat liberated by cooling condensed steam from temperature t to $T = m(t - T) \times 1$.

Heat given to calorimeter $= C(T - t')$.

Heat given to water $= M(T - t') \times 1$.

Heat lost = heat gained.

Hence,

$$mL + m(t - T) \times 1 = M(T - t') \times 1 + C(T - t').$$

$$L = \frac{M(T - t') \times 1 + C(T - t') - m(t - T) \times 1}{m}.$$

Example.—Mass of steam condensed = ¼ lb.
Initial mass of water in calorimeter = 6.8 lb.
Initial temperature of water in calorimeter = 42°F.
Final temperature of water in calorimeter = 82°F.
What is latênt heat of steam in British thermal units?
Heat gained by water in calorimeter

= mass of water × change in temperature × sp. heat of water
$= 6.8 \times (82 - 42) \times 1 = 272$ B.t.u.

Heat lost by condensed steam in cooling from 212 to 82°F.

= mass of steam × change in temperature × sp. heat of water
$= \frac{1}{4} \times 130° \times 1 = 32.5$ B.t.u.

Heat lost by steam in condensation = mass of steam × latent heat
$= \frac{1}{4} \times L$ B.t.u.

Hence,

$$\frac{1}{4} \times L + 32.5 = 272.$$

$$\frac{1}{4}L = 239.5.$$

$$L = 958 \text{ B.t.u. per pound.}$$

$$\text{Correct value} = 966 \text{ B.t.u. per pound.}$$

312. Cooling by Evaporation.—In considering the heat of evaporation, it was seen that in order to cause 1 g. of liquid to change into the vapor state, it was necessary to supply a certain quantity of heat known as the heat of vaporization. If the liquid is evaporating, this heat will be taken from the remaining liquid and the surrounding bodies. The withdrawal of this heat from the surrounding bodies will cause their temperatures to decrease. For this reason the evaporation of the water sprinkled on the sidewalk causes the air to become cooler, and the evaporation of perspiration cools the body and makes sunstrokes less likely.

313. Evaporation from Soils.—If one of two soils which are exactly alike in their composition and structure is kept continuously wet so that large amounts of water are evaporating from its surface, while the other is kept as dry as possible, it will be found that the former soil is much cooler than the latter because of the fact that the heat necessary to evaporate the water is absorbed from the soil, and the temperature is thus lowered. It often happens that in the spring differences of temperature of as much as 12°F. are observed in soils on account of differences in drainage. Since clay soils retain the moisture better than sandy soils, there will be a difference in the temperatures of two such soils which are equally well drained.

The amount of evaporation from soils may be much decreased by covering the soil with some sort of protective coating as straw or dead leaves. Repeated shallow plowing by which a few inches of loose soil are always kept on the surface is very effective and is much followed in hot climates.

314. Evaporation from Leaves.—Evaporation takes place from the leaves of a growing plant whenever the amount of energy received from the sun and the state of the dryness of the atmosphere are such that water can be changed into vapor.

315. Boiling Point of Water.—Fill a flask half full of water and insert a thermometer in one of the holes in the stopper. In the other hole insert a short glass tube through which the steam may escape. Heat the flask over a flame until the water boils. By reading the thermometer from time to time, it will be found that no matter how rapidly the heat is applied, the temperature does not rise above 100°C. It will be noticed that at a certain temperature bubbles forming at the bottom of the flask rise to the surface, growing in size as they rise. That temperature at which the bubbles begin to reach the surface of any given liquid is called the **boiling point** or **the boiling temperature.**

The boiling point can be defined in another way. The pressure on the surface of the liquid is the pressure of the atmosphere. This pressure is, according to Pascal's principle, exerted throughout the liquid. Hence, the pressure which a bubble of vapor inside the liquid must sustain is also equal to atmospheric pressure plus the hydrostatic pressure due to the weight of the liquid above it. Until the pressure of the vapor in the bubble becomes as large as atmospheric pressure, the bubbles will collapse before they reach the surface of the liquid. The boiling point can then be defined as the temperature at which the pressure of the saturated vapor of the liquid is equal to the pressure of the atmosphere on the surface of the liquid.

316. Effect of Pressure on the Boiling Point.—Since the boiling point of a liquid is the temperature at which the vapor pressure of the liquid is the same as the outside pressure on it, it follows at once that when the outside pressure is changed, the boiling point will also change. This is easily understood if we recall that ordinarily the pressure of the atmosphere is about 15 lb. to the square inch. If this pressure is decreased, it will not be necessary to raise the temperature so high in order to allow the bubbles to form. When the pressure is raised, it will be necessary to raise the temperature still higher in order to produce bubbles. The bubbles will form only when the pressure of the vapor in the bubble is equal to the pressure on the surface of the liquid.

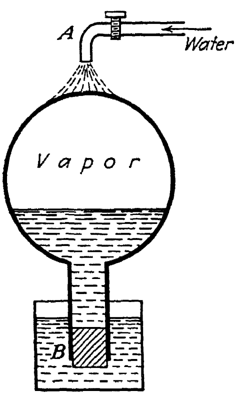


FIG. 284.—The boiling point is lowered by the reduction of the pressure above the liquid.

The influence of pressure on the boiling point can be shown by filling a flask (Fig. 284) half full of water and boiling it vigorously for some time to remove the air from above the water. Insert a rubber stopper in the flask, rendering the flask air-tight. Remove the flask immediately from the flame. Invert the flask and pour cold water on the bottom. This cold water will cause some of the vapor in the flask to condense, and the pressure on the hot water in the flask will be reduced sufficiently to allow the water to begin boiling again. The effect of pressure on the boiling point is also seen in Fig. 285. The water under the bell jar boils at a lower and lower temperature as the air is more and more removed from the jar.

On the top of high mountains the atmospheric pressure is less than at the base. The temperature to which the water can be raised before it boils will be lower on such mountains than at their bases. For this reason, vegetables like potatoes will cook more slowly at high elevations than at sea level. If the altitudes are very high, it may be impossible to cook such vegetables in open vessels because a sufficiently high temperature cannot be reached in this way.

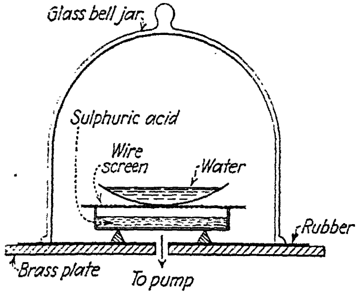


FIG. 285.—Evacuation of the bell jar causes the liquid to boil at room temperature.

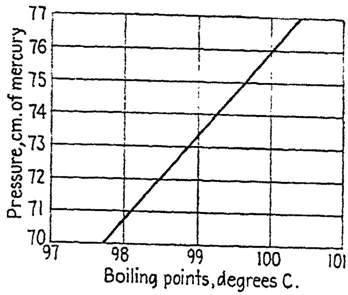


FIG. 286.—Change of boiling point with barometric pressure. At high altitudes boiling points are below normal.

In Fig. 286 is shown a curve giving the relation between the barometric pressure and the boiling point of water. The pressures have been plotted on the vertical axis and the temperatures on the horizontal axis (see tables on pages 768 and 769).

The effect of pressure on the boiling point of water is well illustrated in the action of geysers (Figs. 287 and 288).

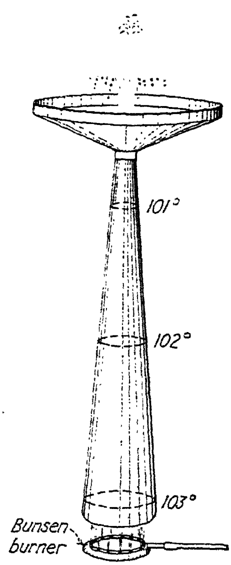


FIG. 287.—The pressure due to the weight of the water raises the boiling point of the water in the geyser.

317. Pressure Cooker.—In canning and also in cooking meats, it is often desirable to cook at temperatures higher than 100°C. Pressure cookers are used for this purpose. They are essentially small closed steam boilers (Fig. 289) provided with a pressure gauge and a safety valve. As the temperature of the water in the cooker is raised, the pressure of the water vapor increases just as in the ordinary steam boiler. By continuing to increase the temperature of the water, any desired pressure

may be developed in the boiler. In this way, any desired temperature can be obtained in the cooker so that foods may be cooked at temperatures above the boiling point of water under atmospheric pressure. The safety valve is adjusted to release at any pressure above that desired in the cooker.

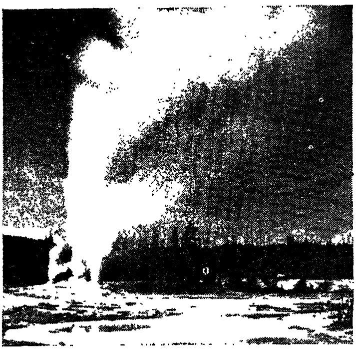


FIG. 288.

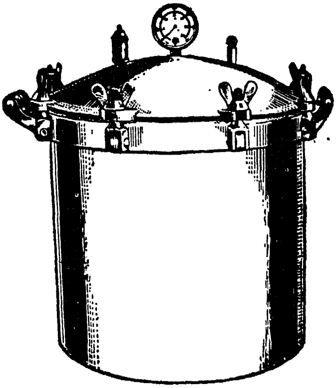


FIG. 289.

FIG. 288.—Union Geyser, in the Shoshone Basin, erupts from three sinter cones simultaneously. The highest jet reaches an altitude of 114 feet. The intermittent flow arises from a periodic increase in the pressure and the temperature of the water. (*Courtesy Carnegie Institution of Washington.*)

FIG. 289.—Pressure cooker. Higher pressures mean higher temperatures.

The pressure gauge indicates the pressure in the cooker in excess of the atmospheric pressure.

318. Vacuum Pans.—In many cases, substances which are to be vaporized are injured by heating to their normal boiling point. Such is the case with

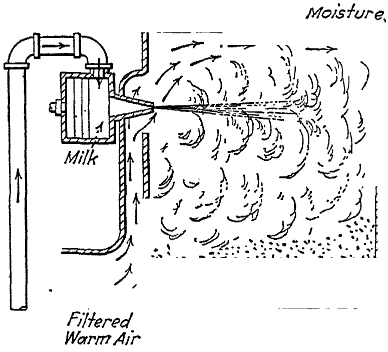


FIG. 290.—Removing the moisture from milk by the circulation of warm air.

sirups or milks. These liquids must then be evaporated at temperatures below their normal boiling points. In order to accomplish this, use is made of vacuum pans which operate on the principle that by reducing the pressure on the liquid, its boiling point may be lowered to some desired temperature.

These pans, which are used in the manufacture of condensed milk, consist essentially of a closed boiler from which the air can be removed to any desired extent. As the pressure of the air on the liquid is reduced, the boiling point is lowered and evaporation takes place at these low temperatures without heating the liquids to the higher temperatures which would be necessary under atmospheric pressure. By this evaporation the water contained in the milk or in the sirup is removed, and the solid matter remains behind in the pan. Figure 290 shows another way by which the water may be removed from the milk and the powdered milk left behind.

319. Boiling Point of Solutions.—When water has some foreign substance like sugar dissolved in it, the temperature at which it boils is raised. The amount the boiling point is raised is proportional to the amount of the substance dissolved in the water. In making candies or sirups, the temperature is taken as a means of determining the concentration of the solution. By observing the temperature at which the solution is boiling, it is possible to know when the candy or sirup has been boiled sufficiently long. When vegetables, fruits, meats, etc., are boiled in water, some of their contents dissolve in the water. This raises the boiling point slightly, so that the water boils above 212°F. The effect of a dissolved substance on the vapor pressure and the boiling point is shown in Fig. 291. The continuous curve gives the relation between the temperature and the vapor pressure of the pure solvent, and the dotted curve the same relation for the solution.

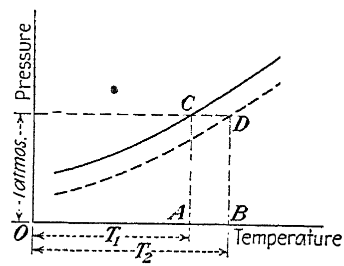


FIG. 291.—Boiling points of solutions. The addition of salts raises the boiling point.

320. Distillation.—When solids, like salt or sugar, are dissolved in liquids, the vapor which rises above the liquid does not contain any of the dissolved substance. In order to obtain pure water from water containing solids in solution, it is only necessary to evaporate the water first and then condense the vapor. The solids which were in solution will be left behind in the vessel. The way in which this is ordinarily done is represented in Fig. 292. Pure water vapor rises from the water which is being boiled in the flask D. This vapor passes into the tube A where it is condensed by the water flowing in the tube C. The condensed water drops into the vessel placed under the end of the tube A. In the manufacture of sugar, it is the solid sugar in solution which is of interest. The sugar would remain behind in the flask. It is not necessary in this case to condense the water vapor which rises from

the flask or vessel containing the solution. The lower the pressure, the lower the temperature at which the boiling takes place. In case it is not desired to heat the solution to the normal boiling point, the space above the liquid in the flask is kept exhausted by means of an air pump, as previously stated in the discussion of vacuum pans.

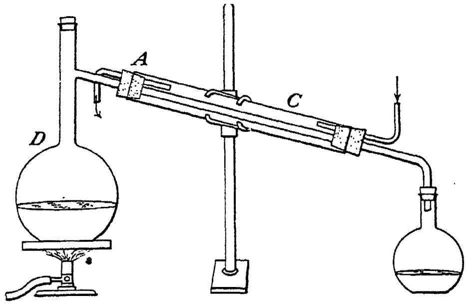


FIG. 292.—Distillation of liquids. The more volatile liquids evaporate first.

321. Household Still.—Where it is desirable to have distilled water in the household, a form of still shown in Fig. 293 may be used. Water from the boiler *B* which is located on the stove is vaporized. The vapor passes into the condenser *J*. On the inside of this condenser, there is a hollow tube *N* through which a current of cold water is flowing continuously. By this means, the vapor is condensed and drops to the bottom of the condenser from which it may be taken through the tap *R*.

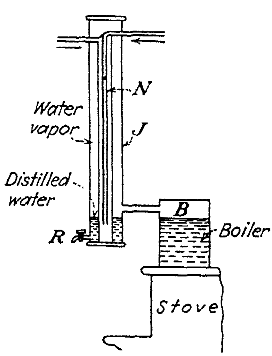


FIG. 293.—Household still. Water is purified by distillation.

322. Sublimation.—If a substance, like solid camphor or solid iodine, be left for some time in a closed vessel, the sides of the vessel become covered with small crystals of the substance. This is due to the fact that vapor which is given off by the solid is afterwards condensed on the sides of the vessel. This formation of vapor directly from a solid without passing through the liquid state is called **sublimation.** In like manner, the vapor may go directly from the vapor state to the solid state without passing through the liquid state. For most solids, the vapor pressure is nearly zero at ordinary temperatures, but in many cases the sense of smell tells us that some vapor is being given off. For ice at 0°C., the vapor pressure is 4.6 mm. of mercury; but as the temperature decreases, the vapor pressure becomes smaller.

The heat of sublimation is the heat necessary to change one gram of the substance from the solid to the vapor state without change of temperature. A vapor-pressure curve showing the vapor pressure when a solid is in equilibrium with its vapor may be plotted for this case as for the case of a liquid in equilibrium with its vapor. Such a curve may be called a **sublimation curve.**

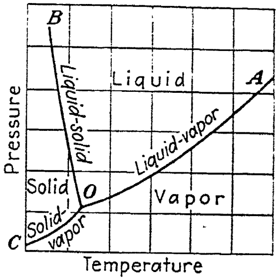


FIG. 294.—The triple point. Liquid, vapor, and solid are in equilibrium at the triple point.

323. Triple Point.—It has been seen that, under certain conditions of temperature and pressure, a liquid may be in equilibrium with its vapor. On a temperature-pressure diagram these conditions are represented by a curve called the **vapor-tension** curve. It was also seen that, under suitable conditions, a solid may be in equilibrium with its vapor, and that these conditions are represented by the sublimation curve. In like manner, a curve may be drawn showing the relations between the temperatures and pressures at which a solid is in equilibrium with the liquid. If all of these curves are drawn for a substance like water (Fig. 294), they intersect at a point which is called the **triple point.** It indicates the temperature and pressure at which the solid, vapor, and liquid can exist together without one of them gaining in mass at the expense of the others. At this temperature and pressure, the solid, liquid, and vapor are in equilibrium. There is only one such point for a simple substance like water. For water this point is at a pressure of 4.6 mm. of mercury and at a temperature of 0.0076°C.

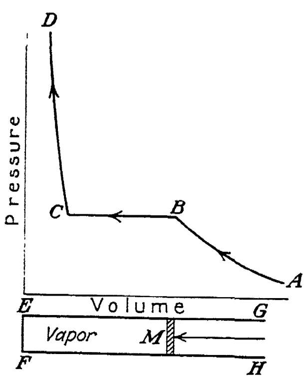


FIG. 295.—Isothermal of water and its vapor.

324. The Critical Point.—If a vapor is enclosed in a cylinder $HFEG$ (Fig. 295) and the piston M forced in while the temperature of the vapor remains constant, the pressure of the vapor at first increases as the volume is decreased. The relation between the pressure and the volume is represented by the curve AB. When the volume and pressure corresponding to the point B are

reached, the pressure of the vapor ceases to increase, and liquefaction begins. As the piston is forced in still farther, the quantity of vapor decreases and the quantity of liquid increases. This continues at constant pressure until the volume corresponding to the point *C* is reached where liquefaction is complete. A further application of pressure to the piston causes a small decrease in the volume of the liquid, and the curve *CD* representing the relation between the pressure and the volume of the liquid becomes very steep. The whole curve *ABCD* is an **isothermal** which shows the relation between the volume and the pressure when the temperature is kept constant.

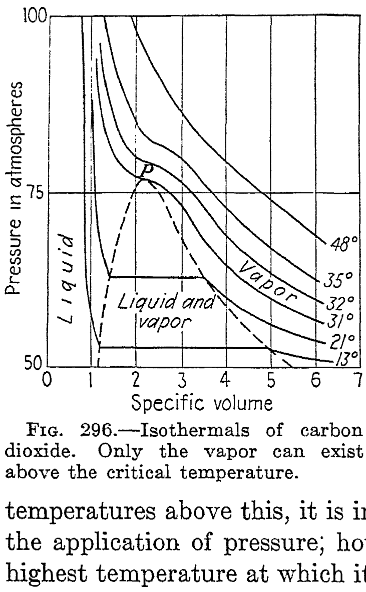


FIG. 296.—Isothermals of carbon dioxide. Only the vapor can exist above the critical temperature.

If this process be carried out at higher and higher temperatures (Fig. 296), the horizontal part of the curve becomes shorter and shorter until a temperature is finally reached at which the horizontal portion disappears and is replaced by a slight bend in the curve. At a temperature just below this, it is possible to liquefy the vapor by the application of pressure alone; but at temperatures above this, it is impossible to liquefy the vapor by the application of pressure; however great that pressure. This highest temperature at which it is possible to liquefy a vapor by the application of pressure is called the **critical temperature.** The point *P* (Fig. 296) at which the horizontal part of the isothermal just disappears is known as the **critical point.** The pressure and specific volume of the vapor at the point *P* are known as the **critical pressure** and **critical volume,** respectively.

The isothermals drawn in Fig. 296 refer to carbon dioxide. Its critical temperature is 31.1°C. and its critical pressure 77 atmospheres. Above this critical temperature, carbon dioxide cannot be liquefied by the application of pressure. If the carbon dioxide vapor is at a temperature higher than 31.1°C., its temperature must be lowered as well as its pressure increased to produce liquefaction.

325. Transition through the Critical Point.—If a heavy-walled glass tube (Fig. 297) closed at both ends is partly filled with liquid carbon dioxide, the remainder of the tube being filled with the saturated vapor of carbon dioxide, for any temperature there is equilibrium between the liquid and its vapor. As the temperature of the tube and its contents is increased, the volume of the liquid carbon dioxide increases and the volume of the saturated vapor decreases. The mass of the liquid, however, decreases while the mass of the saturated vapor increases. The density of the vapor increases and the density of the liquid decreases. The pressure exerted by the vapor also increases. As the temperature is raised more and more, the meniscus marking the boundary between the liquid and the vapor becomes more indistinct and finally disappears. The tube is now filled with a homogeneous substance. At this point, it is impossible to distinguish between the liquid and vapor states. That temperature at which the meniscus disappears is the critical temperature.

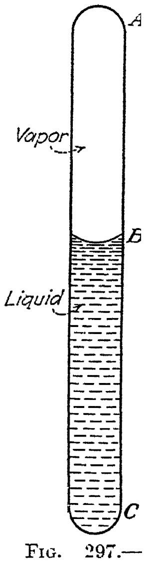


FIG. 297.—The surface of separation disappears at the critical point.

When the temperature is again decreased, a hazy cloud forms in the upper part of the tube, the meniscus or boundary between liquid and vapor reappears, and there are now again two states, liquid and vapor, in the tube. For carbon dioxide the temperature at which the meniscus disappears is 31.1°C., and the pressure exerted by the saturated vapor is 77 atmospheres.

The behavior of the density of the liquid and vapor carbon dioxide is shown in Fig. 298. In this figure, the densities of the vapor and liquid have been plotted as functions of the temperature. It is observed that at the point P, where the temperature is 31.1°C., the two densities have become equal. With lowering temperature, the density of the liquid increases and that of the vapor decreases. Near the critical point P, the change in the density of both the vapor and the liquid is very rapid. This makes the accurate determination of the critical volume difficult.

326. Liquefaction of Gases.—To liquefy a gas, it is necessary to cool it below its critical temperature and then apply to it

sufficient pressure to produce liquefaction. Most of the so-called permanent gases have low critical temperatures and require the application of large pressures. The necessary pressures are obtained by means of pumps which compress the gas to be liquefied in three or four stages so that the final pressure is between 170 and 200 atmospheres.

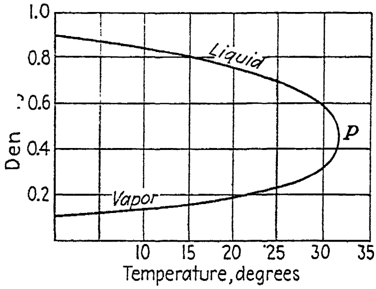


FIG. 298.—Densities near the critical point. Vapor and liquid have the same density at the critical point.

Different methods have been used to produce the necessary lowering of the temperature to bring the gas below its critical temperature. The most common of these is the regenerative process. It is based on the fact that gases cool on expansion (Sec. 297). The gas, having been compressed to about 200 atmospheres by means of the air pump A (Fig. 299), is led through the cooling bath B to the liquefier C. The liquefier, consisting

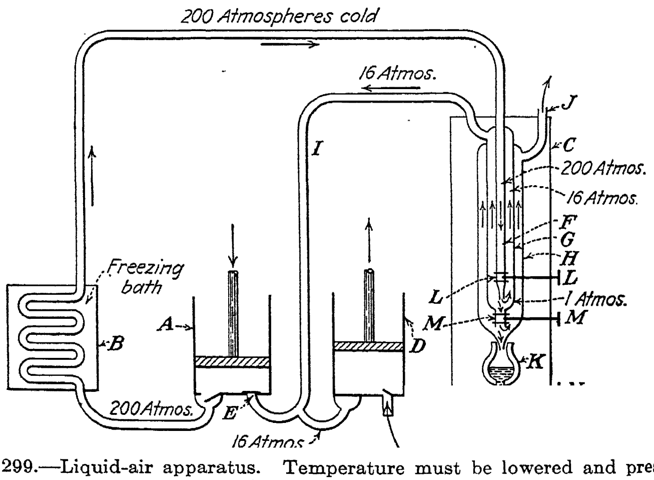


FIG. 299.—Liquid-air apparatus. Temperature must be lowered and pressure raised to liquefy air.

essentially of three concentric copper pipes or tubes, F, G, and H, is usually wound in the form of a compact spiral. The air passes from the central tube F through the valve L, and its pressure is reduced from 200 to 16 atmospheres. This expansion cools the

gas. Part of it flows upward through G and further cools the stream of air flowing downward in F. This air, then, flows back through I to the pump A where it is again compressed. The remainder of the air which has escaped through the valve L flows directly through the valve M, and its pressure decreases from 16 to 1 atmosphere. By this second expansion, its temperature is still further decreased. The air thus passing through the valve M

Name of gas	Freezing point, degrees centigrade	Boiling point, degrees centigrade	Critical temperature, degrees centigrade
Helium...	−272.0	−268.8	−268.0
Hydrogen	−259.0	−252.7	−234.5
Argon....	−188.0	−186.0	−117.4
Nitrogen.	−210.0	−195.7	−146.0
Oxygen...	−219.0	−182.9	−118.0

rises through the tube H and is led from J back to the pump D where it is recompressed. The air rising through the tube H cools the air flowing upward in G. This progressive cooling continues until the cooling of the air in passing the needle valve M causes part of the air to be liquefied. This liquefied air collects in the Dewar flask K, and the remainder is led from J back to the compressor D.

The table on this page shows the freezing point, boiling point, and critical temperature of some of the common gases.

Problems

1. A condenser receives steam at 100°C. and changes it to water at 45°C. The cooling is done by water which enters at 15°C. and leaves at 45°C. How much cooling water is required for each kilogram of steam condensed?

2. How much steam at 150°C. will be needed to turn 120 g. of ice at −15° to water at 40°C.? Specific heat of ice = 0.5, specific heat of steam = 0.48.

3. What will be the final state and temperature of a mixture of 200 g. of ice at 0°C.; 300 g. water at 40°; 50 g. steam at 100°?

4. A steel sphere with a mass of 162.82 g. is placed in an atmosphere of steam at 100°C. and after reaching equilibrium is weighed with the water condensed on it. The observed weight is 164.04 g. What was the original temperature of the sphere?

5. A sterilizer is filled with steam at a gauge pressure of 25 lb. per square inch. The gauge reads 0 at atmospheric pressure. What is the approximate temperature of the sterilizer?

6. A thermometer is placed in live steam at atmospheric pressure when the barometer stands at 740 mm., and the thermometer indicates 98.9°C. What is the error of the thermometer in degrees?

7. Into a copper calorimeter which weighs 200 g. there is passed 18 g. of steam at 150°C. The calorimeter contains 200 g. of water and a certain amount of ice. The ice and water were at 0°C. The final temperature was 35°C. How much ice was in the calorimeter?

8. Ten grams of steam at a temperature of 100°C. were passed into a calorimeter containing a given amount of water. The initial temperature of the water was 12°C., its final temperature 38°C., and the mass of the calorimeter was 250 g. Find the mass of the water in the copper calorimeter.

9. Find the temperature of a sterilizer when filled with steam, if the pressure indicated by the gauge is 30 lb. per square inch. Assume the gauge reads zero when the pressure is 1 atmosphere.

10. One thousand grams of steam superheated to 170°C. was introduced into a calorimeter in which there was a piece of ice weighing 4,500 g. If the copper calorimeter containing the ice weighed 1,500 g. and was at the same temperature as the ice, what was the final temperature of the water in the calorimeter? Specific heat of steam is 0.48 cal. per gram and that of ice 0.50 cal. per gram. Assume the temperature of the ice is 0°C.

CHAPTER XXVIII

ATMOSPHERIC HUMIDITY

327. Water Vapor in the Air.—If a jar filled with water is left open, the water evaporates after a time. The molecules of water escape from the surface and pass off into the air where they move about freely just as the molecules of air do. The molecules of vapor strike each other and also strike molecules of air. In this irregular motion, the molecules of vapor may occasionally strike the surface of the liquid and be held fast. Most of the molecules pass out of the jar and never return. When the temperature of the water is increased, the quantity evaporated in a given time increases. If the wind is blowing, it increases the rate of evaporation by decreasing the opportunity of the molecules to return to the surface of the liquid.

If the water is in a closed vessel, none of the molecules can escape and more of them will strike back into the liquid and be held fast. After a while the number which is held fast will be equal to the number which escapes in the same time. The air above the water in the vessel is then said to be saturated. The number of molecules necessary to saturate the air will depend on the temperature of the liquid, for the number of molecules which can escape from the surface of the liquid in a given time is greater at high than at low temperatures. Hence, the number of molecules of water vapor in air which is saturated is greater at high than at low temperatures. When warm saturated air is cooled, some of the water vapor condenses and reappears as water. The air will still be saturated, but not so much water vapor is necessary to saturate it.

328. Clouds, Fog, Dew, Etc.—When the air above the surface of the earth is saturated and is then cooled, the water vapor in it forms minute drops about particles or ions which are present. As the air is still further cooled, these drops grow in size and fall to the earth as rain. If these drops in falling to the earth pass through a layer of air at a temperature below freezing, they

are frozen and fall to the earth as hail. If the temperature of the air is below freezing when the water vapor condenses from it, crystals are formed which fall to the earth as snow.

When the sun goes down at night and the surface of the earth begins to cool, some objects lose heat more rapidly than others. When warm moist air comes in contact with these objects, some of the moisture from the air is deposited. This moisture appears on the objects in the form of dew. If the air cools below the temperature at which it is saturated, moisture not only collects on objects but forms minute particles in the air. This collection of water particles is called a fog. It does not differ from a cloud except that fogs are near the surface of the earth, and clouds may have any height above the surface.

329. Dew Point.—The temperature to which it is necessary to cool the air before condensation begins is called the dew point. This temperature may be found by partly filling a polished vessel with ether and then inserting in the stopper a thermometer and a tube through which air can be bubbled. The ether is made to evaporate by blowing air through it, and in the process of evaporation it absorbs heat and cools the remaining ether and its containing vessel. By and by, a temperature is reached at which moisture begins to form on the surface of the containing vessel. The temperature at which this deposit begins to form is called the **dew point.** The vessel is polished because it is easier to detect the formation of dew on a polished surface.

330. Relative and Absolute Humidity.—It is often important to know the degree of saturation of the air. On such information depends the likelihood of rain or frost, the proper growth of plants, and proper hygienic living conditions. **The degree of saturation or relative humidity is defined as the ratio obtained by dividing the mass of moisture actually present in the air by the mass of moisture necessary to saturate the air at that temperature.** Since the amount of water vapor is proportional to the pressure which it exerts, the relative humidity is also the ratio between the pressure of the water vapor in the air and its pressure if the air were saturated at that temperature.

Let t = the observed temperature of the atmosphere.
t' = the dew point.
p = the vapor pressure of water at t.
p' = the vapor pressure of water at t'.

Then

$$\text{Relative humidity} = \frac{p'}{p},$$

or

$$\text{Relative humidity} = \frac{\text{mass of water vapor per unit volume of air at observed temperature}}{\text{mass of water vapor per unit volume of air when saturated at same temperature}}.$$

Example.—Find the dew point, when the relative humidity of the air in a room is 40 per cent and the temperature of the air is 25°C.

Let p = the vapor pressure of water at 25° = 23.76 mm. of mercury.
p' = the vapor pressure of water at the dew point.

$$\text{Relative humidity} = \frac{p'}{p} = 0.40.$$

$$p' = 0.40 \times 23.76 = 9.504 \text{ mm. of mercury.}$$

This vapor pressure corresponds to about 10°C., as can be seen by referring to a table giving the vapor pressure of water. At 10°C. the air in the room would be saturated so that 10°C. is the dew point.

The absolute humidity is the mass of water vapor per unit volume in the atmosphere at a given temperature.

331. Measurement of Humidity.—For the measurement of the humidity of the air a number of different types of instruments is used. Some of these depend on the hygroscopic properties of hair or a thin flat strip of some material. This type of instrument depends for its action on the property of a substance to absorb moisture from the air and to change its length on account of this absorption. This change in length is indicated by a hand which moves over a dial. On this dial, the words dry, moist, etc., are recorded, and the position of the hand on the dial is thus made to indicate the humidity of the air. These are very convenient but not very accurate instruments.

The wet- and dry-bulb hygrometer (Fig. 300) is the simplest of the reliable instruments. It consists of two thermometers, one of which gives the true temperature of the air. The other thermometer is covered with a wet cloth and gives a temperature that varies with the rate of evaporation of the water into the air. This wet bulb is generally kept moist by allowing the wick to dip into a cup of water. The drier the air, the greater is the evaporation from the wet bulb and the lower the temperature which is indicated by that thermometer. When the air is saturated, there

is no evaporation from the wet bulb, and both thermometers indicate the same temperature. A chart showing the relation between the relative humidity and the difference between the readings of the two thermometers must be used with this instrument. This chart must be prepared by a calibration of this kind of instrument under the conditions under which it is to be used. In Fig. 301 provision is made for whirling the thermometers.

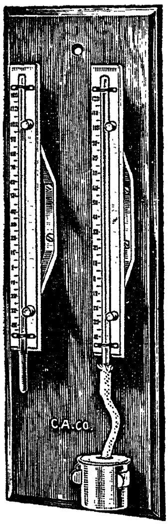


FIG. 300.

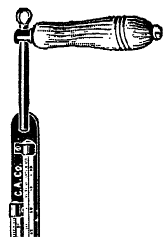


FIG. 301.

FIG. 300.—Wet- and dry-bulb hygrometer. Evaporation from the wet bulb lowers the reading of that thermometer.

FIG. 301.—Psychrometer of U. S. Weather Bureau. The rotation of the thermometer aids evaporation. (*Courtesy of the Chicago Apparatus Company.*)

332. Chemical Hygrometer.—When it is desired to know the **absolute humidity or the mass of water vapor in unit volume of the air,** the chemical hygrometer is used. In this instrument (Fig. 302), a known volume of air is passed through a series of U-tubes which are filled with calcium chloride or some other substance which absorbs water vapor but does not absorb air. These tubes are weighed before and after passing the air through them. The increase in weight is the mass of water vapor absorbed by the substance in the tubes and is, therefore, the mass of water vapor originally in the volume of air which passed through the tubes. The amount of water vapor per unit volume is obtained

by dividing the water vapor absorbed in the tubes by the volume of air passing through the tubes. This result gives the absolute humidity of the air. It is expressed in grams per cubic centimeter or in pounds per cubic foot.

333. Importance of Atmospheric Humidity.—The moisture in the air plays an important part in the health and comfort of everyone. It also has a marked effect on the physical conditions and behavior of many materials, such as wood, wool, and cloth. The lack of moisture often causes woodwork and furniture to become damaged. On the other hand, an excessive amount of moisture is just as serious. The low humidity which is often found in houses in winter is injurious to health. It seems a well-

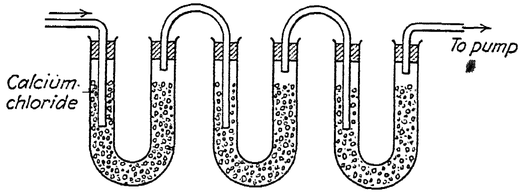


FIG. 302.—Chemical hygrometer. The water vapor is absorbed by the calcium chloride.

established principle that the extremes of heat and cold are felt less and are less injurious where the humidity is low. In many forms of manufacture, it is necessary to control by artificial means the humidity in the factory. Thus, in the manufacture of bread or in the manufacture of cigars, it is desirable that the humidity remain constant and that it be neither too high nor too low.

Problems

1. A chemical hygrometer gains 0.14 g. in weight by having 6 l. of air at 18° passed through it. Calculate the absolute humidity of the air.

2. How great is the pressure of water in a room at 20°C. if the humidity is 25 per cent?

3. Taking the density of water vapor at 10°C. as 9.3 g. per cubic meter, find the amount of water vapor which will be contained in a room which is saturated at 10°C., if the room has a volume of 1,000 cu. m.

4. A room is 8 m. long, 6 m. wide, and 3 m. high. The temperature is 20°C. How much water must be evaporated in the room to raise the relative humidity from 15 to 30 per cent?

5. Air which is saturated at 75°F. rises vertically until its volume is doubled. At the same time, its temperature decreases until it is 38°F. Find the number of grams of water which will be condensed out of each cubic meter of the air, measured at the original temperature and pressure.

6. The relative humidity of an auditorium which has a volume of 600 cu. m. is 20 per cent at the beginning of a concert and 65 per cent at the end of the concert. If the temperature of the room is assumed to remain 24°C. throughout the concert, how many grams of water have been added to the room?

CHAPTER XXIX

TRANSFER OF HEAT

334. Convection.—The simplest way in which heat may be transferred from one place to another is by the motion of the heated substance. Such a movement of heat is known as **convection** (Figs. 303 and 304). The movement of the heated substance depends in this case on the change in density which takes place when the substance is heated. For example, when a gas or a liquid is heated, it expands and becomes lighter than the cold gas or liquid. When water is heated in a vessel on a stove, the liquid in the bottom of the vessel is hotter than that on the top. The density at

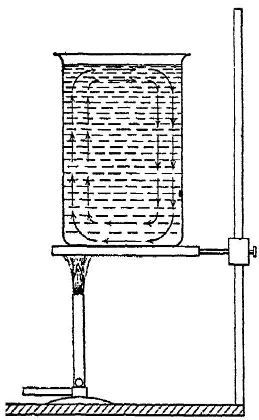


FIG. 303.

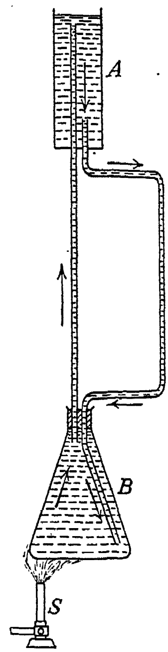


FIG. 304.

FIG. 303.—Convection currents in a beaker. The less dense liquid rises.
FIG. 304.—Convection currents in connected vessels caused by the difference in density of the liquid at different temperatures.

the bottom is less than that near the top. The cool liquid sinks down and forces the warmer liquid to a higher level. The currents of water thus set up in the liquid are known as convection currents. In case a block of ice is placed on the top of a vessel of warm water, the convection currents are reversed from what they were in the preceding case.

Consider the convection currents in a lake as winter approaches. The layers of water near the surface are first cooled and these layers, being heavier than the lower layers, sink. Convection currents are set up which persist until all the water in the lake has been cooled to 4°C., the temperature at which the density of the water is greatest. A further cooling of the layers of water at the surface will make the water at the surface lighter than that at the bottom. Since in this case the lighter water is already at the surface, no convection currents will be set up. The colder water thus tends to remain at the surface. The deeper layers of liquid are thus protected from further cooling. This fact keeps the water some distance below the surface from becoming cold enough to destroy all forms of animal life in winter.

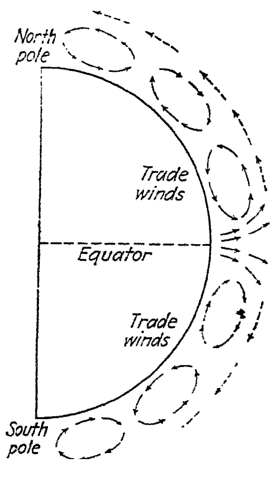


Fig. 305.—Trade winds are caused by convection currents.

335. Draft in a Stove.—The draft in a stove or a chimney is produced by convection. The air in the neighborhood of the fire is heated and expands. It thus becomes lighter than the air higher up in the chimney or stove. The heavier air sinks and forces the lighter air up the chimney. Since there is a greater difference of pressure between the top and the bottom of a chimney when it is high than when it is low, tall chimneys draw better than low ones.

336. Trade Winds (Fig. 305).—The trade winds which are found in regions a few degrees north and south of the equator are convection currents in the atmosphere. These winds blow in the same direction for a long time. They are produced by the considerable heating of the atmosphere in the neighborhood of the equator. This heated atmosphere rises and the colder atmosphere from the north and south of the equator flows in to take its place. If the earth did not rotate on its axis and if it were perfectly smooth, these winds would blow from the north toward the equator in the northern hemisphere and from the south toward the equator in the southern hemisphere. On account of the rotation of the earth, these currents are somewhat deflected.

337. The Hot-water Tank.—The hot-water tank (Fig. 306) is heated by convection currents in the water. The water in the portion of the heater which is within the stove becomes heated and rises to the top of the tank. Cold water comes in from the water mains and replaces the water which has risen after being heated. A continuous flow of water is kept up by this unequal heating. The direction of this flow is indicated by the arrows in Fig. 306.

338. The Hot-water Furnace.—One of the best methods of heating a house is by means of a hot-water heating plant (Fig. 307). Such a plant transfers heat from the furnace to the radiators by means of convection. The plant consists of a number of pipes holding water. These pipes surround the fire box and the water in them is heated and rises from the top of the furnace through a supply pipe to the radiator. The water loses some of its heat, cools, and thus contracts. The increase in density arising from this cooling causes the water to be heavier than the incoming water. It

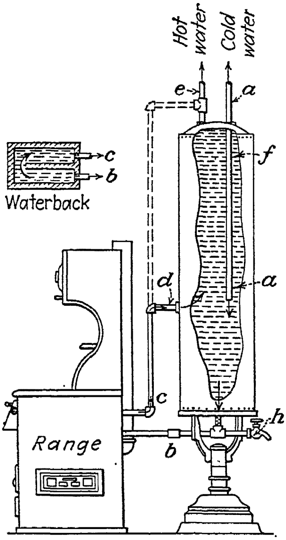


FIG. 306.—A range boiler for heating water. The circulation of the water is caused by convection currents.

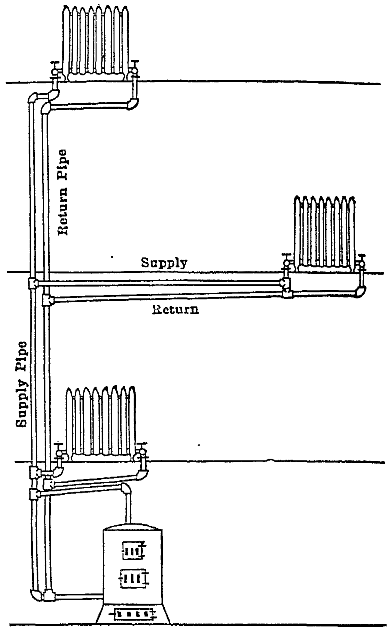


FIG. 307.—Hot-water furnace. The rising hot water is replaced by water cooled in the radiators.

therefore sinks to the bottom of the furnace through the return pipe where it is again heated and the process is repeated. In order to take care of the excess expansion arising out of the expansion of the water in heating, an expansion tank (Fig. 307) is provided. This tank is usually located in the attic at a height greater than the height of the highest radiator.

339. Ventilating System.—In schools and homes, as well as elsewhere, a constant supply of fresh air is important for the health of those occupying the rooms. A common method of admitting cold air to rooms to produce ventilation depends entirely on convection currents in the air. The hot air enters the room through the radiator in the floor. The cold air enters through an opening in the side of the house. The cold air being heavier

than the warm air sinks to the floor, while the warm air rises to the ceiling. This provides a continuous circulation of air in the room.

340. Radiator of a Gasoline Engine.—A gasoline engine, when in operation, becomes very hot. The most common method of cooling the engine is by means of circulating water. The cylinder is surrounded by a hollow chamber filled with water (Fig. 308). In this water jacket the water is heated by contact with the hot cylinder. After being thus heated, the water rises through a pipe at the top of the cylinder and is replaced by cold water which enters through a pipe at the bottom of the water jacket. The hot water, after being cooled in the radiator, again passes around the circuit and serves again to cool the engine.

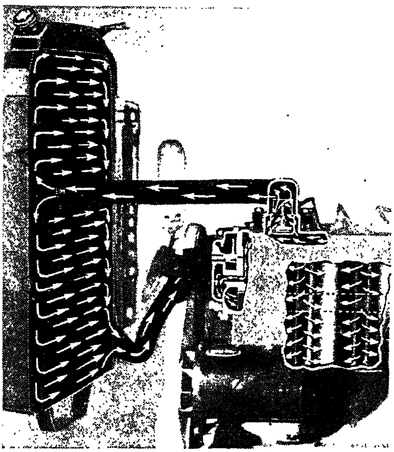


Fig. 308.—Circulation of water in radiator of an automobile by natural and forced convection.

341. Conduction.—When a metal rod is held in the fire, the heat travels along the rod and after a time the rod becomes too hot to hold. If a rod of wood or glass is held in the flame, the heat travels along it much less readily. In this process the heat which consists of the vibrations of the molecules of which the body is made is handed on from molecule to molecule. The layer of molecules in contact with the fire is heated first and thus made to vibrate more rapidly. This layer hands the motion on to the adjacent layer, because each layer is bound to the adjacent layer by certain cohesive forces. It is thus impossible for the molecules in one layer to vibrate without setting the molecules in the neighboring layers in vibration. As this process goes on, the entire medium is heated after a time. When heat, as in this case, is transferred from one part of the body to another without any

progressive motion of the parts of the substance, the heat is said to be transferred by **conduction.**

Steam pipes are covered with non-conducting substances (Fig. 309) to prevent loss of heat.

Liquids are poor conductors of heat. This fact can be shown in the following way. Nearly fill a test tube with water. Into the water drop a piece of ice. Hold the ice at the bottom of the tube with a weight of some kind. By holding the top of the tube in the flame of a Bunsen burner the liquid on the top of the tube may be made to boil without melting the ice in the bottom of the tube.

Let a spherical bulb filled with air be inserted in a stopper and the bulb then covered with liquid. The end of the tube to which the spherical bulb is attached is immersed in a beaker of water. If ether is poured on the top of the water and lighted, it will burn, but the air in the bulb will

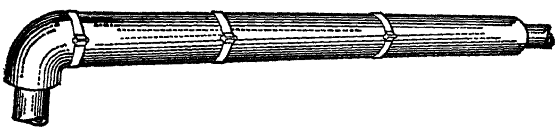


FIG. 309.—Covering of pipes to prevent loss of heat. The thermal conductivity of asbestos is small.

not expand sufficiently to force any air out of the bulb through the water in the beaker. This experiment shows that a liquid like water is a poor conductor.

342. Measurement of Thermal Conductivity.—In order to study more carefully the law according to which heat is conducted, consider the case in which heat flows along a uniform bar whose ends are kept at uniform temperatures so that the fall of temperature in the bar remains always the same. The quantity of heat Q which will be conducted down this bar depends on the following relationships:

1. **The quantity of heat passing through the conductor in a given time is in direct proportion to the area of the conductor.** In this case, just as in the case of a water pipe, the larger the area through which the flow takes place, the greater the quantity transferred.

2. **The quantity of heat is also directly proportional to the difference in temperature between the ends of the bar.** This difference in temperature may be compared to the difference in pressure which forces the water through the pipe. No flow of water occurs unless there is a difference in pressure, and the

greater the difference in pressure the greater the flow. Likewise, in conduction there is no flow of heat unless there is a difference in temperature between the ends of the bar; and the greater this difference, the greater is the flow of heat.

3. **The quantity of heat decreases as the length of the path or the distance between the ends of the bar is increased.** This is also analogous to the flow of water in a pipe in which it is found that the quantity of water flowing is inversely proportional to the length of the pipe under a given set of conditions.

4. **The quantity of heat is directly proportional to the time of flow.**

These facts may be expressed compactly in the following equation:

$$Q = \frac{kA(t' - t)T}{d},$$

where Q = the quantity of heat flowing through the bar.
A = the area of cross section of the bar.
d = the distance between the ends of the bar.
T = the time during which heat flows.
t' = the temperature of the hot end of the bar.
t = the temperature of the cold end of the bar.
k = a constant called the coefficient of thermal conductivity. (Appendix D-12.)

The physical meaning of the coefficient of thermal conductivity is seen by considering a special case in which it is assumed that the area of cross section of the bar = 1 sq. cm.; the length of the bar = 1 cm.; the difference in temperature between the ends of the bar = 1°C.; and the time = 1 sec. In this case,

$$Q = k.$$

Hence, k, the coefficient of thermal conductivity, may be defined as **the quantity of heat which will flow through unit cross section of a bar in 1 sec. when the difference of temperature between the ends of the bar is 1°C. and the bar is 1 cm. long.**

In the metric system,

$$\left.\begin{array}{c}\text{Coefficient of}\\ \text{thermal conductivity}\end{array}\right\} = \frac{\text{calories per second} \times \text{thickness in centimeters}}{\text{area of surface in square centimeters} \times \text{temperature difference in degrees centigrade}}.$$

In English system of units,

$$\left.\begin{array}{r}\text{Coefficient of}\\ \text{thermal conductivity}\end{array}\right\} = \frac{\text{B.t.u. per hour} \times \text{thickness in inches}}{\text{area of surface in square feet} \times \text{difference in temperature in degrees Fahrenheit}}.$$

An apparatus for measuring the coefficient of thermal conductivity is represented in Fig. 310. It consists of the copper rod AB, the coefficient of thermal conductivity of which is to be determined. This bar should be

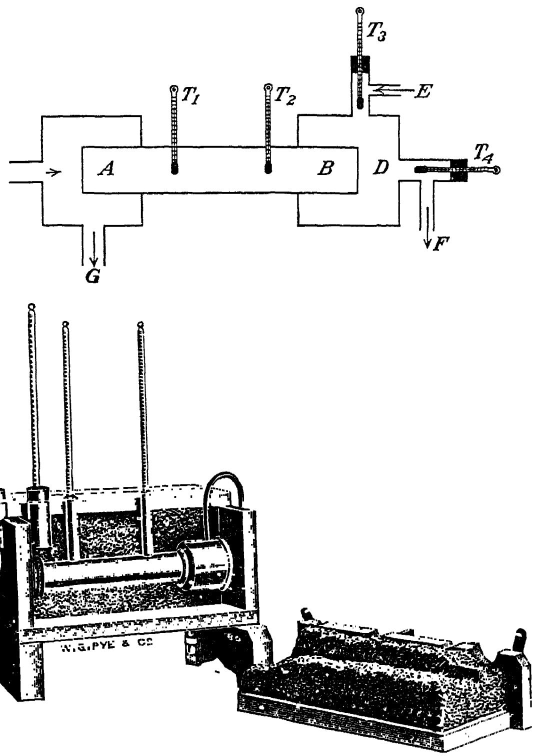


FIG. 310.—Method of measuring thermal conductivity. The heat flowing down the copper rod is absorbed by circulating water.

about 30 cm. long and have a diameter of about 3 cm. On one end is soldered a copper box G through which steam may be passed to make the temperature of that end of the rod about 100°C. Over the other end of the rod is soldered a second copper box through which water is made to circulate from some supply of water at constant pressure. The temperature of this water as it enters this box is found by means of the thermometer T_3, and its temperature as it leaves the box is found from the reading of the

thermometer T_4. The difference in temperature between the ingoing and the outgoing water multiplied by the weight of water gives the quantity of heat which has flowed down the rod. The boxes and the rod are carefully lagged with felt to prevent losses of heat. Two small holes drilled at right angles to the axis of the rod contain thermometers by which the fall of temperature in the rod is measured. From the distance between these holes and the readings of the thermometers in them, the fall in temperature per centimeter is at once obtained.

Let W = the weight of water delivered in T sec.
T_3 = the temperature of ingoing water.
T_4 = the temperature of outgoing water.
T_1 = the higher temperature measured on the rod.
T_2 = the lower temperature measured on the rod.
d = the distance between the thermometers inserted in the holes in the rod.

$$Q = W(T_4 - T_3) = k\frac{T_1 - T_2}{d}AT.$$

Example.—In measuring the thermal conductivity of a rod, the following data were obtained. Find the coefficient of thermal conductivity.

Temperature of ingoing water = 20.0°C.
Temperature of outgoing water = 30.0°C.
Higher temperature on rod = T_1 = 80.0°C.
Lower temperature on rod = T_2 = 60.0°C.
Area of rod = 20 sq. cm.
Distance between thermometers = 10.0 cm.
Weight of water flowing through box = 650 g.
Time of flow = 3 min. = 180 sec.
Heat gained by water = mass of water × temp. change × sp. heat
= (650 × 10) × 1 = 6,500 cal.

$$\text{Temperature fall per centimeter} = \frac{80 - 60}{10} = 2.0\text{°C. per centimeter.}$$

$$Q = kA\frac{T_1 - T_2}{d}T.$$

$$6{,}500 = k \times 20\frac{20}{10} \times 180 = k \times 20 \times 2 \times 180.$$

$$k = \frac{6{,}500}{20 \times 2 \times 180} = \frac{6{,}500}{7{,}200} = 0.9$$ cal. per square centimeter per second when temperature difference is 1°C. per centimeter.

Problems

1. Find the coefficient of thermal conductivity for asbestos paper if it is found that 116 cal. flow per minute through a slab 4 mm. thick and 20 sq. cm. in area, when the temperature of one face is maintained at 100°C. and that of the other 36°C.

2. A refrigerator has in it a window with an area of 1.5 sq. m. If the window is 1.2 cm. thick and the inside temperature is 8°C., how many calories per day are lost when the outside temperature is 30°C.?

3. How much water would be evaporated per hour per square foot by the heat which flows through a boiler plate which is made of iron ¼ in. thick, when there is a difference of temperature of 175°F. between the faces of the plate? Coefficient of thermal conductivity of iron is 160 B.t.u.

4. Heat sufficient to evaporate 1.8 kg. of boiling water per hour passes through the aluminum bottom of a pan, 2 mm. thick and 250 sq. cm. in area, with a certain flame under the pan. What is the temperature of the bottom of the pan on the side next to the flame?

5. A boiler with a copper bottom which is 1.5 mm. thick rests on a hot stove. The area of the bottom of the boiler is 1,500 sq. cm. The water inside the boiler is at a temperature 100°C., and 0.75 kg. is evaporated every 25 min. Find the temperature of the lower surface of the copper bottom, that is, the surface in contact with the stove. Thermal conductivity of copper is 0.91 cal. per centimeter per second.

6. An iron bar has a length of 60 cm. and a cross section of 5 sq. cm. One end is kept in steam at 100°C. and the other in ice at 0°C. Neglecting losses due to radiation, find the number of grams of ice melted in 15 min.

7. A glass window has an area of 4 sq. m. and is 0.4 cm. in thickness. The outer surface is at 0°C. and the inner surface at 22°C. How much heat will flow through the window each hour?

CHAPTER XXX

RADIATION

343. Transfer of Heat as Radiation.—A person sitting in front of a stove receives heat from the stove although the air in the room is cold. In like manner, sunlight falling on a body will warm it above the temperature of the surrounding air. This method of transfer of heat is distinguished from convection and conduction by the fact that the medium through which the transfer occurs is not heated. Thus, the earth receives great quantities of heat from the sun although the space which separates the sun from the earth is very cold. The fact that the earth receives such quantities of heat from the sun shows that this heat can pass through the empty space between the sun and the atmosphere which surrounds the earth. It is difficult to see how energy can be transferred from one object to another unless some medium connects these objects. For this reason it is often helpful to assume that there is a medium known as the **ether** which fills all space, even those portions of space which are occupied by ordinary matter. It is through this medium[1] that the energy travels from the sun to the earth. This energy is assumed to travel as transverse waves in the ether. These waves travel from the sun to the earth. When they reach the earth, they cause the molecules of the body on which they fall to vibrate more rapidly, and the body is thus heated. This movement of heat from one place to another by means of waves in the ether is known as **transfer of heat by radiation.** These waves are similar to light waves and travel with the velocity of light. They do not heat the medium through which they pass, but they heat any substance on which they fall and by which they are absorbed. The transfer of heat by radiation is, therefore, a twofold process: **the conversion of the heat energy of the hot body into a wave motion of the ether, and a**

[1] In modern physics, the hypothesis of an ether has been discarded but it is still a convenient way to describe many phenomena associated with radiation.

reconversion of the wave motion into heat by the body on which it falls.

344. Exchange of Radiations.—All bodies whether cold or hot will radiate some heat. If the bodies are exactly alike in every particular, the hot bodies will radiate more heat than the colder ones. Whatever the temperature and whatever the surroundings, a certain amount of heat will be radiated by each body. At the same time, each body will be receiving heat from the neighboring bodies. Some of this heat will be absorbed and thus go to raise the temperature of the body absorbing it. Whether a body will decrease or increase in temperature under these conditions will be determined by the amount of heat which it radiates in comparison with the amount which it absorbs. If the amount radiated exceeds the amount absorbed, the temperature will decrease. If, on the other hand, the amount absorbed exceeds the amount radiated, the temperature will increase. If the heat radiated is equal to the amount absorbed, the temperature will remain unchanged.

345. Character of the Radiating Surface.—The amount of heat radiated depends not only on the temperature of the body but also on the character and area of the radiating surface. Some surfaces are good radiators, while others are poor radiators. Generally surfaces which are rough radiate best, while polished surfaces are poor radiators. For example, lampblack is an excellent radiating surface as compared to other surfaces. This surface is such a good radiator that it is customary to regard it as a perfect radiator and use it as a standard of comparison for other radiating surfaces. The amount of heat radiated from a surface increases with the area of the radiating surface. For this reason milk coolers are sometimes built of a series of hollow disks.

346. Absorption of Radiation.—The facility with which a surface radiates heat depends on the character of the surface. The same is true of the facility with which it absorbs heat. There is an intimate relation between the rate at which a surface radiates heat, and the rate at which the same surface under the same conditions absorbs heat. **This fact is stated by saying that a good radiator of heat is also a good absorber of heat.** In other words, surfaces which radiate slowly will also absorb slowly. A surface covered with lampblack is found to be an

excellent radiator. If this same surface is exposed to radiations, they will be readily absorbed by it.

347. Absorption of Radiations by Soils of Different Colors.—If sunshine falls on a number of bodies of different colors, the amount which is absorbed by these bodies is determined largely by the color of the bodies. This fact has an important application in the temperature of soils. The amount of heat absorbed by a soil exposed to the solar radiation depends much on the color of the soil.

If the same soil is mixed in one case with lampblack and in the other case with powdered chalk so that the one soil is black and the other is as white as possible, the black soil absorbs more radiation than the white one and for that reason becomes 10 or 12°F. hotter. On the other hand, at sunset when radiation is no longer being received by the soils, the black soil will ordinarily lose its energy more rapidly than the white one and on this account cool more rapidly; an illustration of the fact that good absorbers of energy are also good radiators of energy.

348. Absorption of Radiation by the Air.—The radiations emitted by the warm earth are almost completely absorbed by the moist atmosphere above the earth. Hence, a moist atmosphere above a portion of the surface of the earth prevents it from cooling by radiation. For this reason frosts occur only when the air is clear. Since the humidity of the air is greater in the neighborhood of a body of water, there is less danger of frost in the regions which lie near bodies of water.

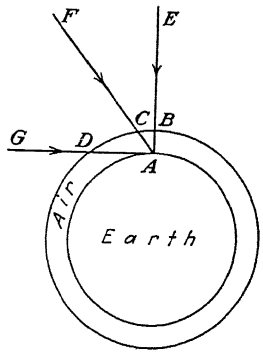


FIG. 311.—Absorption of radiation by the atmosphere. The greater the thickness of the atmosphere, the greater the amount of absorption.

One reason for the diminishing heating effect of the sun's rays as the sun approaches the horizon is the fact that at the horizon the rays must travel through a greater layer of the earth's atmosphere. There occurs in the atmosphere a considerable absorption of radiation, and the thicker the layer of air through which these rays travel the greater is the absorption. In Fig. 311 is shown a representation of the air surrounding the earth. From this figure it is seen that the nearer the sun is to the horizon, the greater is the thickness of the layer of air through which the rays pass and the greater the absorption.

349. Reflection of Radiations.—The wave motion which constitutes radiation is reflected by certain surfaces in the same way in which light is reflected. The reflection of radiations from two spherical mirrors is indicated in Fig. 312. The amount of radiation which is reflected from a surface depends on the character of the surface. Polished surfaces reflect much of the radiation while rough or blackened surfaces reflect little.

Surfaces which are good absorbers would, of course, be poor reflectors. In order to prevent a body from losing heat by radiation and to prevent it from gaining heat by absorption, it is often silvered. The surface is thus made into a mirror which reflects most of the radiation which falls on it.

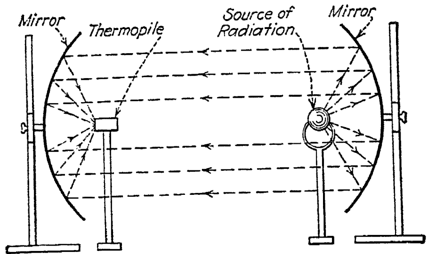


FIG. 312.—Reflection of radiation by mirrors. The radiation is focused on the thermopile.

350. Dewar Flask or Thermos Bottle.—A useful device for keeping things either hot or cold is the vacuum or thermos bottle (Figs. 313 and 314). This bottle is made up of one silvered glass bottle inside a second glass bottle which is also silvered. The space between the two bottles is free from air or any other gas. Since the space between the glass walls of the bottle

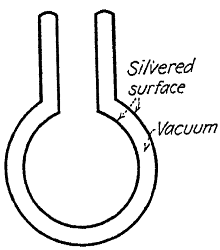


FIG. 313.

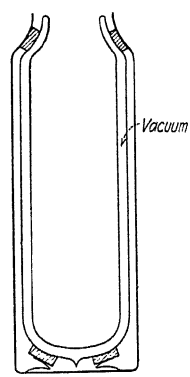


FIG. 314.

FIG. 313.—Dewar flask. Conduction and convection are reduced by evacuation. Radiation is reduced by silvering the surfaces.
FIG. 314.—Thermos bottle.

is a vacuum, there is little opportunity for heat to flow across this space. The residual air in this space is a very poor conductor of heat. When the surfaces are silvered, they become good reflectors of whatever radiation falls on them. This very much decreases the amount of energy which might enter the flask by radiation and also the amount of energy which the flask might

lose by radiation. If hot bodies are placed in such a bottle, there is little chance for the heat to escape from them by either conduction, radiation, or convection. Hence, the substances in the bottle remain hot for a long time. On the other hand, if a cold body is placed in the bottle, there is little chance for heat to flow into it from the outside, and the substance inside the bottle will remain cool for a long time.

351. Solar Constant.—The amount of heat received per square centimeter per minute on a surface which is perpendicular to the rays from the sun is called the solar constant. Some of this radiation is absorbed by the earth's atmosphere, so that the amount received on a surface above the earth's atmosphere is greater than the amount received at the surface of the earth. It has been found that about one-third of the total amount of radiation received is absorbed by the atmosphere of the earth. The latest determinations of the quantity received per square centimeter on a surface above the atmosphere is about 2 cal. per minute when the surface is perpendicular to the direction of the rays.

352. Measuring the Solar Constant.—A thermometer with a blackened bulb indicates a higher temperature when exposed to the rays from the sun than the temperature indicated by a thermometer with a polished surface under similar conditions. The radiation falling on the blackened surface is absorbed to a greater extent than that falling on the polished surface. Hence, the temperature of the mercury in the bulb with the blackened surface is higher than it is in the polished bulb. In like manner, if a vessel with blackened surface is filled with water and is insulated in such a way that heat cannot escape by radiation, the temperature of the water rises when the sun's rays are allowed to fall on the surface of the vessel. By observing the rate at which the temperature of the water rises, it is possible to calculate the rate at which heat is received from the sun. If all the energy received from the rays from the sun were absorbed by the water, and none of it escaped from the vessel, this method would be a very simple one for determining the solar constant. As a matter of fact, as soon as the vessel becomes warmer than its surroundings, it begins to lose heat by radiation. The rate of loss of heat increases as the temperature of the water rises. After a time, the temperature of the water in the vessel attains a steady

temperature at which the loss of heat is just equal to the gain of heat from the sun. The temperature of the water above its surroundings gives a measure of the rate at which heat is being received from the sun. Such a method is a very inaccurate one for determining the rate at which energy is received from the sun.

An apparatus devised by Langley yields much more accurate results. It consists of a blackened strip of platinum foil, A (Fig. 315), placed in front of a slit through which radiation from the sun is allowed to pass in such a way that it falls directly on the blackened platinum foil. A second strip of platinum, B, precisely like the first one, is placed near the first strip in such a way that the sunlight does not fall on it. The strips are made the arms of a Wheatstone bridge with a galvanometer and a battery connected in the usual manner. As the temperature of the blackened strip of platinum increases, its resist-

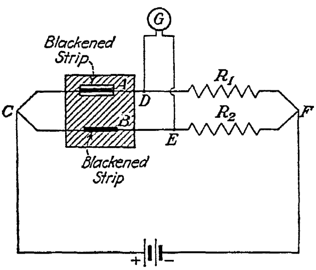


FIG. 315.—Bolometer for measuring intensity of radiation. The radiation is absorbed by the blackened strip.

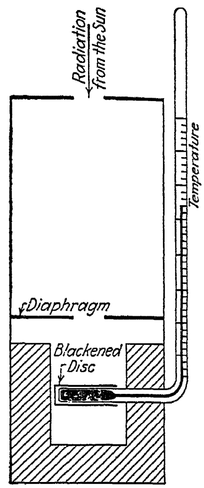


FIG. 316.—Abbot's pyroheliometer for measuring the intensity of solar radiation.

ance also increases, and this fact is indicated by the deflection of the galvanometer. By observing the change of resistance of the blackened platinum strip, the energy received from the sun by each square centimeter of the blackened surface can be calculated. The apparatus is a very delicate electrical thermometer capable of indicating very small changes of temperature.

Dr. Abbot of the Smithsonian Institution has devised a pyroheliometer (Fig. 316) which still further increases the accuracy of radiation measurements. It measures the radiation by noting the rise of the temperature of a blackened disk in contact with a thermometer.

Figure 317 shows the way in which the solar radiation changes from hour to hour for three typical days of the year. These curves show that the amount of solar radiation reaching the northern hemisphere of the earth in December is small in comparison with the amount reaching it in June.

353. Influence of Slope on Temperature of Soils.—If the rays from the sun strike the surface of the earth at an angle other than 90 deg., the energy

in a given bundle of rays is spread out over a larger surface, so that the amount of energy received per unit of area is less than it would have been if the rays had been perpendicular to the surface of the earth. This spreading of energy in the rays of the sun over a larger area and the consequent decrease in the amount received per unit area are indicated in Fig. 318. This fact is of much consequence in determining the temperature of soils which are differently exposed to the sun's rays. Consider the northern

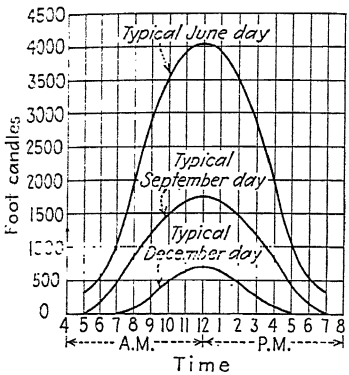


FIG. 317.—Variations in solar radiations with the time of day. Solar constant also varies with season of year, latitude, and altitude. For definition of foot-candle see page 547.

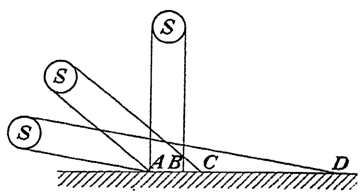


FIG. 318.—Position of sun and intensity of radiation. Intensity decreases as sun moves toward the horizon.

and the southern exposure shown in Fig. 319. If a bundle of rays from the sun be represented in width by the line *DG*, and this bundle be then divided into two equal parts *DE* and *EG*, it is seen that the southern slope *BH* receives more energy than the equal northern slope *HC*. For this reason, southern exposures warm up earlier in the spring than do northern exposures.

354. Sources of Solar Radiation.—It is interesting to inquire how a body like the sun can radiate heat at such a tremendous rate for millions and millions of years without an appreciable decrease in its temperature. A number of hypotheses have been advanced to account for this remarkable phenomenon. None of them are thoroughly satisfactory.

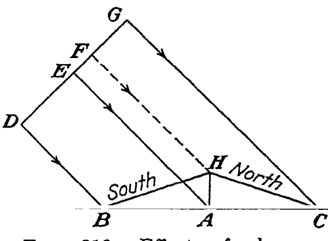


FIG. 319.—Effect of slope on intensity of radiation. The greater the slope, the less the intensity.

1. *Meteoric Hypothesis.*—It is well known that streams of meteors fall into the sun. The number of these meteors is very great and they range in size from grains of sand to very large masses. An estimate of the number which falls to the earth can be made. The number which falls to the sun must be many times larger than the number which falls to the earth. The kinetic energy associated with each of these meteors would be converted into heat when the meteor was stopped by the sun. After making the most liberal assumptions about the number, the velocity, and the mass of these meteors, it is impossible to account for the energy radiated by the sun on this hypothesis.

2. *Contraction Hypothesis.*—If heat is withdrawn from a solid, liquid, or gas under ordinary conditions, it decreases in volume. If the total amount of heat in the sun is diminishing, the volume of the sun would be growing smaller and smaller. Because of this contraction, the energy which the sun possesses because of the gravitational forces between its different parts would decrease. The gravitational energy which disappears might be released as heat and thus account for the heat liberated as radiation by the sun. Again, the amount of energy which could be liberated by this process is insufficient to account for the energy radiated by the sun.

3. *Radioactive Hypothesis.*—There are certain chemical elements, like radium, which are in the process of spontaneous disintegration. In this process of disintegration, two particles, one positively charged and the other negatively charged, are emitted. The positively charged particle has nearly the mass of the helium atom and the negatively charged particle the mass of an electron. Each of these particles moves with very great speed and they have associated with them a definite amount of kinetic energy. When these particles are stopped, their kinetic energies are transformed into heat, so that the temperature of a radioactive substance remains above the temperature of its surroundings. If it is assumed that there is a large supply of some radioactive substance like radium in the sun, and that this radium is in a process of disintegration as it is on the earth, there would be a definite amount of heat supplied to the sun by this process, and this supply of heat would tend to keep the temperature of the sun from decreasing although the sun continually radiates heat. Rutherford has, however, shown that if the entire mass of the sun were originally made of uranium and its derivatives, like radium, the generation of heat from this source would not be sufficient to supply the heat radiated by the sun.

4. *Transformation of Matter into Energy.*—The atomic weight of hydrogen is 1.008 and the atomic weight of helium is 4.00. If four atoms of hydrogen could be made to combine to form one atom of helium, there would be a decrease in mass of **0.032.** Theoretical considerations show that this change of mass would be radiated as energy. If, therefore, in the sun, hydrogen is being transformed into helium with a decrease in mass, there would be a liberation of energy which would tend to keep up the temperature of the sun. The spectroscope shows that both hydrogen and helium are present in the sun, and it is a plausible assumption that a transformation of hydrogen into helium goes on in the sun. Calculations show that, on reasonable assumptions, the energy released in this way would account for the energy radiated by the sun. At all events, this is the most plausible hypothesis at the present time to account for the fact that the sun continually radiates heat without appreciably decreasing in temperature.

Problems

1. At a time when the solar constant is 2 cal. per minute per square centimeter, find the area which receives energy at the rate of 1 hp.

2. Every square centimeter of the earth's surface receives 1.93 cal. per minute. How many horsepower are received per square yard of the earth's surface?

CHAPTER XXXI

HEAT AND WORK

355. Nature of Heat.—It is now believed that heat is the energy which a body possesses by virtue of the fact that its molecules are in motion. In solids, these molecules do not much alter their relative positions but vibrate back and forth through positions of equilibrium. The distances through which these molecules vibrate are increased by the addition of heat, and the energy of the molecules is also increased. There are ways by which these molecules can be made to vibrate more rapidly and to increase their energy. If two sticks are rubbed together vigorously, they become hot. Drills or augers become too hot to hold in the hand when used to bore hard wood or metal. A grindstone is kept wet with water to keep the tools which are being ground from becoming too hot. In all such cases, work is done on the body and heat is developed in consequence of this work. From such experiments, it is possible to conclude that work can be transformed into heat.

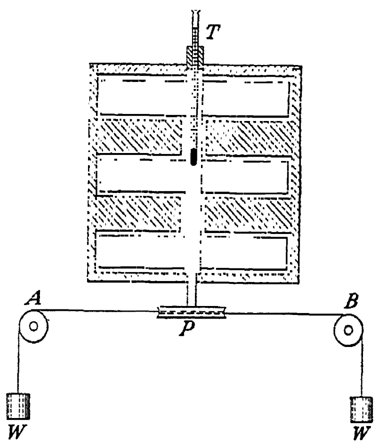


FIG. 320.—Apparatus for measuring the mechanical equivalent of heat. Work done = heat produced, measured in work units.

356. Mechanical Equivalent of Heat.—Since heat is a form of energy, it may be converted into other forms of energy, and other forms of energy may be converted into heat. According to the law of conservation of energy, it is not possible to create or destroy energy. Hence, a definite quantity of work is required to produce a given amount of heat. Experiments have shown that this amount of work is the same under all conditions.

The following experiment first performed by Joule shows one method of determining the amount of mechanical work necessary

to generate one unit of heat. The apparatus which was used in this experiment is shown in Fig. 320. It consists of a calorimeter in which revolves a series of paddle wheels. The calorimeter was fitted with baffle plates having spaces cut in them to allow the paddle wheels to pass. By this means, the water in the calorimeter was thoroughly churned. The paddle wheels were driven by two weights which were hung over two pulleys. When the weights were allowed to descend through a given distance, the paddle wheels revolved and did work on the water in the calorimeter. This work caused the water in the calorimeter to increase in temperature. By observing the weights and the distance through which they descend, the work done on the water is easily calculated from the relation,

$$\text{Work} = \text{force} \times \text{distance} = \text{weight in pounds} \times \text{feet}.$$

By measuring the rise in the temperature of the water in the calorimeter, the heat generated is calculated from this rise in temperature together with the mass of water and the thermal capacity of the calorimeter.

Heat generated = (mass of water + water equivalent of calorimeter) × change of temperature × specific heat of water.

Let M = the mass of water in pounds.
m = the mass of calorimeter.
s = the specific heat of material out of which the calorimeter is made.
t = the initial temperature of the water.
T = the final temperature.
h = the distance through which weights descend.
$2W$ = the sum of weights driving the paddle wheels.

$$\text{Work done on paddle wheels} = 2W \times h \text{ ft.-lb.}$$
$$\text{Heat generated} = M(T - t) \times 1 + ms(T - t) \text{ in B.t.u.}$$
$$\text{Work to generate 1 B.t.u.} = \frac{2Wh}{M(T - t) \times 1 + ms(T - t)} \text{ ft.-lb.}$$

Example.—In an experiment on the determination of the amount of work necessary to generate 1 B.t.u. the following data were taken. Find the number of foot-pounds to generate 1 B.t.u. Neglect heat given to calorimeter.

Weight hanging over each pulley = W = 100 lb.
Distance each weight descended = 20 ft.

Initial temperature of water = 65°F.
Final temperature of water = 75°F.
Mass of water = 0.52 lb.
Work done = $2 \times 100 \times 20$ = 4,000 ft.-lb.
Heat generated = $0.52(75 - 65) \times 1$ = 5.2 B.t.u.

$$\text{Work per B.t.u.} = \frac{4{,}000 \text{ ft.-lb.}}{5.2} = 770 \text{ ft.-lb.}$$

Mechanical equivalent of heat is defined as the number of units of work necessary to generate one unit of heat. In the English system, the mechanical equivalent of heat is the number of foot-pounds necessary to generate 1 B.t.u. In the metric system, it is the number of gram-centimeters or kilogram-meters or ergs required to develop 1 cal. To generate 1 B.t.u., according to the latest determinations, requires 778 ft.-lb., and to generate 1 cal. requires 4.26×10^4 g.-cm. or 4.18×10^7 ergs.

Example.—How much heat is generated each second by an electric mixer which makes 15 revolutions per second and generates a torque equal to 3×10^7 dyne-cm.?

Work per second in ergs = torque in dyne-centimeters × angular velocity in radians per second

$$\text{Heat per second in calories} = \frac{\text{work per second in ergs}}{4.2 \times 10^7}$$
$$= \frac{3 \times 10^7 \text{ dyne-centimeters} \times 2\pi \times 15}{4.2 \times 10^7}$$
$$= 67 \text{ cal.}$$

357. Transformation of Heat into Work.—The heat engines which play such a large part in modern life depend on the reverse of the operation described in Joule's experiment,— that is, on the transformation of heat into work. The heated steam in the cylinder of a steam engine does work in pushing the piston back. This work is available for driving the machinery connected to the engine. A gasoline engine can drive an automobile or a tractor only when it is supplied constantly with heat from the exploding gasoline in the cylinders. In these cases, heat is transformed into work.

358. First Law of Thermodynamics.—The first law of thermodynamics is a special case of the law of conservation of energy. It is implied in the definition of the mechanical equivalent of heat and may be expressed by the equation

$$W = JH,$$

where W = the work measured in work units.
H = heat measured in heat units.
J = the mechanical equivalent of heat.

More specifically, the law states that when any mechanical change occurs in an isolated system, the energy of the system remains constant. Heat may be transformed into work or work into heat, but the total energy of the system remains unchanged. In other words, the **first law of thermodynamics states that in the transformation of work into other forms of energy or in the transformation of one form of energy into other forms of energy, no energy is ever created or destroyed.** The energy before and after the transformation is always the same. This law in its general form cannot be proved by experiment but conclusions based on it have always been confirmed by experiment.

359. Second Law of Thermodynamics.—The second law of thermodynamics states the conditions under which heat may be transferred from one body to another. It is in effect a statement of the fact that heat naturally flows from a place of higher to one of lower temperature but never in the reverse direction. An analogue may make the meaning clearer. Water may flow from a higher to a lower level with the performance of work. Heat may flow from a higher to a lower temperature with the performance of work. To cause water to flow from a lower to a higher level requires that external work be done on it. To cause heat to flow from a lower to a higher temperature also requires the performance of external work. The natural tendency of heat to flow from a higher to a lower temperature makes it possible for a heat engine to transform heat into work. On the contrary, a mechanical refrigerating machine must transfer heat from a colder to a hotter body. Work must be done on such a machine to make this transfer. The following is one form of statement of the law:

It is impossible for any kind of a machine working in a cycle to transfer heat from a lower to a higher temperature unless external work is done on it. A similar statement of the water analogy would be: It is impossible for a pump working in a cycle to transfer water from a lower to a higher level unless external work is done on it. The law cannot be proved by direct experiment. It states that a certain process or change cannot take place. It is a generalization based on the fact that in all human experience no contradictions of the law have been found. It merely states that heat of itself can only flow from higher to lower temperatures and no exceptions to this rule are known.

360. Steam Engine.—The simplest form of the cylinder of a steam engine is that shown in Fig. 321. It is a cylinder in which moves a closely fitting piston. This cylinder is connected to the steam chest by means of two pipes, *A* and *B*. These pipes, which are provided with valves, serve alternately as inlet and exhaust for the steam. As the piston moves forward, steam enters through *A* and the used steam is forced out through *B*. When the piston moves in the opposite direction, steam enters the cylinder at *B* and used steam is forced out at *A*. With this simple arrangement, the steam would leave the cylinder on exhaust at a temperature nearly as high as that at which it entered it. A considerable quantity of heat would thus be carried to the condenser or the outside air and lost so far as

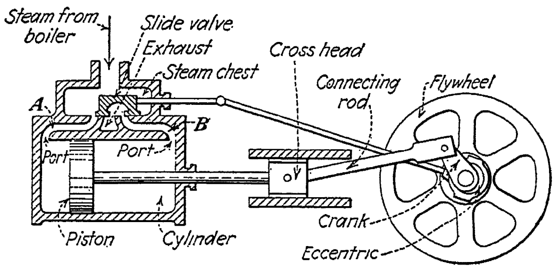


FIG. 321.—Steam-engine cylinder and plane slide valve. A case of transformation of heat into work.

useful work is concerned. The efficiency of the engine is thus quite low. In order to prevent as far as possible this waste, an automatic cut-off is provided. When the piston has moved through about one-fourth of its stroke, this slide valve automatically cuts off the supply of steam.

After this cutting off of the steam from the steam chest, the steam that has already entered the cylinder expands and pushes the piston forward, through the remainder of the stroke. During this expansion the piston is doing work, the pressure of the steam is being reduced, and its temperature lowered. The heat contained in the steam is thus converted into useful work and the efficiency of the engine is increased; for it must be remembered that this heat would otherwise have been lost to the outside air without producing any useful work.

When the piston has reached the end of this stroke, the slide valve opens *A* and connects *B* to the steam chest. Live steam

is now again admitted to the cylinder behind the piston and it pushes the piston toward the left. The dead steam in front of the piston is forced through A. When, as before, the piston has made about one-fourth of its stroke, the slide valve closes B, and the steam behind the piston expands until the piston has reached the end of its stroke. The cycle is then repeated.

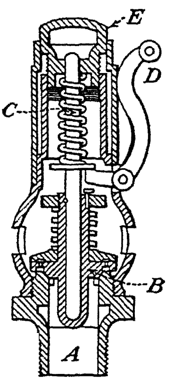


FIG. 322.—Cross section of pop valve.

The pressure of the steam in the boiler is regulated by means of a pop valve (Fig. 322) which allows the steam to escape when the pressure exceeds a certain value.

361. Gas Engine.—Gas engines and gasoline engines operate on the same principle. In each case the energy is derived from the explosion of a mixture of air and gas or gasoline vapor. The gasoline engine is now the most common type of engine. It is used to drive motor cars, motor boats, tractors, etc.

A diagram of the common four-stroke cycle gasoline engine is shown in Fig. 323. This engine makes four strokes or two revolutions of the flywheel for each power stroke. The first is the charging stroke, the second the compression stroke, the third the power stroke, and the fourth the exhaust stroke. The charging stroke is represented in diagram 1 of Fig. 323. The flywheel pulls the piston forward, and a mixture of gasoline vapor and air enters through the intake valve A. When the cylinder has been filled, the valve closes and the piston moves back and compresses the mixture of air and gasoline vapor in the cylinder to about one-fifth of its original volume. When the compression stroke is near its end, a spark from an induction coil or magneto ignites this mixture and causes an explosion which forces the piston forward with considerable velocity. The valve B is opened and the burnt gases are forced out through this valve. The momentum of the flywheel is sufficient to carry the piston through three

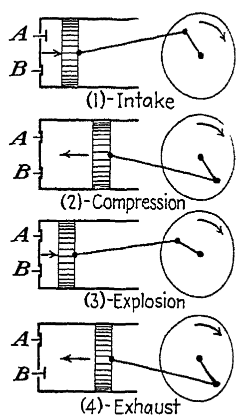


FIG. 323.—Four-stroke cycle gas engine.

of these strokes. After the burned gases have been forced out of the cylinder, the cycle is again repeated. Out of four strokes such an engine gives only one working stroke.

362. The Two-stroke Cycle Engine.—The principle of the two-stroke cycle internal-combustion engine is shown in Fig. 324. On the upward stroke of the piston P the pressure is reduced in the crank case C, and the explosive mixture of air and gasoline vapor is drawn in through the valve at A. At the same time, a quantity of this mixture previously taken into the space above the piston P is compressed. Near the end of this compression, the explosive mixture is fired by means of the spark from the magneto. The spark plug is shown at S. This explosion produces an increase of pressure in the cylinder behind the piston P. This pressure drives the piston down, and this is the working stroke of the piston. As the piston

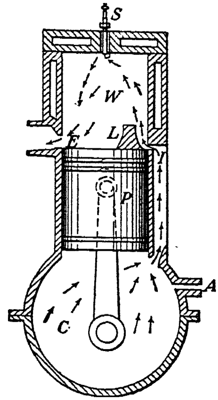


FIG. 324.—Two-stroke cycle engine.

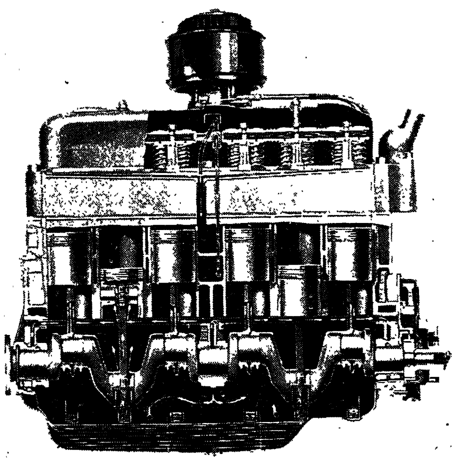


FIG. 325.—Crank shaft, pistons, etc., in gasoline engine. (*Courtesy General Motors.*)

descends, it compresses the mixture in the crank case. During this downward stroke the admission valve A is closed, and the exhaust valve E is also closed. Near the end of the downward

stroke, the exhaust valve E is opened, and the burned gases are allowed to escape to the atmosphere. The piston continues its downward stroke and soon opens the port I through which the slightly compressed gases in the crank case are forced into the space W above the piston.

The port I and the exhaust port E are opened and closed by the piston. The inlet valve in the crank case is operated like the valves in the other type of engine.

363. Work Done by a Gas Expanding at Constant Pressure.— Let a volume of gas be enclosed behind a piston (Fig. 326) which is air-tight and moves without friction. Let the pressure acting on the piston be denoted by p and the area of the piston by A. The total force acting on the piston pushing it backward is

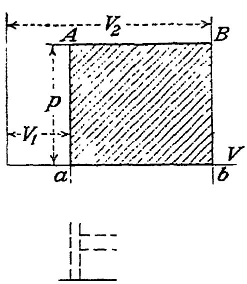


FIG. 326.—Work done by a gas at constant pressure = pressure × change of volume.

$$F = pA.$$

If, now, the gas in the cylinder is heated, it may be allowed to expand without any change in its pressure, and the piston moves back through a distance d.

$$\begin{aligned}\text{Work done on piston} &= \text{force} \times \text{distance}\\ &= p \times A \times d.\end{aligned}$$

Now $A \times d$ = increase of volume of the gas during expansion. Hence,

$$\text{Work} = p \times \text{change in volume.}$$

It is convenient to represent the work done by the gas by plotting the pressure of the gas on the vertical axis and the volume of the gas on the horizontal axis. In this case, the pressure is constant for all volumes. Hence, AB represents the relation between the volume and the pressure. If V_1 denotes the original volume and V_2 the final volume, the length of the line ab represents the change in volume during expansion. The product of the change in volume and the pressure is represented by the rectangle $ABba$. This area then stands for the work done by the gas during its expansion; and since this work is equal to the heat supplied to the gas during expansion, this rectangle also represents the heat taken in by the gas during its expansion.

364. Work Done by a Gas Expanding at Variable Pressure.—If the gas expands under a variable pressure, the work which it performs may be represented by a diagram similar to that by which it was represented for a gas expanding at constant pressure. In this case the line AB (Fig. 327), instead of being horizontal as in the preceding case, slopes toward the horizontal axis along which the volumes are plotted. Nevertheless, the area under AB will represent the work done by the gas as it expanded under the action of a changing pressure. It would be possible to construct a rectangle having the same base as the figure $ABba$ and the same area. The height of this rectangle would be the average pressure of the gas during its expansion. If the varying pressure acting on the piston has been replaced by a constant pressure equal to the average pressure, the work done by the gas would remain unchanged. **The average pressure may then be defined as the constant pressure by which a varying pressure may be replaced without changing the amount of work done on the piston for the same stroke.**

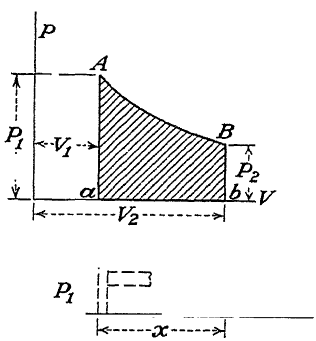


FIG. 327.—Work done by a gas at variable pressure = average pressure × change of volume.

365. Positive and Negative Work.—In the preceding cases we have discussed the work done on a piston by the expanding gas. Such work must be considered **positive,** for it is work delivered by the engine.

In order to force the dead steam or the burned gases out of the cylinder, the piston on its return stroke must exert a force and must, therefore, do work. This is the work done to return the piston to its initial position ready for a new forward stroke. It is work done by the engine on the useless gases. It represents a waste or loss which must take place in order to make the working stroke of the piston possible. To distinguish this work from the work done on the piston by the gases in the forward stroke, it is called **negative.** This work done on the gases by the piston must be subtracted from the work done on the piston by the gases in order to get the useful work done by the piston in one complete cycle.

This work done on the gases by the piston may also be represented by an area, such as that used to represent the work done by the gases on the piston. One of these areas, however, represents the work done on the piston and the other represents work done by the piston. The net work will be the difference between these areas. If the gas expands at constant pressure and is later compressed at a lower constant pressure, the net work done by the piston is represented by the difference between the areas $ABba$ and the area $DCba$, *i.e.*, by the rectangle $ABCD$ (Fig. 328).

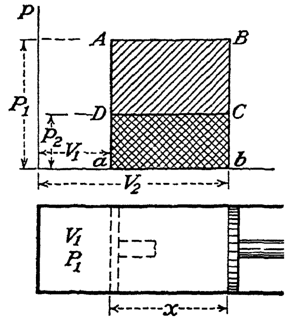


FIG. 328.—Positive and negative work. Work done by the gas is positive; work done on the gas is negative.

366. Efficiency.—Since heat and work are convertible, the most important thing to know about any device for this purpose is its efficiency, which gives a measure of the amount of heat which can be transformed into work under a given set of conditions. **The ratio of the work obtained from the machine to heat put into it is called the efficiency of the machine.** Both the heat and the work must be measured in the same units. In finding, for example, the efficiency of a burner used to heat a kettle of water, it is necessary to find the amount of gas consumed by the burner and the amount of heat thus developed. It is next necessary to find the amount of heat which gets into the water in the kettle. The ratio obtained by dividing the heat which gets into the water by the heat developed by the burning of the gas is called the efficiency of the burner. The efficiency may be defined as the fraction which tells what portion of the total heat supplied is used for the purpose for which it was intended.

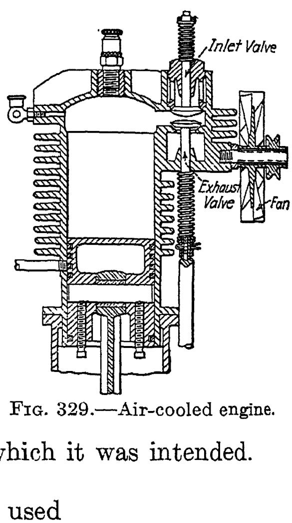


FIG. 329.—Air-cooled engine.

$$\text{Efficiency} = \frac{\text{heat used}}{\text{heat supplied}}.$$

The operation of transforming heat into work is inefficient because only a small fraction of the heat developed by the combustion of the fuel is converted into work. In a steam engine a part of the heat must pass over with the exhaust steam and be lost in the condenser. This loss is so great that even the best steam engines do not use more than about 18 per cent of the heat of combustion of the fuel. An ordinary locomotive uses about 8 per cent of the heat generated by the combustion of its fuel. In a gas engine, provision must be made for removing the excess heat. This is done by circulating air or water. An air-cooled engine is represented in Fig. 329.

Example.—If 1 lb. of coal which has a heating value of 8,000 B.t.u. was burned in a machine which raised 500 gal. of water 100 ft., what percentage of the heat from the coal was converted into useful work?

Heat supplied = 8,000 B.t.u.

Work done = 500 $\times$ weight of water per gallon $\times$ 100

= 500 $\times$ 8.3 $\times$ 100 = 415,000 ft.-lb. = 533 B.t.u.

$$\text{Efficiency} \quad \frac{\text{heat used}}{\text{heat supplied}}$$

$$\frac{533}{8,000} = 6.7 \text{ per cent.}$$

367. Isothermal Changes.—A gas may be expanded or compressed and at the same time the temperature of the gas may remain constant. In order that such a change may be possible, heat must be removed from the gas in case it is being compressed, and heat must be added to the gas in case the gas is expanding. If the gas is compressed without at the same time removing heat from it, the temperature of the gas will increase. If, on the other hand, the gas is expanded without heat being supplied to it, the temperature of the gas will decrease. In the case of a gas enclosed in a cylinder, for example, the work done in compressing the gas reappears as heat energy in the gas, but in case the gas in the cylinder is expanding the work done on the piston is at the expense of the heat energy in the gas, and the temperature of the gas will decrease.

A process which is carried out in such a way that the temperature of the substance—gas, liquid, or solid—remains constant is called an isothermal process. If the walls of the cylinder are good conductors of heat, and if the compression or expansion of the gas is carried out slowly so that there is a chance for the heat to flow out of the gas during the compression or into the gas

during the expansion, the process will be nearly isothermal. The relation between the pressure and volume of a gas during isothermal expansion and compression is shown in Fig. 330 by the line ST. The equation of this curve is, PV = constant, where P is the pressure of the gas, and V is its volume.

368. Adiabatic Changes.—If a gas or any other substance is compressed or expanded in such a way that no heat is allowed to enter it or to escape from it, the temperature of the substance increases in the case of compression and decreases in the case of expansion. The work necessary to push back the piston if the gas is in a cylinder closed by a piston, as in Fig. 328, is obtained at the expense of the heat energy of the gas. Hence, the temperature of the gas decreases during the expansion unless heat flows into the gas from some external source. If, on the other hand, the gas is compressed and no heat allowed to escape during the compression, the work done on the piston appears as heat energy in the gas, and the temperature of the gas is increased. Such an expansion or compression may be realized experimentally by enclosing the gas in a cylinder which is surrounded by non-conducting materials and then compressing or expanding the gas rapidly so that there is little time for heat to flow into the gas or out of it. **A process in which there is no exchange of heat between the substance and its surroundings is called an adiabatic process.**

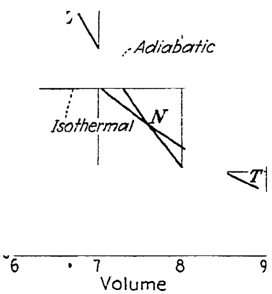


FIG. 330.—A comparison of the isothermals and adiabatics of a gas. The slope of the adiabatic is greater than the slope of the isothermal.

The relation between the volume and the pressure of a gas during an adiabatic expansion is shown by the curve LM in Fig. 330. The curve for the adiabatic process is steeper than the corresponding curve for the isothermal process. If the pressure, temperature, and volume of two gases are the same at the beginning of an adiabatic and an isothermal change, and if they expand to the same volume, the temperature and pressure of the gas which has expanded adiabatically will be less than the corresponding temperature and pressure of the gas which has expanded isothermally. The work done on the gas during

the isothermal compression is less than the corresponding work for the adiabatic compression. The temperature and pressure at the end of the adiabatic compression are greater than they are at the end of the isothermal compression. In other words, it is easier to compress a gas isothermally than it is to compress it adiabatically. The heat generated during the adiabatic compression increases the pressure of the gas, so that more work is done in compressing it.

369. Carnot Cycle.—An ideal engine was devised by Carnot to illustrate and analyze the fundamental principles involved in heat engines. This engine was imagined to consist of a cylinder filled with gas and closed by a movable piston. By allowing the gas in the cylinder to expand isothermally and then adiabatically, and later compressing the gas isothermally and then adiabatically, the gas in the cylinder is carried through a cycle and made to yield work.

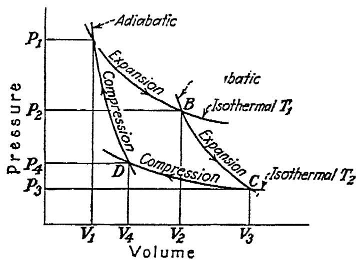


FIG. 331.—Carnot's cycle, bounded by two isothermals and two adiabatics.

Suppose the gas in the cylinder has a volume and a pressure represented by the point A in Fig. 331. If now the gas is allowed to expand isothermally, the temperature remains constant, the volume increases, and the pressure decreases, until the volume and pressure represented by the point B on the curve are reached. The area under the curve AB represents the work done by the gas during this expansion. At the point B, suppose the conditions surrounding the cylinder are changed so that the gas expands adiabatically until the pressure and volume have the values indicated by the point C. During this adiabatic change, the temperature of the gas has changed, so that the temperature is now T_2, instead of T_1. The area under the curve BC represents the work done by the gas during this adiabatic expansion.

Now conditions are again changed, and the gas is compressed isothermally at the temperature T_2 until the gas has the pressure and volume indicated by the point D. This point D is so chosen that the cycle can be closed by an adiabatic compression along the curve DA. The area under CD represents the work done on the gas during the isothermal compression. This work is regarded as negative since it is work done on the gas by external forces and not work done by the gas.

Again change the thermal conditions surrounding the cylinder, so that the gas may be compressed adiabatically until it has the volume and pressure represented by the point A. The temperature, pressure, and volume of the gas are now the same as they were at the beginning of the cycle, and the gas has been returned to its original condition. The area under the curve AD represents the work done on the gas during the adiabatic compression. Again this work is negative, since it is work done on the gas by external forces.

The net work of the cycle is represented by the area enclosed in the figure *ABCD*. It is the difference between the positive work done during the isothermal expansion *AB* and the adiabatic expansion *BC*, and the negative work done on the gas during the isothermal compression *CD* and the adiabatic compression *DA*.

During the isothermal expansion along the line *AB* at the temperature T_1, H_1 units of heat flowed into the gas. During the adiabatic expansion along the line *BC*, no heat flowed into the gas or out of it. During the isothermal compression along the line *CD*, H_2 units of heat were rejected from the gas at a temperature T_2. During the adiabatic compression along the line *DA*, no heat flowed into the gas or out of it.

During the cycle $(H_1 - H_2)$ units of heat have been transformed into mechanical energy. If W denotes the work done by the engine during this cycle,

$$W = J(H_1 - H_2),$$

where J is the number of units of work produced from one unit of heat (Appendix E-7).

370. The Efficiency of a Carnot Engine.—The efficiency of a Carnot engine depends only on the temperatures between which it works. Now, the efficiency of any engine has been defined as the ratio between the heat transformed into useful work by the engine and the heat taken in from external sources. Hence, for a Carnot engine,

$$\text{Efficiency} \quad \frac{H_1 - H_2}{H_1}.$$

In the case of a Carnot engine, it may be shown that the efficiency is also given by the expression,

$$E \quad \frac{T_1 - T_2}{T_1}.$$

where T_1 is the temperature of the hot body from which the heat is taken and T_2 is the temperature of the cold body to which the heat not transformed into useful work is delivered. In both cases, these temperatures are measured on the absolute scale of temperatures. No engine can have a greater efficiency than a Carnot engine working between the same temperatures.

Example.—Find the efficiency of a Carnot engine working between a temperature of 1527°C and a temperature of 627°C.

$$T_1 = 1527° + 273° = 1800°.$$
$$T_2 = 627° + 273° = 900°.$$
$$E = \frac{1{,}800 - 900}{1{,}800}$$
$$= 0.50 \text{ per cent.}$$

371. Indicator Card.—If the gases expand and are compressed under varying pressures, the net work may be obtained as follows. The area under the line *ABC* (Fig. 332) represents

the work done on the piston by the gases during the expansion, and the area under the line ADC represents the work done by the piston on the gases during compression. The useful or net work obtained from the cycle is represented by the area of the irregular figure $ABCDA$. It is the work which is left after the work of returning the piston is deducted. Such a figure is called an **indicator card.** The indicator card for a steam engine is essentially like Fig. 332, and in Fig. 333 is given an indicator card of a four-stroke gas engine. It gives a measure of the work performed by the engine in one cycle.

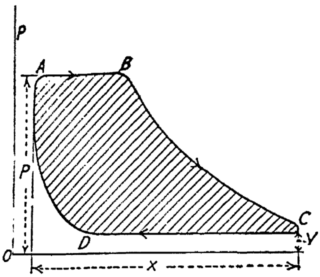


FIG. 332.—Indicator card for steam engine. The area shows the work done by the engine in one cycle.

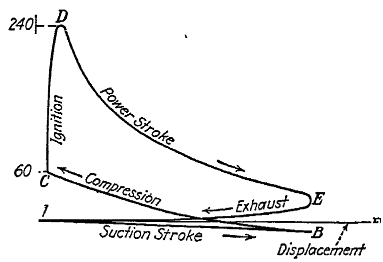


FIG. 333.—Indicator card for a gas engine, showing work done in one cycle.

372. Indicator.—In order to make an indicator card from which to determine the horsepower of an engine, a device known as an **indicator** is used. The essential part of such an instrument is a small vertical cylinder (Fig. 334) containing a movable piston. This cylinder is in communication with the steam in the cylinder of the engine. This piston is held down by a spring which will be compressed as the steam exerts its pressure on the piston. The compression of the spring is made to raise an arm at the end of which is a pencil that records its movements on a revolving drum. The drum is covered with paper and its motion is controlled by the motion of the piston rod. In this way, a diagram showing the unbalanced pressure in the cylinder of the engine for each part of the stroke is obtained. From the average pressure and the length of the stroke, the horsepower of the engine can be calculated.

373. Indicated Horsepower of an Engine.—In order to measure the horsepower of an engine, it is necessary to know: (1) the length

of the stroke in feet, (2) the number of revolutions made each minute, (3) the area of the piston, and (4) the average unbalanced pressure in pounds per square foot on the piston. The length of the stroke and the area of the piston are easily measured and the number of revolutions per minute counted. The average unbalanced pressure of the steam in the cylinder is found from the indicator card in the following manner. Take the area of the figure *ABCDA* (Fig. 332) and find its mean height. This is equivalent to finding a rectangle having the same base as *ABCDA*

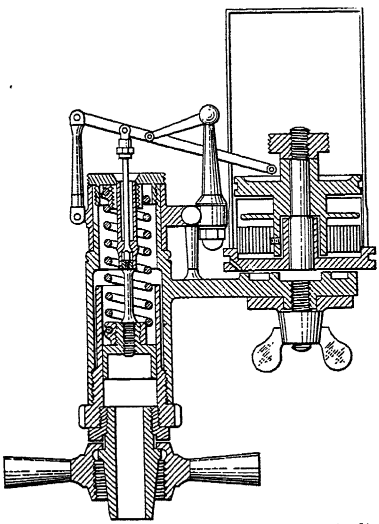


FIG. 334.—Steam-engine indicator for making indicator cards.

and the same area. The height of this equivalent rectangle is the mean effective pressure, which means that if the pressures actually operating in the cylinder had been replaced by a constant pressure equal to this mean effective pressure the work done by the engine would have been unchanged. To find the horsepower, multiply the mean effective pressure in pounds per square foot by the area of the piston in square feet. This gives the average force acting on the piston. Multiply this force by the length of the stroke. This gives the work done in each stroke in foot-pounds. Multiply the work done in each stroke by the number of strokes per minute. The result is the work done per minute. By dividing this work by 33,000, the number of foot-pounds per

minute to make 1 hp., the indicated horsepower of the engine is obtained.

$$\text{Horsepower} = \frac{\text{average pressure} \times \text{length of stroke} \times \text{area of piston} \times \text{number of revolutions}}{33{,}000}$$

$$\frac{PLAN}{33{,}000}.$$

This formula applies to the simple case of a single-acting engine. In practice, most engines are of the double-acting type. The horsepower of a double-acting engine can be obtained by inserting the factor 2 in the expression for the horsepower of the single-acting engine. The horsepower of a double-acting engine thus becomes

$$\text{Horsepower} = \frac{2\,PLAN}{33{,}000}.$$

Example.—In a steam engine the average pressure was 40 lb. per square inch, the length of the stroke was 12 in., the number of revolutions per minute 300, and the area of the piston 125 sq. in. Find the horsepower of the engine.

Force on piston = pressure × area = 40 × 125 = 5,000 lb.
Work per stroke = force × length of stroke = 5,000 × 1 ft. = 5,000 ft.-lb.
Work per minute = work per stroke × number of strokes per minute.
= 5,000 × 300 = 1,500,000 ft.-lb.

$$\text{Power} = \frac{1{,}500{,}000}{33{,}000} = 45.3 \text{ hp.}$$

374. Brake Horsepower.—The brake horsepower measures the power which the motor or engine can deliver at the shaft or

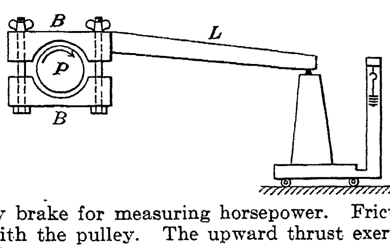


FIG. 335.—Prony brake for measuring horsepower. Friction tends to make the brake rotate with the pulley. The upward thrust exerted by the balance overcomes this tendency.

belt for work such as driving machinery. This power is measured by what is known as a Prony brake. This brake, which is shown in Fig. 335, consists of two wooden blocks *BB* which are clamped around the pulley *P* by means of thumbscrews. From one of these blocks projects a lever *L* which rests on a pair of platform

scales. When the shaft P is rotating, there is a tendency to drag the blocks with it, and the lever presses on the platform scales with a certain force which depends on the force exerted on the blocks by means of the pulley. The force of the pulley on the blocks is better seen in Fig. 336, which represents a belt passing around the pulley. The difference between the readings on the spring balance is equal to the force of friction between the belt and the pulley. The horsepower of the engine is calculated from the weight registered on the scales in the following way: Let L be the length of the brake arm in feet, W the weight registered on the scale expressed in pounds, and N the number of revolutions which the engine or motor makes per minute.

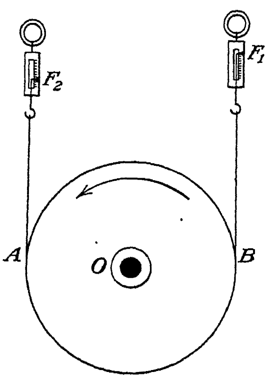


FIG. 336.—With a strap brake the friction between the pulley and the strap is given by the difference between the readings of the spring balances F_1 and F_2.

$$\text{Brake horsepower} = \frac{2\pi L \times W \times N}{33{,}000}.$$

Example.—The reading of the scale of a Prony brake when the engine was making 300 revolutions per minute was 75 lb. The length of the brake arm was 4 ft.

Find the horsepower of the engine.

$$\text{Brake horsepower} = \frac{2 \times 3.1416 \times 4 \times 75 \times 300}{33{,}000} \quad 1.71 \text{ hp.}$$

375. Refrigerating Machines.—The process of reducing the temperature of a body below that of its surroundings is essentially the reverse of the process employed in the steam engine or in the gasoline engine. In the engine, the steam passes from the boiler to the cylinder at high temperature. In the cylinder, it gives up some of its heat. This heat is transformed into work which drives the piston forward. In a refrigerating machine, the working substance is taken into the cylinder at a low temperature. Here it is compressed and thus heated in consequence of the work done on it during compression. This heat is removed by water circulating around the cylinder in which the compression takes place. The working substance then expands or vaporizes, and during this expansion or vaporization it takes heat from the surrounding bodies and thus cools them below their original temperatures.

One of the common forms of refrigerating machines is one in which ammonia is used as the working substance. The cooling effect is in this case obtained from the absorption of heat by the vaporization of the ammonia. A simple diagrammatic sketch

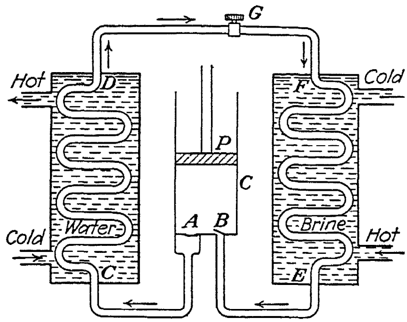


Fig. 337.—Refrigerating machine. It transfers heat from a colder to a hotter body.

of this type of machine is shown in Fig. 337. A force pump C is closed by two valves A and B. One of these opens outward from the cylinder and the other opens into the cylinder. On the upward stroke of the piston, the valve B is opened and ammonia gas is drawn into the cylinder from the pipes immersed in the

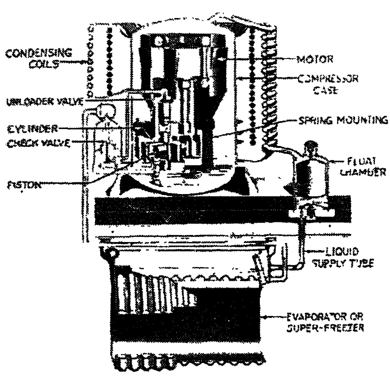


Fig. 338.—Household refrigerator. (*Courtesy General Electric Company.*)

cooling chamber FE. On the downward stroke of the piston, the valve B closes and the valve A is opened. The ammonia gas is compressed, and, on account of the compression, the vapor is heated. In spite of this heating, some of the vapor is changed

into the liquid condition. The mixture of liquid and vapor is forced into the coils *CD* where is it cooled by water circulating around these coils. Here all of the vapor becomes a liquid. After this cooling, the liquid ammonia is allowed to escape through a regulating valve *G* into the coils *FE* which are immersed in the brine to be cooled by the refrigerating machine. In these coils the pressure is low because this space is continually being exhausted by the pump. The liquid ammonia under a reduced pressure in these pipes evaporates and changes back to vapor. In this process of evaporation heat is absorbed. The heat necessary to evaporate the ammonia comes from the coils *FE* and whatever liquids may surround them. The cooling effect in the case described is sufficient to reduce the temperature of the water below its freezing point. It is in this way that artificial ice is manufactured. Household refrigerating machines (Fig. 338) that operate on the principle of removing heat when a liquid is evaporated are becoming increasingly common.

Problems

1. A truck and its load together weigh 6 tons. The brakes are used to bring it to rest from a speed of 30 miles per hour. How much heat is developed, if all the work done against the frictional resistance is converted into heat?

2. If a certain meteor reaches the earth's atmosphere with a velocity of 6.5 miles per minute, how much heat will be generated when it is brought to rest by the frictional resistance of the air? Weight of meteor is 1 g.

3. One cubic foot of water falls from the top to the bottom of Niagara Falls a distance of 160 ft. If all the potential energy lost in the fall is transformed into heat, how much is the temperature of the water raised?

4. A strap brake used for testing a small motor was used under the following conditions: diameter of pulley, 3 in.; reading of spring balances, 10 and 32 lb., respectively; speed, 1,200 revolutions per minute. What power was developed by the motor?

5. The following readings were taken during a test with a Prony brake: revolutions per minute, 840; length of arm, 4 ft. 3 in.; reading of scales, 319 lb. Calculate the horsepower of the motor.

6. Ice is used to cool a Prony brake which is absorbing 8 hp. If the water escapes at a temperature of 10°C., how long will a 40-lb. piece of ice last?

7. How much coal with a heat of combustion of 11,000 B.t.u. will be required per mile of operation by a steam tractor weighing 10 tons against a rolling friction of 16 lb. per ton, assuming that 6 per cent of the energy of the coal is converted into work?

8. The horsepower of an engine was determined by means of a Prony brake. The arm of the brake was 1.8 ft. in length. The engine made 1,200

revolutions per minute, and the reading on the balance at the end of the arm of the brake was 9 lb. What was the horsepower of the engine?

9. A gasoline engine makes 1,800 revolutions per minute. It has eight cylinders and develops 55 hp. The cylinder has a bore which is $3\frac{1}{2}$ in. in diameter, and the length of the stroke is 5.5 in. Find the average pressure which is developed during each working stroke.

10. Suppose that a steam engine develops 3 hp. How many British thermal units of heat must be supplied per min. if the over-all efficiency is only 12 per cent?

11. A brake is applied to the driving shaft of an engine which develops 4 hp. The brake and the shaft are immersed in a calorimeter so that all the work done against friction is transformed into heat and goes to increase the temperature of the water in the calorimeter. If the mass of the water in the calorimeter is 140 lb., how much will its temperature rise per minute?

12. A Carnot engine is working between the temperatures of 250 and 120°C. How much work will be obtained from 1,000 B.t.u.?

PART IV.—MAGNETISM AND ELECTRICITY

CHAPTER XXXII

MAGNETISM

376. Natural Magnets.—A peculiar mineral (Fe_3O_4) called lodestone was found in early times in the neighborhood of Magnesia in Asia Minor. This mineral has the power of attracting small particles of the same mineral and of setting itself in one particular direction when suspended. When a piece of this lodestone is dipped into iron filings, they adhere to it. It is found that there are two places on each piece of this mineral at which the iron filings adhere in greatest quantities. If such a piece of lodestone is suspended by means of a silk thread, it is found that the line joining the places at which the iron filings adhere in greatest quantities points north and south.

377. Artificial Magnets.—When a steel knitting needle is stroked from one end to the other with a piece of lodestone, using for point of contact one of the points at which the iron filings adhere most freely, the needle acquires the property of attracting iron filings and of setting itself north and south when suspended. Such a needle is called an **artificial magnet.** There are other ways in which more powerful artificial magnets can be made. The properties of these magnets do not differ in any way from those of the needle, except that they are much more powerful.

378. Magnetic Poles.—On dipping a magnetized needle into iron filings it is seen that the iron filings adhere most strongly at the ends of the needle. These places at which the tendency of the iron filings to cling to the needle is greatest are called **poles.** If that end of a suspended needle which points north is marked, it will be found that, however, the needle may be disturbed, it will come to rest with the marked end pointing north. For convenience it is desirable to call the end of the magnetic needle which points north **the north-seeking pole or N-pole.** The other end is called the **south-seeking pole or the S-pole.**

379. Force between Poles.—The first law of magnetism states that **like poles repel each other, but unlike poles attract each other.** Thus, two N-poles repel each other, and two S-poles repel each other, but an N-pole and an S-pole attract each other.

380. Law of Force between Magnetic Poles.—Experiments show that the **force with which the poles of two magnets attract or repel each other, in a vacuum, is equal to the product of the pole strength divided by the square of the distance between the poles,** or expressed mathematically,

$$F = mm'/d^2,$$

where F = the force in dynes.
d = the distance in centimeters between the poles.
m = the strength in unit poles of one pole.
m' = the strength in unit poles of the other pole.

381. Unit Pole.—**A unit pole is a pole of such strength that it will repel a similar pole of equal strength with a force of 1 dyne when placed 1 cm away from it in a vacuum.** An isolated north pole is not a physical possibility, since every magnet must have an equal south pole for every north pole. It is possible to realize nearly enough the conditions assumed in this definition by supposing that the magnets are very long, so that the south poles are so far away that their influence is small. The number of unit poles on the end of a magnet is measured by the number of dynes of force which it will exert on a unit pole 1 cm. from it in a vacuum. The force in air differs little from the force in a vacuum. Hence, the distinction between air and vacuum may be neglected.

Example.—Find the force on a N-pole of strength 40 units when placed in air 30 cm. from a like pole of strength 20 units.

$$F = \frac{\quad}{d^2} \quad \frac{40 \times 20}{30^2} = \frac{800}{900} = \frac{8}{9} = 0.89 \text{ dyne.}$$

382. Magnetic Field.—The space outside the magnet in which its influence can be detected is called the **magnetic field.** In the case of powerful magnets, this space extends far from the magnet, but, in the case of feeble magnets, the magnetic field is so weak that it may be considered as confined to a small region near the magnet.

At every point near a magnet or a system of magnets, a free magnetic pole would experience a force tending to drive it in a definite direction. The direction in which a free N-pole would move is called the direction of the magnetic field and the magnitude of the force on unit pole is known as the **intensity of the magnetic field. The intensity of the magnetic field or the magnetic intensity** (Fig. 339) **is thus defined to be the mechanical force measured in dynes which is exerted on unit pole at a point**

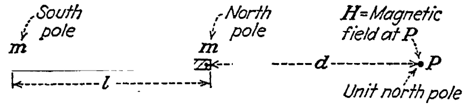


FIG. 339.—Intensity of magnetic field at a point P; $H = \frac{m}{d^2} - \frac{m}{(l+d)^2}$.

in free space. The unit of magnetic intensity is called the **oersted.** If the magnetic field is such that **there is a force of 1 dyne on unit pole, the magnetic field or the magnetic intensity at that point is 1 oersted.** (Formerly the word gauss was used as the name of this unit but gauss is now used exclusively as the unit of magnetic induction.)

383. Magnetic Lines of Force.—In order to make it easier to understand the way in which the magnetic field changes from

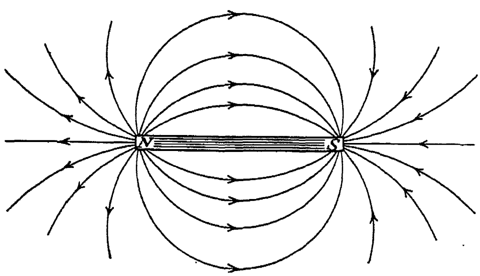


FIG. 340.—Magnetic lines of force around a bar magnet.

point to point, it is convenient to draw certain lines (Fig. 340) which by their direction represent the direction of the magnetic field and by their number represent the intensity of the magnetic field. Such lines are called **lines of magnetic force.** Such a line of magnetic force is the path along which a perfectly free N-pole would travel when left alone in the magnetic field. Since such a free N-pole cannot be obtained in practice, a small compass

needle is used to determine the direction of the magnetic field at a point. When the compass needle comes to rest, its axis points in the direction of the magnetic field at that point

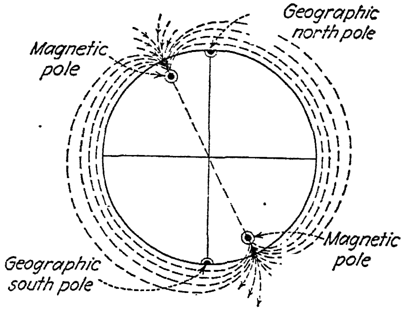


FIG. 341.—The earth as a magnet.

In order to represent the strength of the magnetic field, there is a well-established convention concerning the number of lines of force to be drawn per unit area. The agreement is that **the number of lines of force per square centimeter shall be just equal to the number of dynes with which the field would act on a unit pole.** For example, a magnetic field equal to 10 dynes on a unit pole is represented by drawing 10 lines of force per square centimeter.

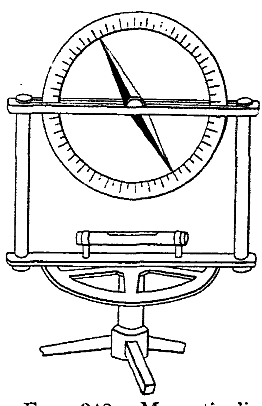


FIG. 342.—Magnetic-dip needle. The plane of the divided circle must be placed in the plane of the earth's magnetic field.

384. The Earth's Magnetic Field.—The action of the compass needle in pointing north and south is explained by the fact that the earth itself acts as if it were a great magnet with an S-pole near the geographical north pole and an N-pole near the geographical south pole (Fig. 341). It is only the horizontal part of the field from this large magnet that is effective in turning the compass needle. The needle itself remains horizontal, because it is ordinarily adjusted in a stirrup or on a needle point until it assumes a horizontal position.

If a needle is carefully balanced (Fig. 342) so that it is free to rotate in a vertical plane and is then magnetized, it will be found

that in the northern hemisphere the N-pole of the needle dips downward and in the southern hemisphere the S-pole dips downward. The earth's magnetic field is, therefore, inclined to the horizontal, and the amount of this inclination varies from point to point on the surface of the earth.

385. Declination.—The early users of the compass were aware that it did not always point exactly north and south, but that the direction of the needle changed as it was moved from point to point on the surface of the earth. The reason for this variation lies in the fact that the earth's magnetic poles do not coincide with the geographical poles (Fig. 341). There are also local causes that influence the direction in which the compass needle points. **The number of degrees between the true north and south line and the axis of the needle is called the declination of the needle.** To determine the declination at any place, it is first necessary to determine the geographical meridian and then the magnetic meridian. The former is determined from astronomical observations and the latter by noting the direction in which the axis of a compass needle points at that place. The angle between these two directions is the declination of the compass needle at that place.

386. Magnetic Dip.—The angle which a magnetic needle free to move about a horizontal axis pointing east and west (Fig. 342) **makes with the horizontal is called the angle of dip.** This angle varies from place to place on the earth. At the earth's magnetic equator it is zero and at the earth's magnetic north pole it is 90 deg. This angle is measured by means of a dip circle. This consists of a vertical circle at the center of which is suspended a magnetic needle so that it is free to rotate about a horizontal axis. When the plane of the circle is placed in the magnetic meridian, the magnetic needle dips below the horizontal. The angle of this dip is read on the divided circle.

387. The Variation of the Earth's Magnetic Field.—The earth's magnetic field is not a fixed quantity. It changes in both magnitude and direction There is a cyclic change in the magnetic declination. This change has a period of approximately 960 years. Such a change is known as a **secular change.** There is also a change from year to year in the magnetic declination. This change is known as an **annual variation.** In addition to these two changes, there is a variation from day to day. This **diurnal variation** is small, but it can be detected by delicate magnetic instruments. There is also a correlation between the erratic variations in the earth's magnetic

field and sun-spot activity. In other words, sun-spot activity is frequently accompanied by magnetic storms on the earth. The magnetic field on the sun is much stronger than it is on the earth. It is approximately 250 times as much on the sun as it is on the earth. The horizontal component of the

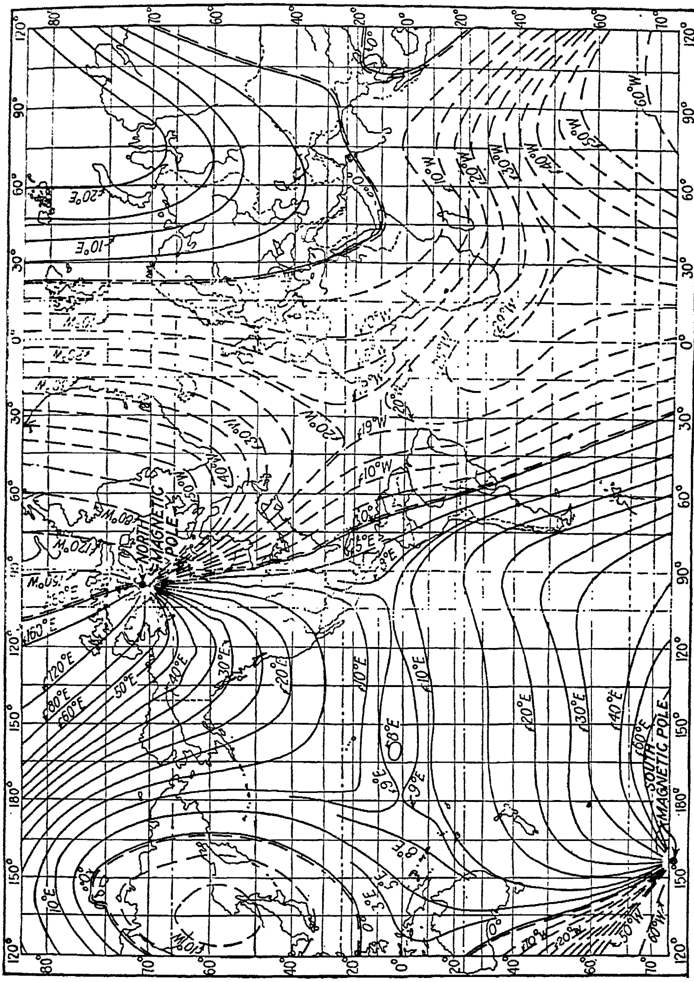


FIG. 343.—Approximate isogonic lines for the year 1930. (*Courtesy Department of Terrestrial Magnetism, Carnegie Institution of Washington.*)

earth's magnetic field is about 0.2 oersted, but on the sun it is about 50 oersteds. In the region of a large sun spot, the magnetic field on the sun may be as much as 3,500 or 4,000 oersteds.

388. Magnetic Survey of the Earth.—An intensive study of terrestrial magnetism is now being made by the Department of Terrestrial Magnetism

of the Carnegie Institution of Washington. Such an intensive study is essential for an understanding of the causes and effects of the magnetism of the earth.

Continuous observations extending over many years are being carried out at a large number of fixed stations. These stations are distributed on both land and sea in such a way as to give data on the magnetic elements at the greatest possible number of widely distributed points on the surface of the earth. At these stations, observations on the magnetic declination, dip, and horizontal intensity are made with great precision. The results can be represented for convenience by means of maps on which lines are drawn through the points for which a particular magnetic element has the same value. *Isogonal lines are lines passing through those points for which the magnetic declination has the same value.* Figure 343 gives the distribution of these isogonic lines for the year 1930. *Those lines for which the magnetic dip has the same value at every point are called isoclinals.* The line where the magnetic dip is zero is the magnetic equator. It follows the general course of the geographic equator.

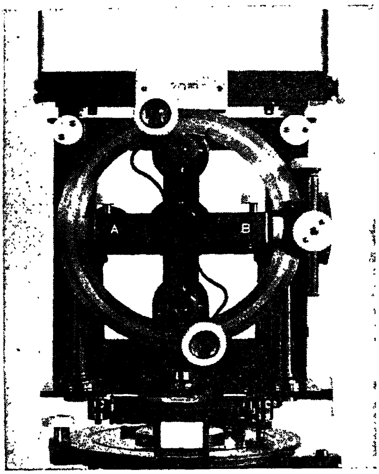


FIG. 344.—Dip circle as modified by the Department of Terrestrial Magnetism. Carnegie Institution of Washington. (Note celluloid caps on parts to be touched with the hands to prevent freezing of fingers.)

To make accurate observations possible under such widely different conditions has required the development of very special instruments by the Department of Terrestrial Magnetism of the Carnegie Institution. Figure 344 shows a type of dip circle used in the Arctic and Antarctic for observing magnetic dip, total intensity, and declination. All screws and parts turned by the hand are provided with celluloid caps to prevent the observer from freezing his fingers where they would otherwise come in contact with metals at low temperatures.

389. Classes of Magnetic Substances.—From the point of view of their magnetic properties, bodies are divided into three groups: ferromagnetic, paramagnetic, and diamagnetic. **Ferromagnetic substances** include those substances whose intensity of magnetization at saturation is of the same order of magnitude as that of iron. In this group are included substances like iron, nickel, cobalt, and Heusler alloys. **Paramagnetic substances** become feebly magnetized in the direction of the magnetic field. Among the paramagnetic bodies are found oxygen,

palladium, manganese, and the salts of various metals. In a magnetic field such substances set themselves so that the longer axis is in the direction of the magnetic field. **Diamagnetic substances** include the greater number of simple bodies and chemical compounds. Bismuth is the best illustration of this group of bodies. When a diamagnetic body is placed in a nonuniform magnetic field, it tends to move from the stronger to the weaker magnetic field and if it is free to rotate, it turns so that the longer axis is perpendicular to the magnetic field (Fig. 345).

390. Molecular Theory of Magnetism.—Every substance which is capable of being rendered a magnet consists of a very large number of small parts which are very small magnets. These parts are sometimes called molecular magnets, for they are probably no larger than the molecules out of which the substances are made. When the substance is unmagnetized, these molecular magnets are not arranged in any particular direction but are oriented indiscriminately. When the substance is magnetized, a larger number of these little magnets are made to point along the axis of the magnet than point in any other direction. In the interior of the magnet, the little north poles lie so close to the little south poles that each destroys the influence of the other. At one end of the bar are free north poles, and at the other end are free south poles. The sum of all these little north poles makes the N-pole of the magnet, and the sum of all the little south poles makes the S-pole of the magnet.

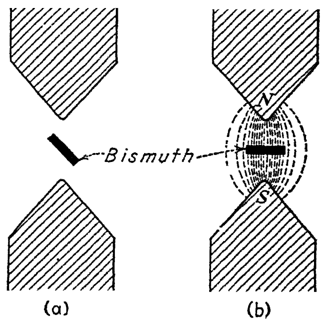


FIG. 345.—Diamagnetic substances such as bismuth set the long axis perpendicular to the magnetic field.

Take a knitting needle which has been magnetized and after marking its north pole break it into two nearly equal parts. Suspend each of these parts in turn and test their polarity. It will be found that each part has become a magnet and that two new poles, an N-pole and an S-pole have been developed by breaking the magnet. A further breaking will produce additional poles, and this process may be continued until the magnets become as short as we please. It would seem from such an experiment that a magnet is made up of a large number of very small magnets which are directed along the axis of the magnet.

391. Electron Theory of Magnetism.—In a later chapter it will be seen that a magnetic field surrounds an electric current. Starting from this observation Ampère proposed a theory of magnetism which assumed that circular electric currents are flowing in small closed paths of molecules. This theory merely accounts for the presence of the elementary magnets assumed in the preceding section and really states that each molecule is an elementary magnet owing to the fact that small electric currents are, in some way, flowing around it. Now the modern electron theory of matter assumes that each atom consists of a nucleus of positive electricity about which negatively charged particles are moving. These moving negatively charged particles called **electrons** are equivalent to the electrical currents which Ampère assumed to flow in the molecules and to give rise to the elementary magnets which were postulated in the **molecular theory of magnetism.**

392. Magnetic Induction.—When a bar magnet is placed near a piece of unmagnetized iron, the elementary magnets in the iron tend to arrange themselves, so that all the N-poles point in one direction and all the S-poles in the opposite direction. The piece of iron thus becomes a magnet so long as it is in the presence of the permanent bar magnet. South-seeking poles are produced near the north-seeking pole of the bar magnet and north-seeking poles at the other end of the piece of soft iron. This process of magnetization is known as **magnetization by induction.**

If a piece of soft iron is held in the direction of the earth's magnetic field and jarred by hitting it with a hammer, it will become magnetized. The lower end will be a north-seeking pole and the upper end a south-seeking pole. When objects made of iron stand for a long time in the magnetic field of the earth, they also become magnetized. For example, the water pipes in a house are usually magnetized. These are all cases of magnetization by induction; that is, magnetization by the orientation of the elementary magnets in the soft iron due to the action of some external magnetic field.

As soon as the soft iron is taken out of the magnetic field which has produced the induced magnetization, the elementary magnets in the soft iron rearrange themselves irregularly and nearly all the induced magnetism disappears. Large masses of iron or very long pieces of iron or steel retain their induced magnetism better than do small pieces. To make what is called a permanent magnet, the steel is first tempered until it is quite hard and brittle. If it is then magnetized by induction in a strong magnetic field, it retains a much greater percentage of the induced magnetism than does a piece of soft iron. For this reason it may be regarded as a *permanent magnet.*

Problems

1. The magnetic declination in Oregon is 20°E. How far from a true north course would a flyer be after traveling 100 miles following the compass

without making a correction for declination? Would he be east or west of his course?

2. A unit magnetic pole is placed 8 cm. from a pole of unknown strength, and a force of 320 dynes is observed. What is the strength of the unknown pole?

3. Two magnetic poles have strengths of $+40$ and -90 units, respectively. At what distance in air will the force of attraction between them be 1 dyne?

4. A long bar of cobalt steel with a mass of 3.5 g. when placed horizontally over a similar bar, both being equally magnetized, remains suspended at a distance of 0.8 cm. above it. What is the pole strength at each end of each bar?

5. A magnet with poles of 480 units separated by a distance of 5 cm. is placed in a uniform field with an intensity of 3,000 lines per square centimeter, so that the magnet is at right angles to the lines of force. What torque does the field exert on the magnet?

6. Find the direction and magnitude of the force on a unit pole placed at a point 5 cm. away from each of the two poles of a magnet 6 cm. long, if the magnet has a pole strength of 120 units.

7. A pole of $+15$ units (N.) is placed at a distance of 12 cm. from a pole of -60 units (S.). How far from the N-pole, on a line drawn through the two poles, will the combined field be zero?

8. At a place where the horizontal component of the earth's field is 0.20, a bar magnet 8 cm. long with a pole strength of 36 units is placed horizontally at right angles with the earth's field, with its N-pole pointing toward the west. Find the direction and intensity of the field at a point 10 cm. west of the N-pole of the magnet.

9. A bar magnet is 10 cm. long, and each pole has a strength of 25 unit poles. Find the intensity of the magnetic field at a point 20 cm. from the S-pole, the distance from the pole to be measured at right angles to the axis of the magnet.

10. What is the intensity of the magnetic field at a point 50 cm. from the center of a magnet which is 12 cm. long? The pole strength of the magnet is 160 units and the point lies on the perpendicular bisector of the line joining the poles.

11. A bar magnet 12 cm. long and with a pole strength of 48 units is placed horizontally at right angles to the earth's magnetic field with its north pole pointing west. Find the intensity and direction of the magnetic field at a point 20 cm. west of the N-pole. Take the horizontal component of the earth's magnetic field as 0.20 oersted.

12. A bar magnet has a pole strength of 400 units. The distance between the poles is 15 cm. Find the force on unit magnetic pole at distance of 30 cm. from each pole.

13. What torque is necessary to hold the axis of a magnet at an angle of 30 deg. with the magnetic meridian where the horizontal component of the earth's magnetic field is 0.20 oersted? The length of the magnet is 20 cm. and its pole strength is 40 units.

CHAPTER XXXIII

ELECTROSTATICS

393. Two Kinds of Electricity.—If a pith ball is hung from a support by a silk thread, and a rubber rod which has been electrified by stroking it with cat's fur is brought near it, the pith ball is at first attracted to the rod. If the pith ball is allowed to come in contact with the rod, it is then found that the ball is repelled by the rod. When a glass rod which has been rubbed with silk is brought near the same pith ball carrying the charge which it received from the rubber rod, the charged pith ball is attracted by the electrified glass rod. There are then two states of electrification, or, as is usually said, two kinds of electricity; that which appears on an ebonite rod when rubbed with cat's fur and that which appears on a glass rod rubbed with silk. These charges differ in one important respect. **Charges which are alike repel each other, and charges which are unlike attract each other.** That kind of electricity which appears on a glass rod that has been rubbed with silk is called **positive electricity** and the kind which appears on an ebonite rod rubbed with cat's fur is called **negative electricity.**

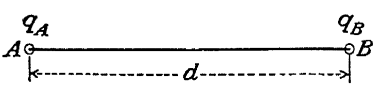


FIG. 346.—Law of electric force between charges.

394. Law of Electric Force. If two point charges of electricity of opposite kind are in the neighborhood of each other (Fig. 346), they will exert attractive forces on each other. If they are of the same kind, they will exert repulsive forces on each other. The force which one exerts on the other is determined by the distance between the charges and the magnitude of the charges. Experiments have shown that the force is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges. Thus,

$$F = K\frac{q_A q_B}{d^2}$$

where F is the force B exerts on A and also the force which A exerts on B, d is the distance between the charges, and q_A and q_B are the charges of electricity located at A and B, respectively. If these charges are alike, that is, both positive or both negative, the force will be a repulsion pushing the charges apart. If the charges are unlike, that is, one positive and the other negative, the force will be an attraction pulling the charges together.

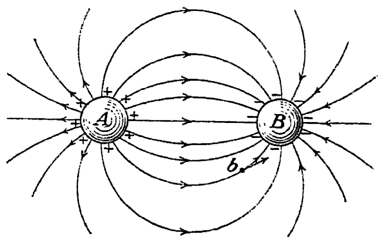


FIG. 347—Lines of force about unlike charges. These lines show the direction in which a free positive charge would move.

This law of force gives a convenient method of defining the unit of electrostatic charge. **Unit electrostatic charge is defined to be that charge which, when placed 1 cm. from an equal charge of like sign in a vacuum, will repel it with a force of 1 dyne; sometimes called a statcoulomb.**

395. Electrical Field of Force.—It has just been seen that if one charged body is brought into the neighborhood of another, there is a force of attraction or repulsion between them and that this force depends on the distance between the bodies, being greatest where the distances are least and least where the distances are greatest. However great the distance, there is always some force from a charged body though this force may be very small. The region surrounding one or more charged bodies is known as the **electrostatic field.** It is frequently represented by drawing lines which represent the direction in which the force on a positive charge acts at different points in the neighborhood of the bodies. Such lines are called **lines of force.** They show the direction in which a positive charge would move if it were placed in the field of force. Lines showing the field of force due to two unlike charges and to two like charges are shown in Figs. 347 and 348, respectively.

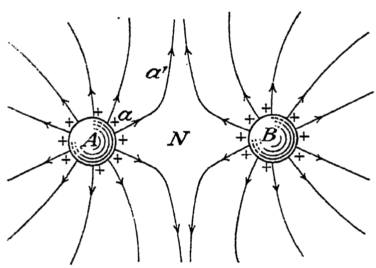


FIG. 348.—Lines of force between like charges.

The intensity of an electric field is determined by the force which unit positive charge experiences when placed in it. Sup-

pose that A (Fig. 349) is a charged sphere having an electrostatic field about it. The intensity of this field at P is the force which would be required to hold unit positive charge in position at P. The force on q units of electricity at P would be q times as great as the force on unit charge. If E denotes the intensity of the electric field at P, and q the number of unit charges located at P, then the force F on this charge is

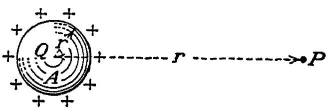


FIG. 349.—Definition of intensity of an electric field.

$$F = Eq \text{ dynes.}$$

An electrostatic field has an intensity of unity when it exerts a force of 1 dyne on unit charge at that point.

396. The Electron.—The preceding elementary experiments indicated that there are two kinds of electricity—positive and negative. These two kinds of electricity are the most important entities in nature, for every atom is built up of a certain number of positive units of electricity together with an equal number of units of negative electricity. It is a matter of first importance, that negative electricity is found in matter in very definite, small, indivisible units. It is possible to have any number of these units, but it is never possible to subdivide one. This is analogous to a monetary system in which the money is made up of a large number of small units, the smallest in America being a penny. But there can be no subdivision of the penny. This requires us to think of electricity as granular in its structure. The size of the units is extremely small. The grains are very fine but are all the same size and absolutely indivisible. This result has been arrived at by most careful experiments which have confirmed one another in every particular.

It is not possible to have electricity apart from matter, and this elementary charge is always associated with matter. It has been most carefully studied in the case of negative electricity, for we know much more about negative than about positive electricity. It is found that with this elementary charge of negative electricity is always associated a mass which is 1/1,845 of the mass of a hydrogen atom. This elementary charge of negative electricity has been named **the electron.** In whatever kind of atom it is found, it always has the same mass and the same charge. In this respect, it differs much from atoms, which differ

from substance to substance, existing in about one hundred different varieties. Means have been found for detaching electrons from a great number of different kinds of atoms and carefully measuring their mass and their charge, so that it becomes possible to say that one of the important constituents of all matter is the electron which is common to all kinds of atoms and is of unvarying charge and mass. The charge on the electron is extremely small, so small that the early theories of electricity regarded electricity as a continuous fluid.

397. Positive and Negative Charges.—The only way to charge a body negatively is to add some electrons to it, and the only way to charge it positively is to take away some electrons, leaving an excess of positive electricity. Each atom in the normal state contains enough positive electricity just to balance the negative electricity on its electrons. Positive electricity never leaves the atom, but negative electricity, *i.e.*, electrons, may be taken away from the atom or added to it. In the former case, the atom becomes charged positively and, in the latter case, it becomes charged negatively. To charge a body positively, then, is to take away some of its electrons, and to charge it negatively is to give to it some additional electrons. The negative electricity is as mobile as the electrons. The positive electricity is as mobile as the atoms. The transfer of positive electricity is always associated with the transfer of some kind of matter. When the rubber rod was charged negatively by rubbing with cat's fur, some electrons passed from the cat's fur to the rubber rod, leaving the cat's fur charged positively and the rubber charged negatively. On the other hand, when the glass rod was charged positively by rubbing with silk, some electrons passed from the glass to the silk, leaving the glass rod charged positively and the silk charged negatively.

In the normal condition, the amount of positive electricity in the atom is just equal to the amount of negative electricity on all its electrons. One or more of these electrons may be detached from the atom, leaving it with an excess of one or more positive charges of electricity. In such a case, the residue which is left after detaching these electrons is what is called a **positively charged ion.** On the other hand, an atom may gain one or more electrons in excess of its quota. It has on it then one or more negative charges and becomes a **negatively**

charged ion. For example, when hydrochloric acid dissociates in solution to form hydrogen ions and chlorine ions, the chlorine takes one more electron than its normal quota, thus giving it one negative charge and making it a negatively charged ion. Since the molecule of hydrochloric acid is originally neutral, the hydrogen is left with one less electron than its quota and is thus charged positively and becomes a positive ion.

398. Conductors and Insulators.—The electrons are more or less loosely bound to their parent atoms. In some atoms the forces holding the electrons to the atoms are not very large, and some of the electrons may be detached temporarily from the atoms and wander about in the vacant space between the atoms. Thus, in copper and silver some of the electrons become detached from the atoms and are free to wander about for longer or shorter times in the interstices between the atoms. Under the action of an electric force, these electrons migrate through the metal. Such substances in which there are free electrons which can be made to migrate through the substance under the action of an impressed electric force are called **conductors.** To this class of bodies belong the metals.

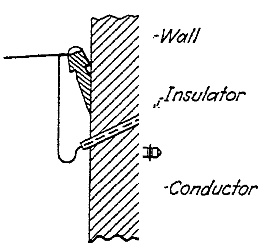


FIG. 350.—Use of conductors and insulators.

If the electrons are more rigidly bound to the atom so that they do not become free except under the action of very large forces, no free electrons will be found in the vacant spaces between the atoms. If an electric force is applied to such a substance, it cannot cause the electrons to migrate through the substance, and there is no flow of electrons from one part of the substance to the other. If an excess number of electrons be placed on one part of such a substance, they will remain there without wandering to other parts of the body. The most that an impressed electric force can do in such a case is to cause a limited displacement of the electrons within the atom without causing the electrons to migrate from atom to atom. Such substances in which the electrons are rigidly bound to the atom and which do not, therefore, have the power to transfer electrons through themselves are called **nonconductors** or **insulators.** To this class of substances belong mica, porcelain, quartz, glass, wood, ebonite,

etc. Figure 350 is an illustration of the use of conductors and insulators.

399. Electrostatic Induction.—If an uncharged conductor B (Fig. 351) is brought into the field of force of a conductor A, charged positively, that is, a conductor in which there is a deficit of electrons, the attractive forces due to the excess positive charges in A will cause the electrons in B to be pulled toward C,

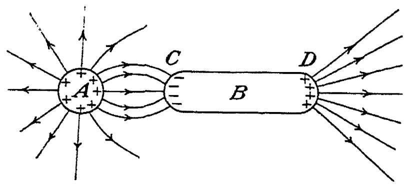


FIG. 351.—Electrostatic induction. Opposite charges are produced at C and D by induction.

leaving the farther end of B with a deficit of electrons and, therefore, charged positively, while that end of B nearest A is charged negatively. Since B was originally uncharged, that is, contained as much positive as negative electricity, this displacement of electrons will still leave it with as much positive as negative electricity on it, no charges of either kind having been added to

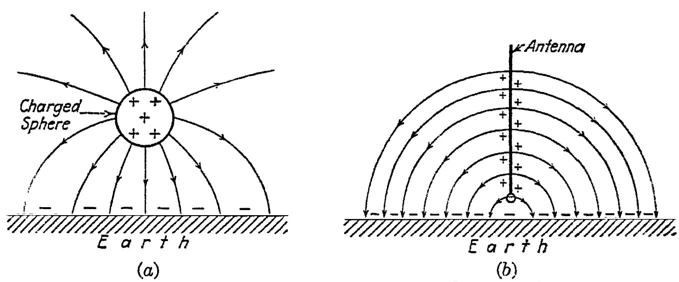


FIG. 352.—The earth is charged negatively by induction, (a) by a positively charged sphere; (b) by a positively charged antenna.

it. Hence, the positive charge on one end is just equal to the negative charge on the other end.

If the conductor B is connected to the earth by means of a wire, enough electrons come from the earth to the conductor B to neutralize the positive charge at B. Meanwhile the electrons on the other end C are held fast by the attractive forces due to the positive charges on A. If the connection to the earth is now broken, and B then removed from the presence of A, it

will, of course, have an excess of electrons and so be charged negatively. If B is now connected again to the earth, this excess of electrons will flow to the earth, leaving B uncharged. A charged sphere in the neighborhood of the surface of the earth (Fig. 352a) induces a charge on the surface of the earth below it. A charged antenna in the neighborhood of the surface of the earth (Fig. 352b) has a similar effect.

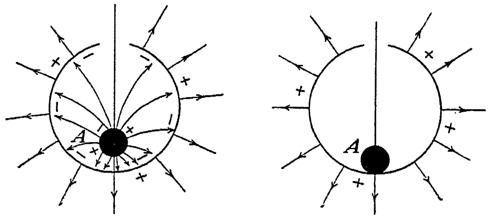


FIG. 353.—Induction in a hollow sphere. Charge inside the cavity is equal to the charge on A.

Another illustration of electrostatic induction is seen in Fig. 353. If a metal sphere, charged positively, is introduced into an uncharged hollow sphere which is insulated, some of the electrons of the hollow sphere are drawn to its inner surface, leaving the outer surface charged positively. If now the metal sphere A is placed in contact with the inner surface of the hollow sphere, the electrons from the inner surface of the hollow sphere go over to the sphere A and just compensate the deficit of electrons which caused its positive charge. This leaves both the sphere A and the inner surface of the hollow sphere without a charge. The outer surface of the hollow sphere has too few electrons and is, therefore, charged positively. If the outer surface is now connected to the earth, it will gain a sufficient number of electrons to compensate for its deficit and it will be left without charge.

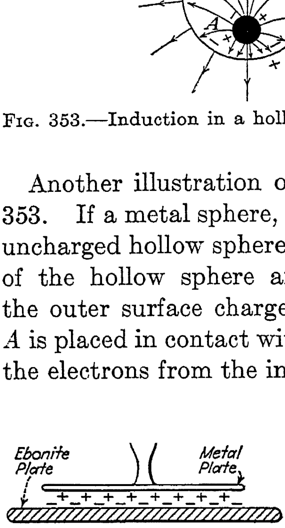


FIG. 354.—An electrophorus. A charge is induced on the metal plate by the charge on the ebonite plate.

If the ebonite plate (Fig. 354) is charged by rubbing it with a suitable substance, a charge of electricity may be induced in the metal plate above it. Proper precautions must be taken to ground the plate correctly.

400. The Electroscope.—One form of gold-leaf electroscope consists of a metal sphere (Fig. 355) which is fastened by means

of a metal rod to two thin gold leaves. The metal rod passes through a sulphur plug by which it is insulated from the glass jar in which the leaves are mounted. When the brass sphere and the leaves are uncharged, the leaves collapse and hang together. When some electrons are removed from the sphere, leaving the sphere and the leaves charged positively, the leaves diverge because of the repulsion between the like positive charges on them. If part of the positive charge is neutralized by adding some electrons, the gold leaves partly collapse. If all of the positive charge is neutralized, the leaves collapse completely. If an excess of electrons is now added to the sphere, electrons will move to the leaves charging them with negative electricity. This causes the leaves again to diverge because of the repulsive forces between the like charges.

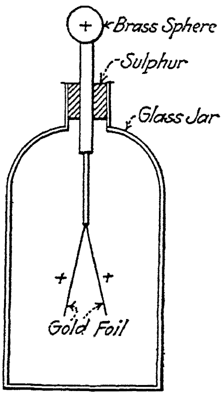


FIG. 355.—Gold-leaf electroscope. The leaves are repelled when charged but collapse when the charge escapes.

401. Lightning.—Lightning results from a case of electrostatic induction on a very large scale. There is present in the atmosphere a number of electrons which have become freed from their parent atoms in various ways. These electrons furnish nuclei about which the water vapor in the air can condense. As this condensation progresses and the cloud accumulates, there is collected together not only a large quantity of water but also a large quantity of electricity. When a cloud thus charged comes near the surface of the earth, an electric charge of opposite sign is induced in the surface of the earth due to the repulsion of electrons from the surface, leaving the earth charged positively. The positive charge on the earth and the negative charge on the cloud try to rush together through the air. When this disruptive discharge occurs, there is a flash of lightning with accompanying thunder. This is the simplest possible case. Discharges from cloud to cloud also occur, and in some cases the cloud may be charged positively instead of negatively. With high-voltage transformers or other sources of high voltage it is possible to produce artificial lightning (Figs. 356*a*, *b*, and *c*).

402. Surface Density.—If a sphere be charged with Q units of electricity, the electricity spreads over the entire surface of the sphere. The amount of electricity per unit area in such a case is known as the surface density. In the case of a sphere, the surface density is uniform and given by the expression,

$$\sigma = \frac{Q}{4\pi R^2},$$

where σ = the surface density = the charge per unit area.
R = the radius of the sphere.
Q = the total charge on the sphere.

If the curvature of the surface varies from point to point, the surface density is not uniform over the entire surface. Where the surface has the greatest curvature, the surface density is the greatest.

When a conductor terminates in a sharp point, the surface density at the point is so great that the molecules of air in the neighborhood of the point

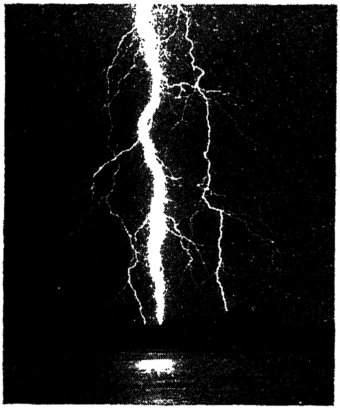


FIG. 356a.

FIG. 356b.

FIG. 356a.—Lightning discharge in the atmosphere. (*Underwood and Underwood.*)

FIG. 356b.—Artificial lightning produced in the high-voltage laboratory of the General Electric Company. (*Underwood and Underwood.*)

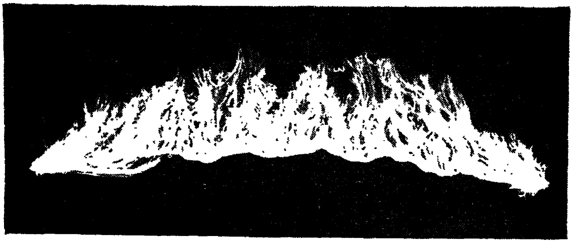


FIG. 356c.—A 60-cycle arc between points 25 ft. apart; 2,000,000 volts were required to arc this distance. Current in the arc was about 1.5 amp. and the duration of the arc about 2 sec. (*Courtesy J. A. Carroll, Stanford University.*)

become charged with electricity. Since like charges repel each other, the charged molecules of air are repelled from the point. These molecules carry away the charges from the point, and the body to which the point is attached is discharged.

If a positively charged insulated sphere L (Fig. 357) is brought near a body M on which there is a pointed conductor, the point becomes charged with negative electricity by induction. This electricity is conducted away by a stream of air which flows from M to L, and in the end the body M is charged with positive electricity.

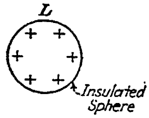


Fig. 357.—Discharge of electricity from points.

The fact that charged points allow the electricity to escape from them is used in electrostatic machines (Fig. 358), where rows of metallic points are used to conduct electricity from moving to fixed parts of the machine. This fact is also important in the design of lightning rods.

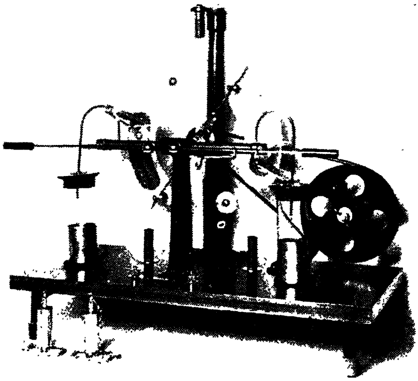


Fig. 358.—An electrostatic machine. (*Courtesy Central Scientific Company.*)

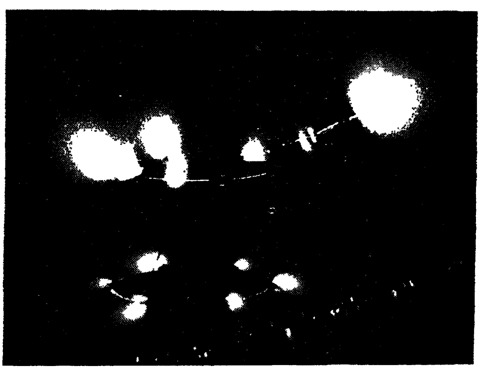


Fig. 359.—Corona discharge, near arcing voltage. (*Courtesy General Electric Company.*)

The pointed conductors on the rod bring about a silent and gradual discharge from the rod to the clouds. The escape of electricity from these points prevents the accumulation of enough electricity on the building on which the lightning rod is mounted to result in a dangerous disruptive discharge.

When voltages are very high there is a corona discharge (Fig. 359) around a conductor.

403. Difference of Potential.—If a sphere (Fig. 360) has had removed from it a given number of electrons so that it is charged positively, and it is desired to carry away from it more negative electricity, the positive charge on the sphere attracts the negative charge which is being removed so that work must be performed to remove the negative electricity from the positive. If, on the other hand, there is on the sphere an excess of electrons so that it is charged negatively and it is desired to bring up additional electrons, the negative electricity on the sphere repels the negative charge on the electrons, making it necessary to perform work to bring up the additional charge. Since the force which one charge exerts on another increases as either of the charges is increased, the amount of work necessary to bring up an additional charge depends on the amount of charge already present on the sphere. As more and more charge is added to the sphere, it requires more and more work to bring up additional charges of electricity of the same kind. **When it requires work to carry a charge from one body to another, or when work is done in allowing a charge to pass from one body to another, these bodies are said to differ in potential,** and the work is used as a measure of this difference of potential. If work is obtained when a unit of negative electricity is carried from B to A, then the potential of A is said to be greater than that of B. If, on the other hand, work must be done in carrying a unit of negative electricity from B to A, then A is said to have a potential which is less than that of B.

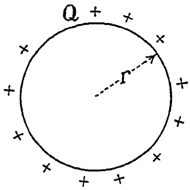


FIG. 360.—Potential at the surface of a sphere equals work necessary to bring unit positive charge from infinity to the sphere.

404. Electrostatic Unit of Difference of Potential.—**When one erg of work is done in moving one electrostatic unit of positive charge from one point to the other, the difference of potential between the two points is one electrostatic unit of difference of potential. It is sometimes called a statvolt.**

405. Concentric Spheres.—In Fig. 361 are represented two concentric spheres A and B. The outer sphere is connected to earth and the inner sphere is charged with positive electricity. The outer sphere is then charged with an equal amount of negative electricity. Electrons will be repelled by B and attracted

by A, and hence go from B to A with the performance of work. The potential of A is, therefore, higher than that of B. The difference of potential between these spheres is measured by the work required to carry unit charge from one to the other. If the quantity of electricity on A and B is increased, the difference of potential between these spheres will be increased, because the potential is proportional to the charge. If the radii of the spheres become more and more nearly equal, the difference in potential becomes less and less for a given charge on the spheres. By inserting some medium other than air between the spheres, the difference of potential between them may also be changed. Hence, the difference in potential between the spheres for a given charge of electricity depends on the distance between their surfaces, the area of the spheres, and the medium which fills the space between them.

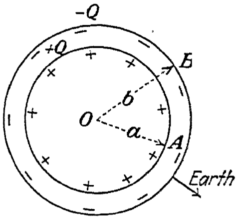


FIG. 361.—Difference of potential between concentric spheres.

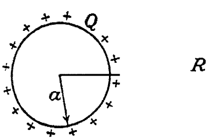


FIG. 362.—Potential at a point due to a charged sphere. $V = Q/R$ = work to bring a unit positive charge from infinity to P.

406. Potential at a Point.—In the vicinity of a small charged body, there exists an electric field whose direction is the direction of the radii of the sphere with the small body at its center. The intensity of this field at a point P (Fig. 362) is given by the equation

$$F = \frac{Q}{R^2},$$

where Q = the charge on the body.
R = the distance from the body to P.
F = the force on unit charge at P.

The work necessary to move unit charge from the point P' to some other point P can be calculated, but since the force varies with the distance this computation requires the use of the calculus. The work to bring unit charge from any point at zero potential—practically the surface of the earth—to the point P, which is at a distance R from the charge Q, can be shown to be

$$W = \frac{Q}{R}.$$

This work is defined as the potential at the point P due to the charge Q; **that is, the potential at a point in an electric field is the work necessary to bring unit charge from an infinite distance to that point.** In the c.g.s. electrostatic system of units, this work is measured in ergs per electrostatic unit of charge (Appendix E-8).

Example.—A small charged sphere has on it 10 statcoulombs of electricity. Find the potential at a point 20 cm. from this charged sphere.

$$\text{Potential at a point} = \frac{\text{charge}}{\text{distance}}.$$

$$V = \frac{Q}{R} = \frac{10}{20} = 0.5 \text{ statvolts}.$$

407. Energy to Charge a Conductor.—By definition, the potential V of a conductor is the work required to carry unit charge from a point where the potential is zero to the conductor whose potential is V. When q units of electricity are carried to the conductor from the place where the potential is zero,

$$\text{Work} = V \times q.$$

If initially the conductor is uncharged, its potential is also zero, and the work to carry unit charge to it is zero; *i.e.*,

$$\text{Work} = 0 \times q.$$

As the charge on the conductor increases, its potential is gradually raised from 0 to the final value V. The average value of the potential of the conductor during the charging is

$$\text{Average value of potential} = \frac{0 + V}{2} = \frac{V}{2}.$$

If Q units of electricity are on the conductor when its potential has become V, the work to charge it to that potential is

$$\begin{aligned} \text{Work} &= Q \times (\text{average potential}) \\ &= \frac{QV}{2}. \end{aligned}$$

where Q = the final charge on the conductor.
V = the final potential of the conductor.
(Appendix E-9.)

408. Atmospheric Electricity.—The electrical conditions over the surface of the earth play an important part in meteorological phenomena. On a clear day the surface of a level field, freely exposed to the sky, is negatively charged so that there is an electrical force tending to move a positively charged body in the atmosphere downward. The force on unit positive charge is known as the potential gradient. In good weather it is of the order of 100 volts per meter. The potential gradient at the surface of the

earth is continually varying. In addition to local variations there are well-defined annual and diurnal variations which differ at different parts of the surface of the earth. As the height above the earth's surface is increased, the potential gradient begins to diminish, and at a height of about 10 km. it becomes nearly independent of the height. This fact means that the lower atmosphere is positively charged with respect to the upper atmosphere. A careful study of the variations in the potential gradient in the atmosphere is being made by the Carnegie Institution of Washington.

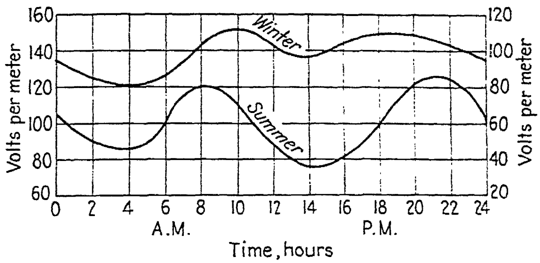


FIG. 363.—Variation of potential gradient in the atmosphere for different hours of the day.

The diurnal variations of the potential gradient for two seasons of the year are shown in Fig. 363.

The normal vertical field in the atmosphere tends to drive negatively charged bodies like electrons upward. The motion of these charged bodies constitutes an electric current which is a negative current from the earth to the atmosphere or a positive current in the opposite direction. The average current from the whole earth to the air amounts to about 1,000 amp. This current carries a positive charge to the earth or a negative charge away from

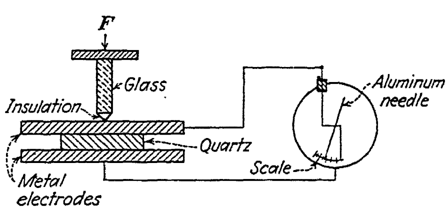


FIG. 364—Piezo-electric cell. Pressure on the crystal produces an electromotive force.

it. Since the earth maintains its negative charge in spite of this fact, there must be some compensating process which continually supplies a negative charge to the surface of the earth.

During thunderstorms, the potential gradient at the ground may be either positive or negative. In such cases, the vertical electrical field may exceed 10,000 volts per meter, and the current from the ground to the atmosphere or in the opposite direction is much greater than it is in fine weather. There are three ways in which a positive or negative charge may pass from the

atmosphere to the ground: (1) The rain or snow may be charged and carry a convection current. (2) A lightning discharge may pass between the cloud and the ground. (3) A continuous-conduction current due to the ionization of the atmosphere may pass between the earth and the neighboring air. In order that there be a flash of lightning, the electric force near a thunder-cloud must have a value which approaches 30,000 volts per centimeter. This is a very large force compared to the largest force observed near the surface of the ground in fine weather.

409. Piezo-electricity.—When certain crystals such as quartz or Rochelle salt are compressed, the opposite faces become charged with positive or negative electricity. If the crystal is elongated instead of compressed, the charges on the faces are reversed. Conversely when two opposite faces of such a crystal are charged oppositely, the crystal is compressed or elongated. If the opposite faces are alternately charged with positive and negative electricity, these faces oscillate with respect to each other. If such a crystal (Fig. 364) is suddenly deformed by applying an impulsive force F, the stresses due to the deformation of the crystal produce an instantaneous difference of potential between the faces of the cell. This difference of potential will be indicated on the electrometer. It may amount to several hundred volts. Piezo-electric cells are used to regulate the frequency of high-frequency electric circuits. When the piezo-electric cell has the same natural frequency as that of the electric circuit, a condition of resonance is produced by which the frequency of the electric circuit is controlled.

410. The Electric Current.—In a metal there is a large number of more or less free electrons which are capable of migrating through the metal under the action of an impressed electric force. When such an electric force is applied to the metal and a stream of these electrons migrates from one part of the metal to another, there is said to be **an electric current** in the conductor. This flow of electricity through a conductor is analogous to the flow of water through a pipe. A difference of pressure at the two ends of the pipe is necessary in order to maintain a flow of water, and, in like manner, a difference of electrical pressure or potential is necessary in order to maintain a flow of electricity in a conductor, that is, an electric current.

Before the discovery of the electron, the convention was adopted of referring to an electric current as a flow of electricity from the positive to the negative end of a conductor, or from the higher to the lower potential. By definition, then, the **conventional current** is said to flow from the positive terminal of the generator through the external circuit to the negative terminal of the source. This designation is usually retained, even though it is inexact. In liquid and gaseous conductors,

positively charged particles actually travel in the direction assigned to the conventional current, but electrons and negatively charged particles travel in the opposite direction. In metallic conductors, on the other hand, no positive particles are free to move, and the entire transfer of charge is due to the motion of electrons in a direction opposite to that of the conventional current. Once this is understood, there need be no difficulty in remembering that a flow of electrons in a wire is described by speaking of a conventional electric current with precisely the opposite direction. Since like charges repel, electrons flow away from the negative terminal of a source. The direction designated as the direction of flow of current is that which would be taken by positive charges if they were free to move.

The migration of electrons through wires occurs with great ease, and the very smallest electric pressures are sufficient to set the electrons in motion. On the other hand, the atoms in a metal are quite firmly fixed in position, with considerable vacant space between them. Whether the electrons pass directly from atom to atom or through the vacant spaces between the atoms, we do not know with certainty. When they hit the atoms, they set the latter into vibration, thus increasing the average atomic motion and thereby increasing the temperature. It is known very definitely that the charges which move in a wire are electrons, but no exact description of the motion can be given. The forces causing the motion may be due to chemical action in a battery, or to other types of forces which will be studied in connection with the dynamo.

Problems

1. Two concentrated equal charges repel each other with a force of 320 dynes at a distance of 4 cm. What is the amount of each charge?

2. A small sphere is charged with 18 statcoulombs of negative electricity. When another similar sphere is brought to a distance of 24 cm. from the first sphere, the second sphere is attracted with a force of 6 dynes. Find the charge on the second sphere.

3. Two small equally charged spheres, each with a mass of 0.15 g., are suspended from the same point by silk fibers 90 cm. long, and the repulsion between them keeps them 6 cm. apart. What is the charge on each sphere?

4. What is the potential energy of a charge of 40 statcoulombs which is at a distance of 25 cm. from a point charge of 25 statcoulombs?

5. Find the potential at a point due to three charges of 20, 30, and 40 statcoulombs which are at a distance of 10, 15, and 25 cm. from the given point.

6. A point charge of 40 statcoulombs is placed at a distance of 20 cm. from a point charge of 30 statcoulombs. What is the potential energy of the system?

7. Two points differ in potential by 750 statvolts. How many ergs of work are required to carry 20 statcoulombs from the point of lower to the point of higher potential?

8. Three charges of +5, −5, and +5 statcoulombs are placed in the same straight line so that they are 10 cm. apart. What force acts on each charge owing to the other two charges?

CHAPTER XXXIV

MAGNETIC FIELDS AROUND ELECTRIC CURRENTS

411. Magnetic Effect of an Electric Current.—In 1819, the Danish physicist, Hans Christian Oersted, discovered that when a wire which is carrying an electric current is brought near a magnetic needle, as in Fig. 365*a*, there is a deflection of the needle such that it tends to set itself at right angles to the wire. Another way to describe this phenomenon is to say that an electric current flowing in a wire or the migration of a stream of electrons through a wire causes a magnetic field around the wire. Later, Rowland

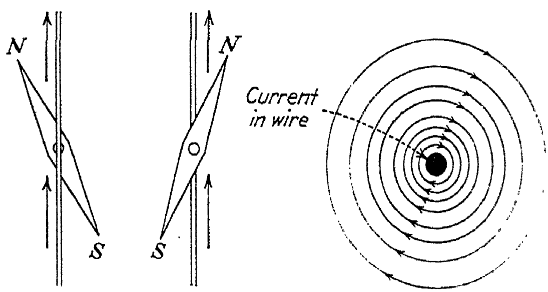


FIG. 365*a*　　FIG. 365*b*.

FIG. 365*a*.—The deflection of a magnetic needle by an electric current. Deflection above and below the wire in opposite directions.

FIG. 365*b*.—Magnetic lines of force around a conductor. Magnetic field decreases as the distance from the wire increases.

found that an electric charge in motion is surrounded by a magnetic field. Hence, we come here upon a phenomenon of major importance, *viz.*, an electric charge in motion produces a magnetic field around it. When the electric charge ceases to move, the magnetic field disappears. This phenomenon is one of the most fundamental relationships yet discovered between electricity and magnetism.

To understand the relation of this magnetic field to the current which produces it, we may draw magnetic lines of force which by their direction and distribution will represent the magnetic field produced by the current. Figure 365*b* represents the magnetic

field around a long straight wire carrying a current of electricity. The wire is assumed to be perpendicular to the plane of the paper. In Fig. 366, the wire is in the plane of the paper. If a compass needle is held above or below the wire, it points in a direction at right angles to the wire. When the direction of the current in the wire is reversed the direction of the deflection of the magnetic needle is also reversed. The intensity of the magnetic

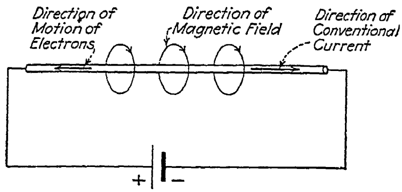


FIG. 366.—Magnetic lines of force around a straight wire. The direction of motion of the electrons determines the direction of the magnetic field.

field varies inversely as the distance from the wire in the case of a long straight wire. For a circular loop Fig. 367 shows the way the lines of force are distributed.

It is convenient to have a rule by which to remember the relation between the direction of the current and the direction of the magnetic field which arises from it. The following handrule serves this purpose. Grasp the wire in which the current is flowing with the right hand, with the thumb lying along the wire and pointing in the direction in which the conventional current is flowing; then, the fingers will point around the wire in the same direction as the magnetic lines of force. Care must be taken in applying this rule to remember that the conventional or positive current [illegible]s supposed to flow in the opposite direction to that in which [illegible] electron current actually flows. Hence, if the electronic c[illegible]nt is considered, the left hand should be used instead of [illegible]ght hand (Appendix E-10).

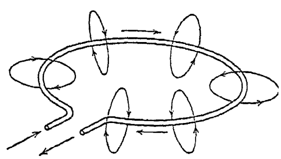


FIG. 367.—Magnetic lines of force around a circular loop.

412. Definition of Unit [illegible]nt.—In order to compare electric currents it is necessary t[illegible] an agreement with respect to the units in terms of which [illegible]are to be measured. There are in use two systems of u[illegible]; [illegible]e practical system and the absolute system. The pra[illegible]it of current is called the **ampere**

and the absolute unit of current is called the **abampere.** To definite a unit of current it is necessary to select some effect which is produced by the electric current, and define the magnitude of the current in terms of this effect. For example, it will be seen later that an electric current produces a heating effect. This heating effect might be used as a measure of the magnitude of the electric current and a unit current defined from this heating effect. In like manner, it will be seen that a current of electricity deposits a metal from an electrolytic solution. The amount of the metal deposited depends on the magnitude of the current and the time it flows. This effect may also be used as a means of measuring an electric current, and hence a unit electric current can be defined from the amount of metal deposited by a current of electricity in a given time. Neither of these effects furnishes the best method of defining a unit electric current.

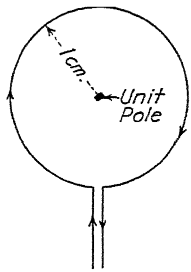


FIG. 368.—Magnetic field at the center of a circle of unit radius. The magnetic field is perpendicular to the plane of the circle.

In the preceding section it was seen that an electric current produces a magnetic field around it and that the intensity of this magnetic field depends on the magnitude of the electric current. It is from the magnetic field produced by an electric current that the absolute unit of electric current is defined.

Consider a current flowing in a loop of wire bent in the form of a circle (Fig. 368). The magnetic field produced by this current is perpendicular to the plane of the circular loop. The intensity of the magnetic field at the center of the loop is proportional to the current in the loop and is inversely proportional to the radius of the loop. If unit magnetic pole is placed at the center of the loop which is carrying the current, it will be acted on by a force H, perpendicular to the plane of the loop. The magnitude of this force is given by the equation,

$$H = \frac{2\pi I}{a} \text{ dynes per unit pole or oersteds,}$$

where I is the current in the loop, and a is the radius of the loop.

Now, suppose the circular loop has a radius of 1 cm. and the current is such that the unit magnetic pole at the center of the loop has a force of 2π dynes perpendicular to the plane of the loop acting on it. Then,

$$H = 2\pi = \frac{2\pi I}{1} \text{ dynes per unit pole or oersteds}$$

and

$$I = 1 \text{ abamp.}$$

An abampere or an absolute unit of current is that current which when flowing in a wire bent in the form of a circle with a radius of 1 cm. will produce a force of 2π dynes on unit pole at the center of the circle. Another form of this definition is: An absolute unit of current is that current which when flowing in a wire bent in the form of an arc of a circle 1 cm. in length and 1 cm. in radius will produce a magnetic field which exerts a force

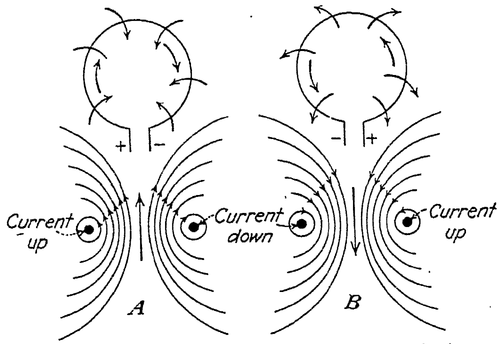


FIG. 369.—Magnetic field at the center of a circular loop of wire. The direction of the field reverses with the reversal of the current.

of 1 dyne on unit pole placed at the center of the arc. The distribution of the lines of force about a circular loop of wire is represented in Fig. 369.

413. Ampere.—The practical unit of current, called the *ampere*, is defined as one-tenth of the absolute unit of current. Hence, 10 amp. = 1 abamp. To calculate the number of absolute units of current divide the number of amperes by 10.

414. Unit of Quantity.—If water is flowing through a pipe, the quantity of water may be measured in gallons, but the current must be expressed in gallons per second. The current is a
of the rate of flow of the water, but the quantity is the
ount of water transferred through the pipe.
ar manner, the quantity of electricity must be dis-
from the rate of flow of the electricity. In practical
of flow of electricity is expressed in amperes. The

corresponding unit of quantity of electricity is called a **coulomb.** The coulomb is defined as the quantity of electricity which is transferred by a current of 1 amp. in 1 sec.

In absolute units, the rate of flow of electricity is expressed or measured in *abamperes*. The corresponding unit of quantity is an **abcoulomb. An abcoulomb is defined to be the quantity of electricity which is transferred by a current of 1 abamp. in 1 sec.** Ten *coulombs* of electricity are equal to 1 *abcoulomb*, just as 10 *amp.* are equal to 1 *abamp.*

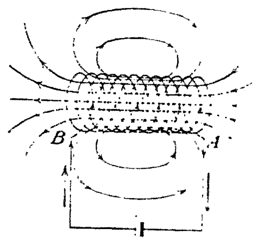


FIG. 370.—Magnetic lines of force in a solenoid. Equivalent to a bar magnet.

415. Magnetic Field around a Coil.—If a wire is bent in a spiral so that the successive turns have the same diameter, this spiral or helix is called a **solenoid** (Fig. 370). The easiest way to make a solenoid is to wind the wire spirally around a cylinder. If a current is maintained in this coil, lines of force link the turns of wire as indicated in Fig. 370. Nearly all the lines of force will thread the entire coil and then return outside the coil to the other end of the solenoid. If a compass needle is brought near one end of such a closely wound solenoid when it is carrying a current, the coil will be seen to behave like a bar magnet. When the current in the solenoid is reversed, the poles of the magnet are reversed, and the direction in which the magnetic needle is deflected is reversed.

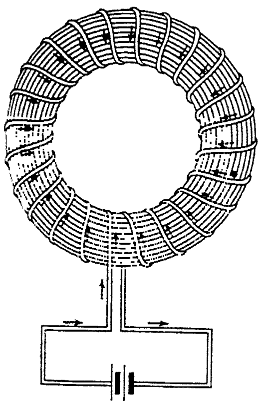


FIG. 371.—Ring solenoid. Magnetic lines of force are entirely within the solenoid.

The magnetic intensity in the in of a solenoid whose length is comparison with its cross section found from the following equation:

$$H = \frac{4\pi nI}{10} \text{ oersteds,}$$

where n = the number of turns per cm. of length of the solenoid.
I = the current in amperes.

This equation also gives the magnetic field inside of a solenoid bent in the form of a ring. The magnetic field in the case of a

ring solenoid (Fig. 371) is entirely confined to the closed space inside the turns of wire forming the ring.

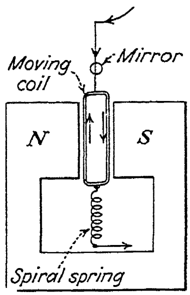


FIG. 372.—Moving-coil galvanometer. The coil behaves like a suspended magnet.

Example.—A solenoid which is 50 cm. long has 1,500 turns of wire on it. What is the magnetic field in the inside of it, when it is carrying a current of 5 amp.?

$$H \quad \frac{4\pi nI}{10}.$$

$$\frac{1,500}{50} \quad 30 \text{ turns per centimeter.}$$

$$H = \frac{4\pi \times 30 \times 5}{10} = 188 \text{ oersteds.}$$

416. Moving-coil Galvanometer.—When a current flows in a coil of wire, the coil will exhibit most of the properties of a magnetic needle. If such a coil is suspended between the poles of a permanent magnet it tends to turn, so that the lines of force due to the coil and those due to the permanent magnet will lie in the same direction. This gives a means of measuring a current by observing the deflection which it produces when it is flowing in a coil of wire between the poles of a

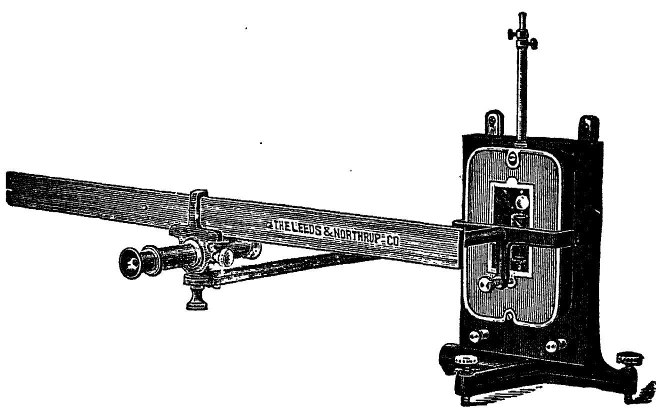


FIG. 373.—D'Arsonval galvanometer. (*Courtesy Leeds and Northrup Company.*)

magnet. The magnet NS (Fig. 372) is usually made in the shape of a horseshoe so that it will be as strong as possible. The coil is wound on a very light frame and is suspended by means of a very fine phosphor-bronze wire between the poles of the magnet. The bottom of the coil is connected to a binding post by means of a

fine wire or ribbon wound in the form of a spiral. The current to be measured enters the coil through the wire by which it is suspended and leaves through the wire or ribbon spiral at the bottom of the coil. Of course, the direction of the current may be reversed. When there is no current in the coil of the galvanometer, the plane of the coil lies in the plane of the poles N and S of the magnet. When a current is produced in the coil, it acts like a small magnet, with its axis perpendicular to the plane of the figure. The coil tries to turn itself, so that its north pole points in the direction of the south pole of the permanent magnet and its south pole in the direction of the north pole of the permanent magnet. The coil turns until the restoring torque due to the torsion in the suspension is as great as the torque produced by the current in the coil. Then the coil comes to rest, and the amount which it has turned is a measure of the current in the coil. A mirror is fastened to the coil so that its deflection can be read with accuracy. The deflection is nearly proportional to the current in the coil.

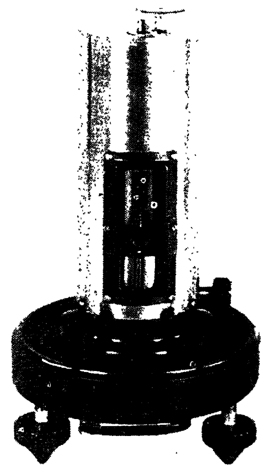
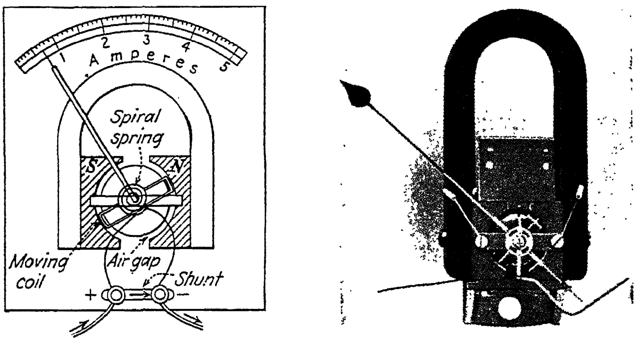


FIG. 374.—A sensitive d'Arsonval galvanometer. (*Courtesy Leeds and Northrup Company.*)

417. Ammeter.—The galvanometer just described is used to measure small currents. An important modification of it is used to measure large currents. This modification is called an **ammeter.** The form in most common use (Fig. 375) consists of a coil of fine copper wire wound on a light frame. This coil is like the coil of a moving-coil galvanometer. Instead of being suspended from a fine wire, it is mounted on jeweled bearings between the poles of a strong horseshoe magnet. When a current flows, the coil rotates between the poles of the magnet. Two spiral springs, one at the top and the other at the bottom, carry the current into and out of the coil. They serve also to keep the coil in position between the poles of the magnet. To the coil is fastened a light pointer which moves over a scale and indicates the current in the ammeter.

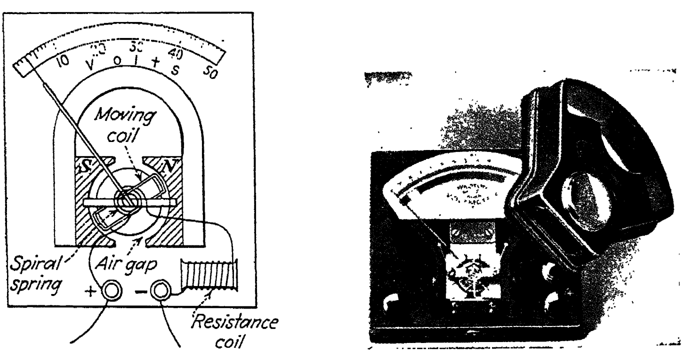
Since it is desired to measure large currents with such an instrument, it is not possible to allow all the current to flow through the coil of fine wire. For this reason, there is placed between the binding posts a short thick piece of wire through



(a) (b)

FIG. 375.—An ammeter consisting essentially of a moving coil in a magnetic field.

which much of this current flows without flowing through the movable coil. This heavy wire or strip of metal is called a **shunt.** Since the current flowing in the movable coil is always a constant fraction of the whole current coming up to the binding post, the



(a) (b)

FIG. 376.—A voltmeter. The current in the moving coil indicates the voltage across the terminals of the instrument.

pointer which is fastened to the movable coil can be made to indicate the entire current coming up to the instrument.

418. Voltmeter.—The voltmeter, which is used to measure electrical pressures, is very similar in construction to the ammeter;

it consists of a moving coil which is mounted between the poles of a horseshoe magnet (Fig. 376). Under the action of the current flowing in the coil, the coil turns as in the ammeter and this rotation is indicated by the movement of a pointer. The voltmeter differs from the ammeter in one important particular. **The ammeter has a low resistance and the voltmeter a high resistance.** The ammeter is placed in series with the resistance in which the current is to be determined. Whatever current goes through the resistance also goes through the ammeter. On the other hand, the voltmeter is placed in parallel with the resistance. It is desired that the fraction of the current which the voltmeter takes be as small as possible, so that the voltage indicated by the voltmeter be the true voltage across the resistance when the voltmeter is not in the circuit. For this reason, the voltmeter has a high resistance. To make its resistance large, a coil of high resistance is placed in series with the moving coil.

Problems

1. A certain current in a circular loop of wire with a diameter of 11 cm. produces a field strength of 8 oersteds at the center of the coil. How much current is flowing: (*a*) in amperes, (*b*) in absolute electromagnetic units of current?

2. A coil with a total of 600 turns is wound on a cylinder 80 cm. long and 1 cm. in diameter. What is the magnetic field intensity at the center of the coil when it is carrying a current of 0.020 amp.?

3. In two concentric solenoids the currents flow in opposite directions. The inner one has 300 turns per centimeter, and the outer one 160 turns per centimeter. What current in the outer one will be necessary in order to have the field at the center zero when the inner coil is carrying 4 amp.?

4. Find the force on a pole of 120 units placed at the center of a circular loop with a radius of 8.8 cm. when a current of 5 amp. is flowing through the loop.

5. What is the intensity of the magnetic field in a coil of wire wound in the form of a ring solenoid having 200 turns to the centimeter, when a current of 4.2 amp. is flowing through the coil?

6. How many turns of wire must be placed on a solenoid 150 cm. long, in order that a current of 4 amp. may produce a magnetic field of 60 oersteds at its center?

7. A cylindrical coil of 1,800 turns, 15 cm. in diameter and 180 cm. long, is placed with its axis parallel to the lines of the earth's field. If the latter has an intensity of 0.20 oersted, what current must flow through the coil in order to make the magnetic field at its center zero?

8. A long straight wire carries a current of 10 amp. A magnetic pole of strength 15 is carried around the wire five times. How much work in ergs is done? (See Appendix 10.)

CHAPTER XXXV

ELECTRICAL RESISTANCE AND OHM'S LAW

419. Electrical Pressure.—In order to cause water to flow in a pipe, it is necessary to have a difference of pressure applied to the two ends of the pipe. This pressure may be obtained by means of water in a standpipe. The weight of the water in the standpipe develops the pressure which causes the water to flow. In order to cause a flow of electricity through a wire, it is necessary to have a difference of electrical pressure between the ends of the wire. This difference of electrical pressure bears the same relation to the flow of electricity that the difference in the hydrostatic pressure bears to the flow of water. In order to obtain a difference in electrical pressure, a number of methods have been adopted. The oldest and most familiar for small currents is the use of a battery. The technical term for a difference of electrical pressure is **difference of potential.** A dynamo, battery, or other source capable of maintaining a difference of potential is said to have an electromotive force.

420. Volt and Abvolt.—The unit in which the difference of potential or electromotive force is measured in the practical system of units is called the **volt.** In the absolute electromagnetic system of units, it is called the **abvolt.** In Sec. 404, it was seen that the difference of potential between two points in the electrostatic system of units is the work required to carry 1 e.s.u. of positive charge from one point to another point against the electric force, and that when 1 erg of work is required to carry 1 e.s.u. of positive electricity from one point to another against the electric forces, the difference of potential between these points is said to be 1 e.s.u. of difference of potential.

The *electromagnetic unit of difference of potential* is defined in a similar way except that in this case 1 e.m.u. of charge must be carried from one point to the other, instead of 1 e.s.u. of charge. The **abvolt** is the absolute electromagnetic unit of difference of potential. It is defined as follows: **An electromagnetic unit of difference of potential exists between two points when it requires**

the expenditure of 1 erg of work to carry 1 e.m.u. of positive electricity from one point to the other against the electric force. This is a small unit of difference of potential.

The volt is defined as 10^8 abvolts. It is the practical unit of difference of potential or electromotive force. The most convenient and accurate standard in terms of which to specify difference of potential or electromotive force is a Weston standard cell which has a difference of potential of 1.0183 volts between its terminals (see Sec. 470).

421. Electric Currents and Water Currents.—There is a definite analogy between the flow of water in pipes and the flow of electricity. Consider Fig. 377, in which two tall open vessels filled with water are connected with a pipe. A paddle wheel which is turned by a falling weight forces water from one of these vessels to the other until the back pressure becomes so great that the paddle wheel is unable to lift the water farther. In this case, the difference in pressure on the two sides of the paddle wheel just balances the tendency of the weight to produce rotation. This difference in level does not depend on the size or shape of the vessel but on the magnitude of the weight hanging from the wheel. This difference in level between the water in the vessels corresponds to the difference in potential or electrical pressure in the battery.

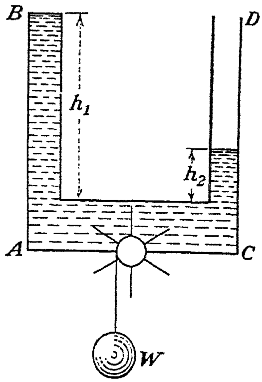


FIG. 377.—Pressure developed by a water wheel when there is no flow of water.

Let the vessels now be connected by a second pipe (Fig. 378). Water will be forced continuously through this pipe by the difference in level between the water in the vessels. After this flow is established, the difference in the level is less than before the flow started. The decrease in back pressure against the wheel being now reduced, the wheel is able to turn and force the water through the circuit. The flow of water thus maintained corresponds to the flow of electricity in the electric circuit. The falling weight represents the chemical action in the cell which produces the electrical pressure driving the electricity around the circuit. The rate at which the water flows depends on the difference in pressure which the falling weight can maintain. In like manner, the rate

at which electricity flows past any point in the circuit depends on the electrical pressure which the battery can establish. The rate at which the water flows in the pipe depends also on the resistance or opposition to flow which the water experiences in the pipe. In the same way, the electrical current depends on the electrical resistance or opposition to electrical flow which the electricity encounters in the wire. The work required to force the water around the circuit comes from the work done by the falling weight. The energy required to send the electricity around the electric circuit comes from the chemical action which takes place in the cell or from the work done in driving the dynamo.

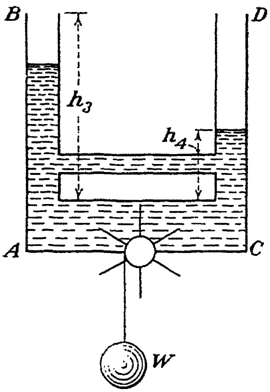


FIG. 378.—Pressure and current from a water wheel. The pressure is less when the current flows.

422. Resistance.—Every conductor offers some opposition to the flow of electricity through it, because the moving electrons which constitute the current collide with the fixed atoms or positive ions. The opposition which a current of electricity encounters in flowing through a wire depends on the length of the wire, on its size, and on the material of which it is made. This opposition to the flow of an electrical current is **electrical resistance.** Its value, which is least in silver, varies greatly from substance to substance. Figure 379 shows a variable resistance for regulating the magnitude of the current.

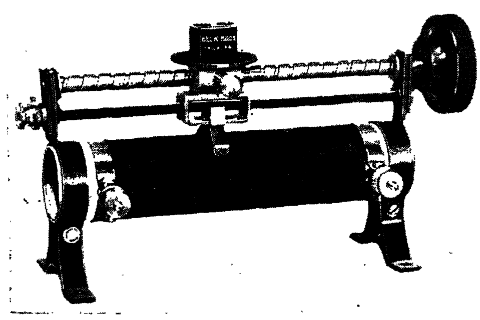


FIG. 379.—A variable rheostat. By means of the sliding contact the resistance can be changed. (*Courtesy Beck Brothers.*)

423. Ohm's Law.—The relation between current, electrical pressure, and the resistance has been stated in what is known as **Ohm's law.** This is one of the most fundamental laws in electricity and has a wide application in electrical machinery. **This law states that the electrical current in a conductor is proportional to the electrical pressure or the electrical pressure is equal to a constant times the electrical resistance.**

$$\text{Current} = \frac{\text{electrical pressure}}{\text{resistance}}.$$

$$I = \frac{E}{R}.$$

where I stands for the current, R the resistance, and E the electrical pressure. The law can be stated in two other forms. Each of these forms states the same facts.

$$\text{Voltage} = \text{current} \times \text{resistance}.$$

$$E = I \times R.$$

$$\text{Resistance} = \frac{\text{voltage}}{\text{current}}.$$

$$R = \frac{E}{I}.$$

424. Units of Resistance.—In order to measure the resistance of a conductor, it is necessary to have some fixed standard. The practical unit of resistance is called the **ohm.** The absolute unit of resistance is called the **abohm.** The unit of resistance can be defined most accurately and fundamentally from the relation between the difference of potential and the current which flows in a conductor.

A conductor has 1 e.m.u. of resistance when 1 e.m.u. of difference of potential will cause 1 e.m.u. of current to flow in it. This unit of resistance is called an **abohm.** It is a small unit of resistance, and a practical unit known as the ohm is defined from it in such a way that,

$$\mathbf{1\ ohm = 10^9\ abohms.}$$

Example.—Find the number of abohms in 100 ohms.

$$1 \text{ ohm} = 10^9 \text{ abohms.}$$
$$100 \text{ ohms} = 100 \times 10^9 \text{ abohms.}$$

A conductor has a resistance of 1 ohm when a difference of potential of 1 volt will cause a current of 1 amp. to flow in it.

A thread of mercury having a cross-sectional area of 1 sq. mm. and a length of 106.3 cm. has a resistance of 1 ohm at 0°C. The resistance of mercury changes rapidly with the temperature, and for this reason the temperature of the mercury must be kept constant. Since a column of mercury is not convenient to use as a standard, it is customary to use standard coils (Fig. 380) made of wire which changes its resistance but slightly with the temperature. This is a great convenience, for it is not necessary to keep the temperature of the coil so nearly constant. Manganin wire is usually used for such resistance coils.

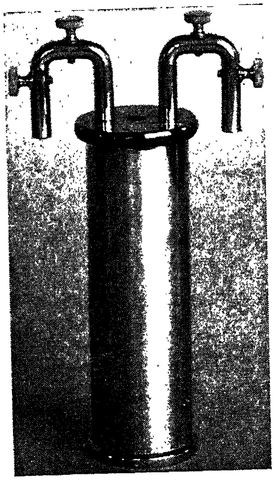


FIG. 380.—Standard resistance made of manganin wire kept at constant temperature. (*Courtesy Leeds and Northrup Company.*)

Example.—In how many ohms can 100 volts maintain a current of 4 amp.?

$$E = 100 \text{ volts.}$$
$$I = 4 \text{ amp.}$$
$$R = \frac{E}{I} \quad \frac{100}{4} = 25 \text{ ohms.}$$

Example.—The pressure in an electrical circuit is 50 volts; the resistance is 12.5 ohms. How many amperes are flowing in it?

$$I = \frac{E}{R}.$$
$$E = 50 \text{ volts.}$$
$$R = 12.5 \text{ ohms.}$$
$$I = \frac{50}{12.5} = 4 \text{ amp.}$$

Example.—How many volts are required to maintain 4 amp. in 25 ohms?

$$I = 4 \text{ amp.}$$
$$R = 25 \text{ ohms.}$$
$$E = R \times I = 4 \times 25 \quad 100 \text{ volts.}$$

425. Resistance Boxes.—It would be inconvenient in experimental work always to have the resistances as individual coils. They are, therefore, joined together in boxes (Fig. 381) in such a way that any number of coils may be joined in series or any one of the coils may be used separately. In this way, a great number of resistances can be obtained from a single box. Each coil is wound on a spool attached to a hard-rubber sheet used as the top of the box. The wire is first doubled and then wound on the spool. The free ends are soldered to brass blocks which can be joined together by means of brass plugs inserted in holes between the brass blocks. When all of the plugs are removed, the coils are all in series. Where a plug is inserted, that coil is short-circuited and the remainder of the coils are in

series. When all the plugs are inserted, the resistance is very nearly zero.

426. Laws of Resistance.—The laws that apply to the resistance of a conductor at constant temperature are as follows:

1. **The resistance of a conductor is proportional to its length.** For example, if 50 ft. of copper wire has a resistance of 2 ohms, then 100 ft. of the same wire has a resistance of 4 ohms.

2. **The resistance of a conductor is inversely proportional to its cross-sectional area.** If the wire has a circular cross section, the resistance is inversely proportional to the square of the

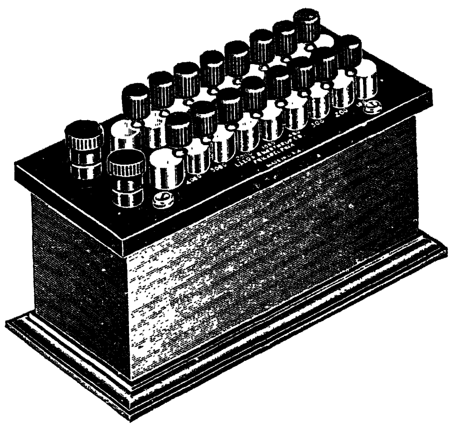


FIG. 381.—Resistance box. (*Courtesy Leeds and Northrup Company.*)

diameter, or radius. If a copper wire having a diameter of 1 mm. has a resistance of 4 ohms, then a copper wire of the same length having a diameter of 2 mm. has a resistance of 1 ohm.

3. **The resistance of a given conductor depends on the material out of which it is made.** Thus, an iron wire, having the same length and cross section as a copper wire, has a resistance about six times as great. It has been found that the four best conducting metals are, in order, silver, copper, gold, and aluminum.

427. Resistivity.—These laws may be brought together into a single formula from which the resistance of wires of different lengths and cross section can be calculated. Let R be the resistance of the wire in ohms, l its length in centimeters, a the area of its cross section in square centimeters, and k a constant depending on the material out of which the wire is made and on the temperature. Then,

$$R = k\frac{l}{a}$$

This constant k is the specific resistance of the wire. **It is equal to the resistance of a wire which is 1 cm. long and has a cross-sectional area of 1 sq. cm.** If the value of this constant k is known, then the resistance of any other wire made out of the same material can be calculated. For example, the specific resistance of silver is 0.0000016 ohm. This means that the resistance of a silver wire 1 cm. long and 1 sq. cm. in cross section is 0.0000016 ohm.

Since wires are usually circular in cross section, engineers have chosen a somewhat different method of calculating the resistance. They call a wire which is 0.001 in. in diameter, **1 mil in diameter,** and the area of its circular cross section is **1 cir. mil.** As in the preceding case, the resistance of any wire can be calculated by comparing it with the resistance of a wire of the same material of standard size and length. Engineers choose for their standard of resistance the resistance of a wire **1 ft. in length and 1 mil in diameter. The resistance of such a piece of wire is the resistance of a mil-foot.** Aluminum has about 17.4 ohms to the mil-foot.

Using this system of units, the resistance of a wire is expressed as

$$R = \frac{Kl}{d^2},$$

where K is the resistance of 1 mil-ft. of the wire, l is the length in feet and d is the diameter in mils.

Example.—The resistance of copper per mil-foot at 20°C. is 10.4 ohms. Find the resistance of a copper wire 500 ft. long with a diameter of 0.021 in.

$$R = \frac{Kl}{d^2} \qquad \frac{10.4 \times 500}{21 \times 21} \qquad 11.8 \text{ ohms.}$$

428. Temperature Coefficient of Resistance.—Whenever the resistance of a metal is given, it is always necessary to state the temperature at which the resistance was measured, for the reason that the resistance of metals increases as the temperature of the metal is raised. The amount which the resistance increases per degree rise in temperature for each ohm in the wire is defined to be the **temperature coefficient of the resistance.** For all pure

metals this coefficient is essentially the same and lies in the neighborhood of 0.004 per degree centigrade.

Taking the resistance at zero as the standard from which to measure changes in resistance, copper wire would increase 0.0042 ohm per degree for each ohm at 0°C.

The relation between the temperature and the resistance is expressed in the form of the equation,

$$R = R_0(1 + 0.0042t),$$

where R = the resistance at a temperature above or below 0°C.
R_0 = the resistance at 0°C.
t = the temperature in degrees centigrade.

Example.—The resistance of a copper wire is 5.25 ohms at 0°C. Find its resistance at 75°C.

$$\begin{aligned} R &= R_0(1 + 0.0042t) \\ &= 5.25(1 + 0.0042 \times 75) \\ &= 5.25 \times 1.315 = 6.90 \text{ ohms.} \end{aligned}$$

429. Temperature Coefficient of Alloys.—When metals are mixed together, the resistance of the mixture may be very different from the resistance of the metals which were used to make the mixture. Alloys always have a higher resistance than that of the best conducting of the metals out of which they were made. On the other hand, the rate at which the resistance changes with the change of temperature is much less in alloys than in pure metals. Manganin, an alloy consisting of copper, nickel, iron, and manganese, has a resistance which is thirty or forty times that of pure copper, but the rate at which its resistance changes with the temperature is practically negligible. For this reason, it is an excellent substance to use for standard resistance coils.

430. Electron Theory of Electrical Conductivity.—The fact that all metals conduct electricity is explained in terms of the electron theory by assuming that each metal contains a large number of more or less free electrons (Fig. 382) which in some way have been at least temporarily detached from the parent atoms. These electrons are assumed to be moving about in the interstices between the atoms or it may be that they can pass easily from one atom to another. It is possible that they are bound more or less loosely to the neighboring atoms in such a way that they can easily be displaced when an electric field is applied to them.

Their exact behavior inside the metal is still unknown. About the essential fact that they move through the wire there can be no doubt.

If a number of wires are connected in series and the free terminals connected to the poles of a battery, the electric force due to the difference of potential between the poles of the battery causes the electrons to stream through the wires in such a way that they approach the positive pole of the battery. Since the electrons are alike in all the wires, the number of those which leave one end of a wire will be just replaced by those which enter it at the other end. There will, for this reason, be no change in the character of the wires. Under the action of the impressed electromotive force, each electron acquires a velocity of drift down the wire. As soon as an electron strikes an atom of the metal, it loses nearly all of this velocity of drift and imparts whatever kinetic energy it possessed to the metal atom. Under the action of the impressed electric field, the electron again regains its lost speed, but again it strikes another metal atom and loses its newly acquired kinetic energy. Since the electrons are continually gaining and losing velocity, the velocity of drift in the conductor under the impressed electromotive force never becomes large.

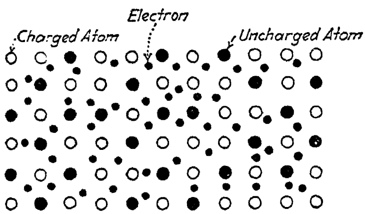


FIG. 382.—Diagrammatic representation of electrons between the charged and uncharged atoms of a metal.

Because of the energy which the electrons give up to the atoms at each collision, the energy of the vibrating atoms is increased. This increase in the energy of the atoms manifests itself as a rise in the temperature of the conductor. The amount of energy imparted to the atoms will be proportional to the number of moving electrons and it will, therefore, increase with the current. On this basis, the resistance which a conductor offers to the passage of an electric current arises out of the fact that the metal atoms stop the electrons. The larger the amplitude of the vibrations of the atoms, the more apt they are to get in the way of a moving electron and the greater the resistance of the conductor. Hence, the resistance of a conductor increases as its temperature is raised. On the other hand, if the atoms

were perfectly at rest, the electrons would experience little opposition to their motion and the resistance of the conductor would be extremely small. Figure 383 shows how the resistance of several metals changes with the temperature at low temperatures.

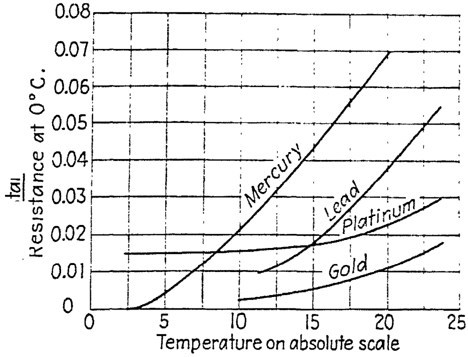


FIG. 383.—Resistance of metals at low temperatures.

431. Superconductivity of Metals.—The knowledge of the conductivity of metals at very low temperatures is due largely to the work of Kammerlingh Onnes and his co-workers at the University of Leiden. The experimental data obtained by these observers showed that in certain metals there is a sudden and rapid decrease in the electrical resistance at very low tempera-

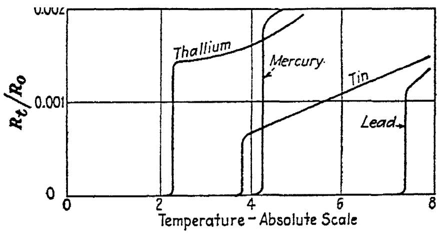


FIG. 384.—Disappearance of resistance at very low temperatures.

tures. In some instances, this decrease is so large that it is impossible to measure the resistance of the metal with any degree of accuracy at these low temperatures because the resistance has nearly vanished. Metals in which the resistance has nearly vanished at very low temperatures are said to be in the *superconducting state.*

The results for mercury, lead, tin, and thallium have been reproduced in Fig. 384. The ratio of the resistance at 0°C. to the resistance at the experimental temperature has been plotted as ordinates in these curves and the experimental temperature as abscissae. The suddenness with which the phenomenon occurs is evident. The temperature at which superconductivity appears in these metals is as shown in the following table.

Substance	Temperature, Degrees Absolute
Lead	7.2
Mercury	4.29
Tin	3.78
Thallium	2.32

The resistance below the transition temperature is a small fraction of the resistance at 0°C. It is too small to be measured

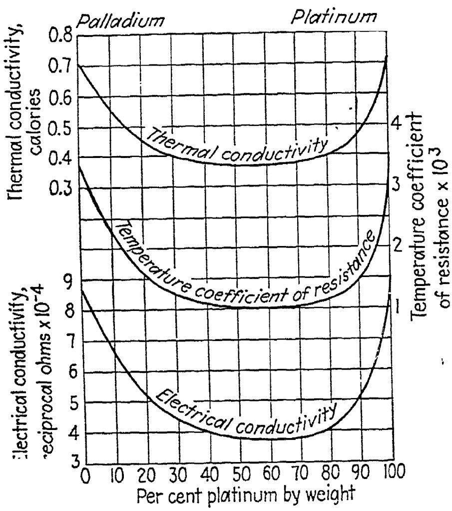


FIG. 385.—Thermal and electrical conductivity of alloys.

accurately so that only an upper limit can be assigned to it. In any case, it is 10^{12} times smaller than the resistance at 0°C. Not all metals become superconductors at very low temperatures. In metals where there is no superconductivity at very low temperatures, the resistance decreases to a minimum and then retains a nearly constant value with a further decrease of temperature. Hence, for these metals at low temperatures the temperature

coefficient of the resistance becomes zero. Metals which belong to this class are gold, platinum, sodium, potassium, and iron.

432. Relation of Electrical to Thermal Conductivity.—It is a well-known fact that good conductors of electricity are also good conductors of heat, and that non-conductors of electricity are poor conductors of heat. This relation has been expressed in what is known as Wiedemann-Franz's law which states that **the ratio of the thermal conductivity to the electrical conductivity of metals is a constant which is independent of the nature of the metal and depends only on the temperature.** The following table shows the value of this ratio for a few metals. This relation is also approximately true for alloys as is evident from Fig. 385.

Metal	$\dfrac{\text{Thermal conductivity at 18°C.}}{\text{Electrical conductivity}}$	Rate of change with temperature
Copper...	6.7×10^{10}	3.9×10^{-3}
Silver.....	6.9×10^{10}	3.7×10^{-3}
Gold.....	7.1×10^{10}	3.7×10^{-3}
Lead.....	7.2×10^{10}	4.0×10^{-3}
Tin......	7.4×10^{10}	3.4×10^{-3}
Platinum.	7.5×10^{10}	4.6×10^{-3}
Palladium	7.5×10^{10}	4.6×10^{-3}
Iron......	8.0×10^{10}	4.3×10^{-3}

433. Resistance Thermometers.—Since the resistance of a wire changes with the temperature, it is possible to infer the change in temperature from observations on the change of resistance.

Let R_0 = the resistance of the wire at 0°C.
R_t = the resistance of the wire at some unknown temperature t.
a = the temperature coefficient of the metal of which the wire is made.
t = the temperature on the centigrade scale.

By definition,

$$a = \frac{R_t - R_0}{R_0 t}$$

$$t = \frac{R_t - R_0}{aR_0}$$

If now, by means of a Wheatstone bridge, the resistance of the wire is measured at 0°C. and again at the unknown temperature t, and if the temperature coefficient of the wire is known from a supplementary experiment, then the unknown temperature t can be immediately calculated.

If a platinum wire is used, such a resistance thermometer can be used to determine the temperature over a wide range of temperatures up to the melting point of platinum. When properly calibrated, such a thermometer gives extreme precision in the measurement of temperatures. Figure 386 gives the essentials of a resistance thermometer.

Example.—In using a resistance thermometer made of platinum wire, the resistance in a mixture of ice and water at 0°C. was found to be 10 ohms, and in a furnace of unknown temperature it was found to be 50 ohms. If

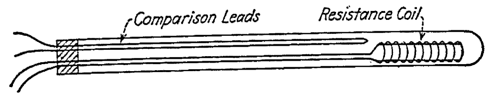


FIG. 386.—Resistance thermometer. Shows change of temperature by the change of its resistance.

the temperature coefficient of the resistance of platinum is 0.004 per degree centigrade, what was the temperature of the furnace?

$$\text{Temperature of the furnace} = t = \frac{R_t - R_0}{0.004 \times 10} = \frac{40}{0.04}$$
$$= 1000°\text{C.}$$

434. Resistance for High-frequency Currents.—When a continuous current has reached its steady value, the current density is uniform over the entire cross section of the conductor. When the current is either rising or falling, the outside of the conductor carries a proportionately larger part of the current than the inside of the conductor. This means that the current density near the axis of the conductor is less than it is near the surface of the conductor. In an alternating-current circuit where the current is continually changing, it is necessary to consider this concentration of the current near the surface of the conductor. This concentration of the current near the surface of the conductor has the effect of increasing the resistance of the conductor for high-frequency currents over its value for steady or low-frequency currents. The higher the frequency of the current, the greater the resistance of the conductor for such currents.

The non-uniformity of the current density over the cross section of a straight wire is not appreciable, unless the wire has a large diameter and the current has a high frequency. The increase of the resistance of a conductor under these conditions depends on the product of the cross section of the conductor and the frequency of the current. This change in resistance also depends on the permeability of the material out of which the wire is made. It is greater for iron than it is for copper.

435. Measurement of Resistance by Ammeter and Voltmeter.—Two resistances R_1 and R_2 (Fig. 387) are connected in series so that the same current goes through both resistances. A voltmeter reads the voltage V_1 across R_1, and a second voltmeter reads the voltage V_2 across R_2. Since the current in each resistance is the same, the resistances are to each other as

the voltages across them; that is, when the fall of potential across R_1 is three times as much as that across R_2, the resistance of R_1 is three times as great as that of R_2. In general,

$$\frac{R_1}{R_2} = \frac{V_1}{V_2}.$$

If a standard resistance is used for R_1 and its resistance given, the magnitude of the resistance R_2 can be calculated as soon as the readings on the voltmeters V_1 and V_2 are observed. In this way, resistances are compared by a direct comparison of the voltages across them.

Example.—A standard resistance of 10 ohms and an unknown resistance are connected as in Fig. 387. The voltmeter across the standard resistance

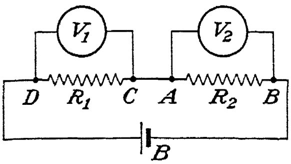


FIG. 387.—Measurement of resistance with two voltmeters. Resistances are proportional to the voltage across them.

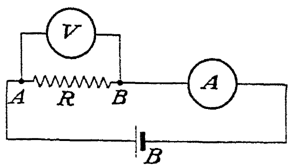


FIG. 388.—Measurement of resistance with a voltmeter and an ammeter.

reads 1.25 volts and that across the unknown resistance reads 3.75 volts. How large is the unknown resistance?

$$\frac{R_2}{R_1} = \frac{V_2}{V_1}.$$

$$\frac{R_2}{10} = \frac{3.75}{1.25} = 3.$$

$$R_2 = 30 \text{ ohms}.$$

Instead of using two voltmeters and a known resistance, the resistance can be determined by means of a voltmeter and an ammeter connected in the circuit as in Fig. 388. A battery B maintains a current through the ammeter and the unknown resistance. The fall of potential over the unknown resistance is given by the voltmeter. If the resistance of the voltmeter is large so that very little current goes through it, the current in the ammeter is very nearly the same as the current in the unknown resistance. If the resistance of the voltmeter is not large in comparison with the resistance to be measured, correction must be made for the fact that some of the current goes through the voltmeter. In the former case where the current in the voltmeter can be neglected, the unknown resistance is given at once by means of Ohm's law from the readings of the voltmeter and the ammeter.

Let V = the reading of the voltmeter.

I = the current in the resistance R.

By Ohm's law,

$$I = \frac{V}{R}$$

and

$$R = \frac{V}{I}$$

Example.—What is the unknown resistance when a voltmeter and an ammeter connected as shown in Fig. 388 indicate 12 volts and 4 amp., respectively?

$$R = \frac{V}{I}$$

$$= \frac{12 \text{ volts}}{4 \text{ amp.}} = 3 \text{ ohms.}$$

436. Wheatstone Bridge.—The most accurate method of measuring resistances is by means of the Wheatstone bridge. In its simplest form it consists of two resistances R and X connected to a wire AB as in Fig. 389. To A, the junction of the wire and the resistance R, is connected one terminal of a battery. To B, the junction of the wire and the unknown resistance X, is connected the other terminal of the battery. This battery sends a current to A which divides, part of it going through R and X and the remainder through the wire AB.

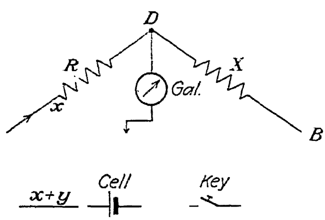


FIG. 389.—Slide-wire Wheatstone bridge for comparing resistances

At D, a point between R and X, one terminal of a galvanometer is connected. The other terminal of this galvanometer is connected to a sliding contact which rests on the bridge wire at C. The sliding contact is moved along until there is no deflection in the galvanometer indicating that there is no difference of potential between D and C. When the bridge is thus balanced, the current in R and X is the same, and the current in both parts of the bridge wire is the same.

Let x = the current in X and R.

y = the current in both parts of the bridge wire.

R_a = the resistance of the bridge wire between A and C.

R_b = the resistance of the bridge wire between C and B.

Fall of potential from A to D = fall of potential from A to C.

$$Rx = R_a y.$$

Fall of potential from D to B = fall of potential from C to B.

$$Xx = R_b y.$$

Dividing one of these equations by the other,

$$\frac{X}{R} = \frac{R_b}{R_a}.$$

$$X = R\frac{R_b}{R_a}.$$

When R is known and the resistances of the bridge wire between A and C and between C and B are known, X can be calculated.

If the bridge wire is made of a single homogeneous material of uniform cross section, the resistance of any part of it is proportional to the length of that part. Hence, the resistance between A and C, as well as the resistance between C and B, is proportional to the lengths of these respective parts. In such a case, instead of the preceding equation, we may write

$$\frac{X}{R} = \frac{b}{a},$$

$$X = \frac{Rb}{a},$$

where b = the length of the bridge wire from B to C.
a = the length of the bridge wire from A to C.

To determine X, it is then only necessary to observe the lengths of the bridge wire when there is no current in the galvanometer and to know the value of the resistance R.

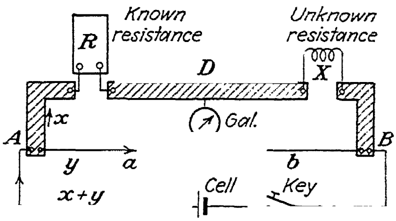


Fig. 390.—Modified slide-wire Wheatstone bridge.

This slide-wire bridge can be modified by using in it a resistance R (Fig. 390) in which the resistance can be varied within certain suitable limits. In this way, it is possible to make the balance point C lie near the middle of the bridge wire. The

accuracy of the observations is thus increased. Figure 391 represents the hydraulic analogue of the Wheatstone bridge. The pump replaces the battery, and when there is no difference of pressure between B and D, no water flows through G.

437. The Post-office-box Bridge.—The accuracy and convenience of the bridge may be still further increased by constructing what is known as a post-office-box bridge

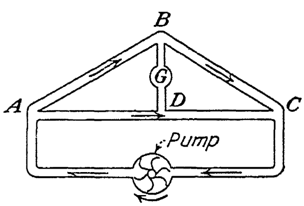


FIG. 391.—Hydraulic analogue of Wheatstone bridge.

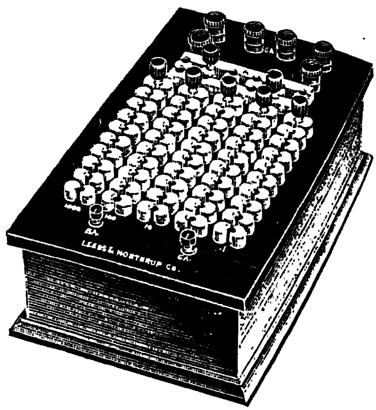


FIG. 392.—Post-office-box bridge for comparing resistances. (*Courtesy Leeds and Northrup Company.*)

(Fig. 392). In this case, the bridge wire is replaced by two known resistances R_3 and R_4. These resistances can be varied within certain limits and their ratio is accurately known. The balance is reached on such a bridge by adjusting the three known resistances R_1, R_3, and R_4 until no current flows through the galvanometer. When this balance is reached,

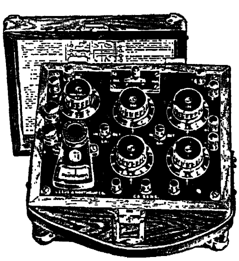


FIG. 393.—A portable Wheatstone bridge. (*Courtesy Leeds and Northrup Company.*)

$$\frac{R_1}{R_3} = \frac{X}{R_4}.$$

$$X = R_4 \frac{R_1}{R_3}.$$

When the ratio of R_1 to R_3 is known and the value of R_4 for a balance has been observed, the value of X can be calculated.

The three known resistances are made up in a single box with provision for connecting to it the battery, the galvanometer, and the unknown resistance.

Figure 393 shows a portable testing apparatus consisting of a compact form of the Wheatstone bridge.

Problems

1. Find the resistance of a thread of mercury with a circular cross section, 0.7 mm. in diameter, with a length of 80 cm.

2. A ribbon of silver, 6 cm. long and 1 mm. wide, is to be made with a resistance of 0.5 ohm. How thick must it be?

3. Tungsten has a specific resistance of 5.51×10^{-6} metric unit. If a lamp filament made of wire with a diameter of 0.08 mm. is to have a resistance of 6.6 ohms, how long must it be?

4. Find the weight of a copper wire 1,200 ft. long with a resistance of 1.8 ohms. What will be the weight of an aluminum wire with the same length and the same resistance?

5. A summer cottage is to be illuminated by current generated at a distance of 180 m. from the cottage. The resistance of the two-wire circuit is to be 1 ohm or less. What is the smallest size of copper wire which can be used?

6. A piece of wire, 8 m. long and 0.5 mm. in diameter, has a resistance of 2 ohms. What length of wire of the same material 0.4 mm. in diameter will have a resistance of 2.5 ohms?

7. Two wires of the same length and material have resistances of 12 and 16 ohms, respectively. If the diameter of the first wire is 0.8 mm., what is the diameter of the second wire?

8. A piece of wire has a resistance of 33.0186 ohms at 35°C. and of 32.1448 ohms at 0°C. What is the temperature coefficient?

9. A device for measuring resistance is capable of measuring with errors not exceeding $\frac{1}{25}$ per cent. How great a change of temperature at 0°C. would produce that much change in the resistance of a copper wire?

10. The resistance of a copper wire which is known to be 18.42 ohms at 0°C. is observed to be 21.08 ohms. What must be the temperature?

11. A copper rod, which was 1 cm. in diameter and 1 m. long, was drawn out into a wire which is 1 mm. in diameter. Find the resistance of the wire produced in this way.

12. In measuring the resistance of a coil of copper wire on a Wheatstone bridge a 2-ohm resistance coil was used in the right-hand gap. The slider was balanced at 37 cm. from the left-hand end of a 100-cm. bridge wire. Calculate the resistance of the coil.

13. Find the current which will flow in a piece of iron wire with a resistance of 2.5 ohms, if it were connected with a storage cell of 2.5 volts.

14. What is the difference of potential between the ends of the feeders for a trolley when a current of 400 amp. is flowing, if the resistance of the feeder is 0.068 ohm?

15. What is the greatest length of copper wire having a resistance of 1.8 ohms per 1,000 ft., which can be used to carry 8 amp. allowing a drop of 4 volts in the wire?

CHAPTER XXXVI

SIMPLE ELECTRIC CIRCUITS

438. Kirchhoff's Laws.—Two extensions of Ohm's law were made by Kirchhoff. These extensions are known as Kirchhoff's laws.

First Law.—The first law states that **at any point in a circuit there is as much current flowing away from the point as there is current flowing to it.** This means that the sum of the currents flowing to a point is just equal to the sum of the currents flowing away from it. If this law were not true, there might be more electricity flowing toward a point than away from it. In that case, there would be an accumulation of electricity at the point. In a system of water mains, no matter how many pipes may lead away from one place the amount of water coming up must equal the amount going away, unless there is a reservoir to receive some of the water or unless water already stored up is discharging from some kind of a reservoir. In similar manner, the electricity going away from a point must just equal that coming up to it, unless there is some accumulation of electricity.

$$I = I_1 + I_2 + I_3 + I_4.$$

The Second Law.—The second law of Kirchhoff states that **the sum of the products of the current by the resistance taken around any closed path in a network of conductors is just equal to the sum of the electromotive forces which one passes in going around the closed circuit.** In going around the circuit in the application of this law, regard must be had for the direction in which the current is flowing. Currents which are flowing in the direction in which the circuit is traced out are regarded as **positive.** Currents flowing in the opposite direction are considered **negative.** Electromotive forces which, if acting alone, would produce a positive current in the circuit are considered positive; and the oppositely directed electromotive forces are considered negative. If there are no electromotive forces in any of the

conductors which comprise the part of the network traced out, then the sum of the IR drops about that circuit is zero.

439. Resistance in Series.—When two or more conductors are arranged as shown in Fig. 394, they are said to be connected in series. Such an arrangement is analogous to several pipes joined as in Fig. 395 so that the same current of water flows through each pipe. Whatever the area of the cross section of these pipes or their length, the flow of water per unit time in each pipe must be the same. There is no opportunity for water to accumulate in a pipe when the flow has once started. In like manner, the current flowing through each of a number of resistances connected in series is always the same.

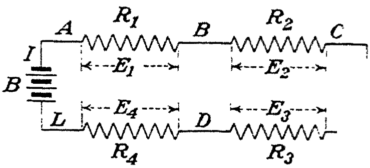


FIG. 394.—Resistances in series. The effective resistance is the sum of the separate resistances.

Let I = the current in each of the resistances.
R = the total resistance between A and L.

The fall of potential from A to $B = E_1 = IR_1$.
The fall of potential from B to $C = E_2 = IR_2$.
The fall of potential from C to $D = E_3 = IR_3$.
The fall of potential from D to $L = E_4 = IR_4$.
Total fall of potential between A and $L = E = IR$.

FIG. 395.—Water pipes in series.

The total fall of potential is the sum of the falls of potential over the individual resistances. Hence,

$$IR = IR_1 + IR_2 + IR_3 + IR_4$$

or

$$R = R_1 + R_2 + R_3 + R_4.$$

Thus, for conductors connected in series the following laws hold:

1. The current in each resistance is the same.
2. The resistance of the entire circuit is equal to the sum of the separate resistances.
3. The voltage across several resistances in series is equal to the sum of the voltages across each of the separate resistances.
4. Since the current in each resistance is the same, the fall of potential over a resistance is proportional to the resistance. Thus, over a large resistance the fall of potential is large, and over a small resistance it is small.

FIG. 396.—Electric lamps in series.

Example.—Three resistances A, B, and C (Fig. 396) one of 10 ohms, another of 5 ohms, and a third of 2 ohms are connected in series. What is the effective resistance of the circuit?

Effective resistance = sum of the individual resistances.

$$R = R_1 + R_2 + R_3.$$
$$R = 10 + 5 + 2 = 17 \text{ ohms}.$$

440. Resistances in Parallel.—When several conductors are joined between two points so that the current divides between them (Fig. 397), they are said to be in parallel or in multiple. This is analogous to connecting a tank M (Fig. 398) to another tank N by means of a number of pipes through each of which water may flow. The head of pressure will evidently be the same whether the water flows through one or more of the pipes at the same time. The amount which flows through each pipe will be determined by the length and diameter of that pipe and will be independent of the amount which flows through any other pipe, so long as the pressure remains unchanged during the flow. The flow through all of the pipes will be equal to the sum of the flows through the individual pipes.

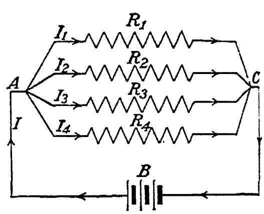


FIG. 397.—Resistances in parallel. The reciprocal of the effective resistance is equal to the sum of the reciprocals of the separate resistances.

In the same way, the current of electricity which flows through R_1 (Fig. 397) will be determined by its resistance, if the potential difference between A and C remains unchanged. The total current entering at A must be equal to the current leaving C, and this total current must be equal to the sum of the currents in the separate resistances.

Let I = the current entering at A.
I_1 = the current in R_1.
I_2 = the current in R_2.
I_3 = the current in R_3.
I_4 = the current in R_4.

Total current = sum of separate currents.

$$I = I_1 + I_2 + I_3 + I_4.$$

The fall of potential from A to C is the same by each of the paths. By Ohm's law the fall of potential is given by multiply-

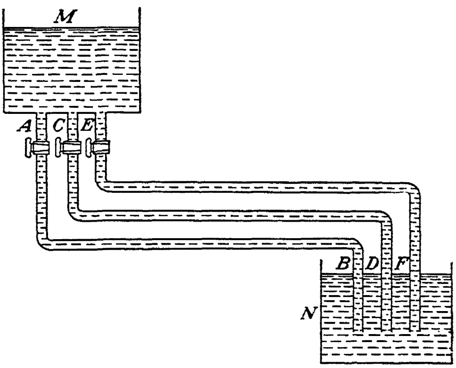


FIG. 398.—Water pipes in parallel.

ing the resistance by the current. Hence, the potential difference between A and C is

$$I_1R_1 = I_2R_2 = I_3R_3 = I_4R_4 = E.$$

Hence,

$$I_1 = \frac{E}{R_1},\ I_2 = \frac{E}{R_2},\ I_3 = \frac{E}{R_3},\ I_4 = \frac{E}{R_4}.$$

Let R denote the equivalent resistance by which these resistances could be replaced without disturbing the fall of potential between A and C or the flow of current between A and C.

Fall of potential between A and $C = IR = E$, or $I = E/R$.

Substituting these values of I, I_1, I_2, I_3, I_4 in the equation $I = I_1 + I_2 + I_3 + I_4$,

$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} + \frac{E}{R_4},$$

and

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}.$$

Example.—Three resistances, A, B, and C (Fig. 399), of 2, 4, and 12 ohms, are connected in parallel; find the equivalent resistance.

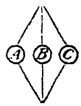


FIG. 399.—Electric lamps in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{12} = \frac{10}{12}.$$

$$R \quad \frac{12}{10} \quad 1.2 \text{ ohms.}$$

Example.—Four resistances, A, B, C, and D, are connected as indicated in Fig. 400; C and D in parallel, and A and B in series with each other and

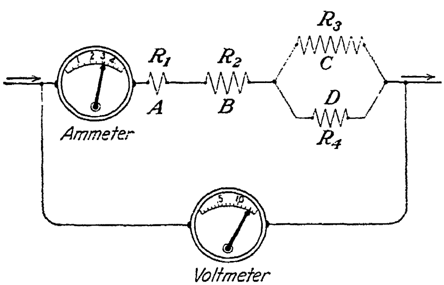


FIG. 400.—Resistances in series and in parallel.

with C and D. The resistance of A is 4 ohms, that of B, 8 ohms, that of C 3 ohms, and that of D, 6 ohms. Find the equivalent resistance.

Let x = resistance of C parallel.

$$\frac{1}{x} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$$

$$x = \frac{6}{3} = 2.$$

Resistance of A, B, and x in series:

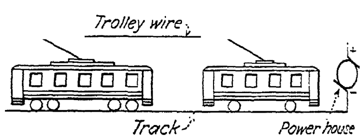


FIG. 401.—Current in a trolley wire and track.

$$R = 4 + 8 + 2 = 14 \text{ ohms.}$$

Shunts.—In many forms of the ammeter the current is too large to be carried by the moving coil in the instrument. A parallel path, of low resistance (Fig. 402) is provided. Most of the current goes through this parallel path which forms a **shunt** for the instrument. Such shunts are also used for other purposes. If X is a shunt for the resistance R and I the total current, the current i in the shunt is given by the following relations:

$$(I - i)R = Xi, \text{ and } i = I\frac{R}{R + X}.$$

441. Cells Connected in Series.—The generators or cells which supply the current may be arranged either in series or in parallel. When cells are connected (Fig. 403) so that the positive pole of one cell is connected to the negative pole of the next, they are said to be joined in series. The electromotive force of such a

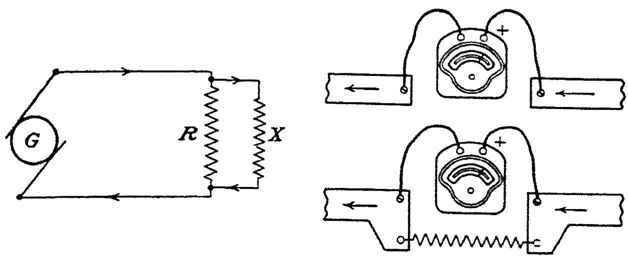


FIG. 402.—A shunt.

combination is the sum of the electromotive forces of the several cells. Since the whole current passes through each cell, the internal resistance of these cells is equal to the sum of the resistances of the individual cells. If these cells are joined so that they send a current through an external resistance, there is only one path for the current, and the current in the circuit is everywhere the same.

Suppose five such cells, each having an electromotive force e and a resistance r, are connected to an external resistance R. The whole electromotive force in the circuit is $5e$, and the whole resistance of the circuit is $R + 5r$.

By Ohm's law,

$$\text{Current} = I = \frac{5e}{R + 5r}.$$

In general,

$$I = \frac{ne}{R + nr},$$

where n equals number of cells.

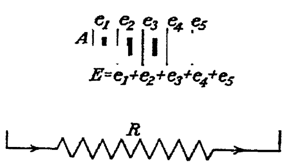


FIG. 403.—Cells in series. The effective e.m.f.-sum of the e.m.fs. of the cells.

Example.—Five cells each having an electromotive force of 1.25 volts and an internal resistance of 0.4 ohm are connected to a resistance of 20 ohms. Find the current in this resistance.

Total electromotive force $= 1.25 \times 5 = 6.25$ volts.

Internal resistance of the cells $= 0.4 \times 5 = 2.0$ ohms.

Total resistance of the circuit = int. resistance of cells + ext. resistance $= 2.0 + 20 = 22.0$ ohms.

$$\text{Current} = \frac{\text{electromotive force}}{\text{total resistance}} = \frac{6.25 \text{ volts}}{22 \text{ ohms}} = 0.28 \text{ amp.}$$

442. Cells Connected in Parallel.—If cells are joined together in such a way that all the positive poles (Fig. 404) are connected together and then all the negative poles are connected, these cells are said to be connected in parallel. The voltage of such a combination of cells is the same as the voltage of each cell. The internal resistance of the combination is equal to the internal resistance of one of the cells divided by the number of cells. The current sent through an external resistance by this combination of cells will be the sum of the currents supplied by the separate cells. By joining cells in this way the electromotive force is unaltered, but the internal resistance of the battery is reduced. It is desirable to connect cells in this way, if the internal resistance of the cells is large in comparison with the external resistance of the circuit.

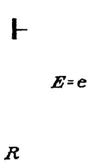


FIG. 404.—Cells in parallel. Total e.m.f. = e.m.f. of one cell.

The current supplied by such a battery is

$$I = \frac{\quad}{R + \frac{r}{n}}$$

where e = the electromotive force.
r = the internal resistance of each cell.
R = the external resistance of the circuit.
n = the number of cells.

Example.—Three cells each having an electromotive force of 2 volts and an internal resistance of 4.5 ohms are connected in parallel to drive a current through an external resistance of 8 ohms. Find the current in the external resistance.

Total electromotive force = electromotive force of one cell = 2 volts.

$$\text{Internal resistance} = \frac{\text{internal resistance of one cell}}{\text{number of cells}} = \frac{4.5}{3} = 1.5 \text{ ohms.}$$

Total resistance of the circuit = external resistance + internal resistance
= 8 + 1.5 = 9.5 ohms.

$$\text{Current} = \frac{\text{total electromotive force}}{\text{resistance of circuit}} = \frac{2 \text{ volts}}{9.5 \text{ ohms.}} = 0.21 \text{ amp.}$$

443. Ohm's Law in Part of Circuit and Brush Potential.—Ohm's law may be applied to a part of a closed circuit as well as to the entire circuit. The law then reads as follows:

The current in a part of the circuit not containing a source of electromotive force is equal to the voltage across that part of the circuit divided by the resistance of that same part of the circuit. It is important to notice that in this case the current, the voltage, and the resistance must be measured in the same part of the circuit.

Example.—A dynamo having a resistance of 2 ohms maintains a current of 11 amp. in two resistances of 8 and 10 ohms in series. The electrical pressure generated by the dynamo is 220 volts. What is the fall of potential over each resistance?

The voltage in the entire circuit = 220 volts.

The current in each resistance is 11 amp.

The voltage across the 8-ohm coil is found by multiplying the resistance of that coil by the current in it.

Voltage across 8-ohm coil = $8 \times 11 = 88$ volts.

In the same way the voltage across the 10-ohm coil is

$$E = 11 \times 10 = 110 \text{ volts.}$$

The fall of potential inside of the generator is

$$E = 11 \times 2 = 22 \text{ volts.}$$

The total potential around the circuit is

$$88 + 110 + 22 = 220 \text{ volts.}$$

Only part of the electromotive force or total voltage generated by the dynamo is available for maintaining the current in the external circuit. Part is used in maintaining the flow of electricity through the generator itself. The actual difference of potential between the brushes of the dynamo is known as **the brush potential or the terminal voltage of the dynamo.** This is the voltage available for maintaining the current through an external resistance. The remainder of the voltage generated in the dynamo or battery appears as a drop of potential inside of the dynamo or battery itself. The brush potential must be carefully distinguished from the total voltage generated by the dynamo.

Let R = the external resistance.

E = the electromotive force of the battery or dynamo.

r = the internal resistance of the battery or dynamo.

$$\text{Current} = \frac{E}{R + r}.$$

$\therefore$ difference of potential between brushes = current $\times$ external resistance.

$$= \left(\frac{E}{r + R}\right) \times R.$$

Entire drop of potential = internal drop of potential + drop of potential over external circuit.

It is thus seen that the voltage generated by a dynamo or battery may be considered as divided into two parts: (1) that which maintains the current in the external circuit, and (2) that which maintains the current in the internal resistance of the dynamo or battery. These two voltages are proportional to the resistance of the corresponding parts of the circuit. Their sum is equal to the voltage produced by the battery or dynamo. On open circuit, the terminal voltage is equal to the electromotive force or total voltage of the generator.

Example.—A battery has a voltage of 1.9 volts on open circuit. It is connected in series with a resistance of 10 ohms, and then the potential difference across its terminals is 1.75 volts. Find the internal resistance of the battery.

Let x = the internal resistance of the battery.
i = the current in the battery when a resistance of 10 ohms is in series with it.

$$i = \left(\frac{1.9}{10 + x}\right).$$

1.90 − 1.75 = 0.15 volt
= drop of potential inside the battery.

$$0.15 = xi.$$

$$i = \frac{0.15}{x}.$$

Hence,

$$\frac{0.15}{x} = \frac{1.9}{10 + x}.$$

$$\frac{x}{0.15} = \frac{10 + x}{1.9}.$$

$$x = 0.86 \text{ ohm}.$$

Problems

1. Three cells of a storage battery, each with a resistance of 0.06 ohm and an electromotive force of 1.4 volts, are connected in series with a coil having a resistance of 3.38 ohms. Find the current which will flow.

2. A group of five dry cells each with an internal resistance of 0.06 ohm and an electromotive force of 1.5 volts sends a current of 5 amp. through an external resistance when connected in series. How great is the external resistance?

3. Eight cells each with an internal resistance of 1.6 ohms are connected in parallel to send a current through an external resistance of 0.2 ohm. How much current will be obtained, if each cell has an electromotive force of 1.1 volts?

4. Twelve cells, each with an internal resistance of 1.2 ohms are connected so as to have four parallel groups of three cells in series. A current of 2.1 amp. is sent through an external resistance of 1.1 ohms. What is the electromotive force of each cell?

5. In a circuit like that in Fig. 388 the voltmeter and ammeter readings were 5.8 volts and 0.12 amp., respectively. The voltmeter had a resistance of 1,200 ohms. Calculate (*a*) the combined resistance of R and the voltmeter in parallel; (*b*) the resistance of R alone.

6. Ten storage batteries, each of three cells with an electromotive force of 2 volts and an internal resistance of 0.015 ohm per cell, are to be charged in series at the rate of 10 amp. from a 110-volt line. How much resistance must be inserted in series with the batteries?

7. A galvanometer has a moving coil with a resistance of 124 ohms and a sensitivity of 1-mm. deflection for 10^{-7} amp. What shunt will be needed to produce 1-mm. deflection for 0.001 amp. in the main circuit?

8. The moving element of a voltmeter has a resistance of 4.8 ohms, and a full deflection is produced when 0.050 volt is applied to the coil. Calculate the series resistances needed to use the voltmeter for 3 volts and for 150 volts, respectively.

9. A galvanometer with a resistance of 120 ohms is shunted by 1 ohm, and a dry cell of 1.5 volts is connected to it through a resistance of 750,000 ohms. A deflection of 180 mm. is read on the scale. Find the current required for 1-mm. deflection.

10. A milliammeter has a resistance of 1 ohm and its full-scale reading is 100 milliamp. Find the resistance of a shunt which must be used in order to convert it into an ammeter on which the full-scale reading is 10 amp.

11. What resistance must be placed in parallel with a resistance of 80 ohms to reduce the resistance to 66 ohms?

12. Eight hundred incandescent lamps are connected in parallel. It is desired to supply each of them with a current of 0.4 amp. at a difference of potential of 110 volts. What must be the resistance of the line from the generator, if the drop of potential over it is 2.2 volts?

13. An ammeter has a resistance of 0.05 ohm. Its full-scale deflection is 2 amp. What shunt must be used so that the full-scale deflection of the instrument may read 10 amp.?

14. A milliammeter has a scale which reads from 0 to 50 milliamp. The resistance of the instrument is 10 ohms. What must be the resistance of a shunt to be used with it in order to make the scale read from 0 to 50 amp.?

15. A divided circuit consists of two parallel branches. One has a resistance of 8 ohms and the other a resistance of 16 ohms. The total current in the circuit is 18 amp. Find the current in each branch.

16. A dynamo has an internal resistance of 0.2 ohm. It generates 120 volts. If it is supplying 8 amp. to a circuit, what is its brush potential?

17. A line with a resistance of 0.08 ohm leads from a generator with a resistance of 1.2 ohms to a set of lamps. A pressure of 112 volts at the lamps is obtained when the current is 1 amp. What will it be when the current is 6 amp.?

18. A circuit consists of a dynamo having a resistance of 0.45 ohm, line wires of resistance 2.2 ohms, and three lamps connected in series, each with a resistance of 12.5 ohms. What is the current if the electromotive force of the dynamo is 120 volts?

19. A circuit has three parallel branches with resistances 30, 40, and 50 ohms, respectively. When a current of $3\frac{1}{3}$ amp. is flowing in the 30-ohm branch, how much current is flowing in each of the other branches?

20. A galvanometer with a resistance of 120 ohms is shunted with a resistance of 0.05 ohm. What fraction of the total current will flow through the galvanometer?

CHAPTER XXXVII

HEAT AND ELECTRIC CURRENTS

444. Heating by Electricity.—The production of heat by an electric current is of much practical importance. On account of its great convenience it is being used more and more. Electric cooking, electric soldering, and electric welding are now very familiar. In an electric furnace, the high temperatures necessary to melt most metals can be produced by an electric current. In an ordinary electric lamp, a filament of tungsten is heated to incandescence by an electric current and becomes a source of light and heat. Electric ranges and electric cookers are in common use.

445. Joule's Law.—In order to determine the heat generated by an electric current, it is necessary to know the electric current and the resistance through which it flows or to measure the electrical current and the electrical pressure which is maintaining it in the resistance.

Joule found experimentally that the heat developed in a conductor by a current of electricity is proportional to (1) *the resistance of the conductor*, (2) *the square of the current*, and (3) *the length of time the current is maintained*. But Joule's law can be derived theoretically from the principle of conservation of energy. It is necessary to recall that the difference of potential between two points in absolute units is the work in ergs that must be done on one electromagnetic unit of charge to carry the charge from the point of lower potential to the point of higher potential, or, conversely, it is the work done by or energy obtained from unit charge when it is allowed to move from the point of high potential to the point of low potential.

Let E = the difference of potential in absolute units between the ends of the conductor, *i.e.*, the work done by unit charge when allowed to move through the wire from high to low potential.

I = the current in absolute units.

t = the time in seconds.

Then

$Q = It =$ the quantity of electricity in absolute units.
$W =$ the work done by the Q units in moving through the conductor.
$W = E \times Q = E \times I \times t$, by the definition of E above.

and since by Ohm's law, $E = I \times R$,

$$W = R \times I^2 \times t \text{ ergs.}$$

By the principle of conservation of energy, the energy given up by the electricity equals the heat energy that appears, and we have

$$\text{Heat energy in ergs} = W = E \times I \times t = R \times I^2 \times t.$$

If the resistance is measured in ohms instead of abohms, the current in amperes instead of abamperes, and the energy in joules instead of ergs, we have

$$\begin{aligned}
I(\text{abamperes}) &= I(\text{amperes}) \times 10^{-1}.\\
R(\text{abohms}) &= R(\text{ohms}) \times 10^{9}.\\
H(\text{ergs}) &= H(\text{joules}) \times 10^{7}.\\
H(\text{joules}) &= R(\text{ohms}) \times I^2(\text{amperes}) \times t(\text{seconds}).
\end{aligned}$$

The rate of supplying energy is the power.
Hence,

$$P = \frac{W}{t} = E \times I.$$

If the current, voltage, and resistance are measured in practical units, this equation gives the power in watts. To reduce this to kilowatts, it is necessary to divide the number of watts by 1,000. To express the energy in kilowatt hours, multiply the number of kilowatts by the time in hours.

Example.—A 25-kw. motor operating on 100 volts is supplied through a line having a resistance of 4 ohms. What power is lost in the line?

$$\begin{aligned}
\text{Current} &= \frac{\text{power}}{\text{voltage}}\\
&= \frac{25{,}000}{100} = 250 \text{ amp}\\
\text{Power lost in line} &= \text{resistance} \times (\text{current})^2\\
&= 4 \times (250)^2\\
&= 250{,}000 \text{ watts}\\
&= 250 \text{ kw.}
\end{aligned}$$

Example.—What power is required to drive a dynamo which delivers 15 amp. at a pressure of 150 volts, disregarding friction?

$$\begin{aligned}\text{Power} &= \text{electrical pressure} \times \text{current}\\ &= \text{volts} \times \text{amperes}\\ &= 150 \times 15 = 2{,}250 \text{ watts}\\ &= \frac{2{,}250}{746} = 3.02 \text{ hp.}\end{aligned}$$

446. To Compute the Power.—When it is desired to measure the power which is being consumed in an electric resistance, an ammeter is inserted and the current in the resistance is measured. At the same time, a voltmeter is attached to the terminals of the resistance and the electrical pressure or voltage across the resistance is measured. This gives sufficient data from which to compute the power from the equation,

$$P = E \times I.$$

If the resistance of the circuit is known, one of the following forms of the equation will be more convenient than another according to the way the data are given.

$$P = EI.$$

But by Ohm's law,

$$E = RI.$$

Hence,

$$P = I(IR) = I^2R.$$

Again,

$$I = \frac{E}{R}.$$

Hence,

$$P = \frac{E}{R} \times E = \frac{E^2}{R}.$$

Example.—Suppose that a dynamo having a voltage of 110 volts sends a current of 2 amp. through a lamp having a resistance of 55 ohms. Find the power expended in the lamp.

$$\begin{aligned}\text{Power in lamp} &= E \times I = 110 \times 2 = 220 \text{ watts.}\\ \text{Power in lamp} &= I^2R = 2 \times 2 \times 55 = 220 \text{ watts.}\\ \text{Power in lamp} &= \frac{E^2}{R} = \frac{110 \times 110}{55} \quad 220 \text{ watts.}\end{aligned}$$

447. Electrical Energy in Heat Units.—Where the heat generated by allowing a current of electricity to flow in a wire is measured by noting the amount it raises the temperature of a given mass of water (Fig. 405), it is convenient to express the electrical energy in calories instead of in joules. By multiplying the current in amperes by the electrical pressure in volts, the electrical power is expressed in watts; but a watt is defined as a

joule per second. Hence, the number of joules generated in the circuit in 1 sec. is found by multiplying the current in amperes by the electrical pressure in volts. In the study of the mechanical equivalent of heat it was found that 4.2 joules generate 1 cal. of heat. **Thus 1 joule is equal to 0.24 cal.** This is another way of saying that a current of 1 amp. flowing through a resistance of 1 ohm, or under an electrical pressure of 1 volt, generates 0.24 cal. each second. It would, therefore, raise the temperature of 1 g. of water 0.24°C. each second. Therefore, the heat generated in an electric circuit may be expressed as

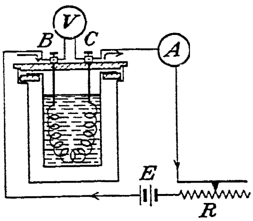


FIG. 405.—An electric calorimeter for measuring heat generated by an electric current.

$$\begin{aligned} H \text{ in calories} &= 0.24 \times I^2R \times t \\ &= 0.24 \times E \times I \times t \\ &= 0.24 \times \frac{E^2}{R} \times t \end{aligned}$$

where H = the heat in calories.
I = the current in amperes.
R = the resistance in ohms.
t = the time in seconds.

Example.—How much heat is generated per hour in an electric iron using 3.5 amp. at 100 volts?

$$\text{Heat in calories} = \frac{\text{volts} \times \text{amperes} \times \text{seconds}}{4.2}.$$

$$\begin{aligned} \text{Heat in calories} = 0.24 \times E \times I \times t &= 0.24 \times 100 \times 3.5 \times 3{,}600 \\ &= 302{,}400 \text{ cal.} \end{aligned}$$

448. Electric Fuses.—It is necessary to have some sort of device to protect electric machines and appliances from excessive currents. One method of furnishing this protection is by means of fuses. These fuses consist essentially of a wire which has a low melting point. Such wires are made by preparing an alloy of tin or lead with some other metals so that the melting point is lower than that of either tin or lead. When an excessive current passes through this fuse wire, the heat generated in accordance with the law just discussed becomes sufficient to melt the wire, and the circuit in which the wire was inserted is opened without harm to the machinery or other electrical appliances. The size of the fuse is so chosen that it melts when the current becomes greater than a certain amount. The fuse is enclosed in some material like asbestos or porcelain so that there is no danger from fire when the fuse is being melted. The manner in which such fuses are inserted in the lighting circuits of a house is shown in Fig. 406.

449. Electric Light Dimmers.—Where it is necessary to use an electric light of small intensity throughout the night or to reduce the intensity of such a light, provision is made for inserting a resistance (Fig. 407) in series with the light. If the voltage applied to the light remains constant, the insertion of a resistance in series with the lamp reduces the current through it and the energy transformed by it. If R is the resistance of the

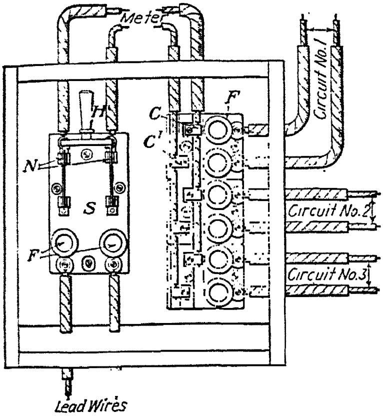


FIG. 406.

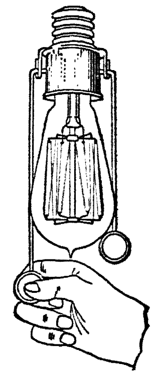


FIG. 407.

FIG. 406.—Electric fuses in a wiring system. The fuses melt when the current is too large.

FIG. 407.—Electric light dimmer. Addition of resistance reduces the brightness of the light.

lamp, I the current in it, and E the electromotive force supplied, then the energy changed to heat in the lamp is

$$P = EI = RI^2 = \frac{E^2}{R}.$$

If now a resistance R_1 is connected in series with the lamp, the new current is

$$I_1 = \frac{E}{R + R_1}.$$

The energy supplied to the light is $P_1 = RI_1^2$, and that supplied to the resistance is $P_2 = R_1I_1^2$. The total energy in this case is

$$P_1 + P_2 = RI_1^2 + R_1I_1^2 = I_1^2(R + R_1) \quad \frac{E^2}{(R + R_1)^2}(R + R_1) = \frac{E^2}{R + R_1}.$$

The total power supplied before the insertion of the resistance was

$$P = \frac{E^2}{R}.$$

Since $R + R_1$ is greater than R, the energy used after the insertion of the resistance is less than before its insertion.

450. Continuous-flow Method for Measuring Specific Heat of Liquids.—An excellent illustration of Joule's law is an electrical method for determining the specific heat of liquids. This method was originally devised by Callendar, and it is suitable for finding the specific heats of liquids over a wide range of temperatures. A simple form of the apparatus is represented in Fig. 408. A narrow glass tube XY is attached at its ends to larger glass tubes A and B. This narrow tube with the wider tubes at its ends is supported on the axis of a glass tube MN of large diameter by means of rubber stoppers. In order to reduce the heat losses as far as possible, the space between the larger glass tube and the inner one is filled with cotton wool or some other heat-insulating substance. A fine platinum wire wound in the form of a spiral is located in the glass tube of small diameter and extends

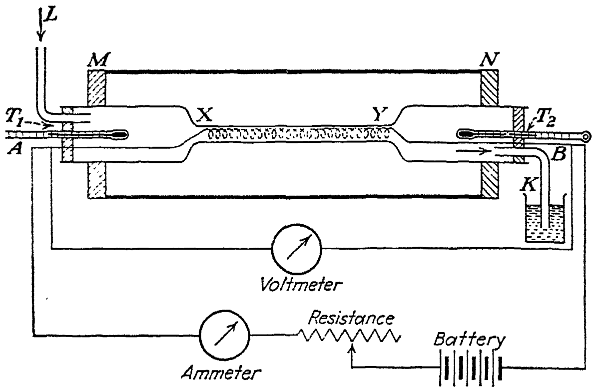


FIG. 408.—Continuous-flow calorimeter for measuring the specific heats of liquids. The heat is supplied by an electric current in a coil of wire.

over the entire length of that tube. By means of heavy lead wires, this spiral is connected through an ammeter and a regulating resistance to the terminals of a battery. To the terminals of this spiral is also connected a voltmeter, by means of which the drop of potential across the spiral can be read. The ends of the larger glass tubes which were attached to the smaller glass tube XY are closed by means of rubber stoppers. Through these stoppers are inserted thermometers T_1 and T_2 by means of which the temperature of the ingoing and the outgoing liquid can be determined.

The liquid to be studied enters the tube AX through a small tube L which is inserted through the rubber stopper at A. The liquid after flowing through the glass tube XY, past the platinum spiral, emerges through a tube and is collected in the beaker K.

When an electric current is caused to flow through the spiral in XY, heat is developed and the liquid flowing past the spiral is heated, since energy is transferred from the wire to the liquid. The thermometer T_1 gives the temperature of the ingoing liquid and the thermometer T_2 the temperature of the outgoing liquid. The mass of the liquid passing through the tube in a given time is determined from weighing the liquid which collects in the

beaker K. The voltmeter gives the electrical pressure across the heating spiral, and the ammeter the current in it. Assuming that all the energy generated in the wire is transferred to the liquid, it is possible to calculate the specific heat of the liquid.

Let m = the mass of the liquid in grams collected in the beaker K.
t = the time in seconds during which the liquid flows.
s = the specific heat of the liquid.
T_1 = the temperature of the ingoing liquid.
T_2 = the temperature of the outgoing liquid.
E = the drop of potential in volts across the spiral.
I = the current in the spiral in amperes.

The heat gained by the liquid in calories $= ms(T_1 - T_2)$.
The energy generated in the wire in joules $= E \cdot I \cdot t$.
Hence,

$$E \cdot I \cdot t = (4.18)m \cdot s(T_1 - T_2).$$

$$s = \frac{E \cdot I \cdot t}{4.18m(T_1 - T_2)}.$$

Example.—With a Callendar continuous-flow calorimeter it was found that when the liquid flowed through the calorimeter, the temperature of the inflowing liquid was 25°C., that of the outflowing liquid was 40°C., the mass of the liquid collected was 100 g., the difference of potential across the heating coil 10 volts, and the current in it 2 amp. The time during which the liquid flowed was 4 min. Find the specific heat of the liquid.

$$s = \frac{10 \times 2 \times 4 \times 60}{(4.18)(100)(40 - 25)} = 0.76 \text{ cal. per gram.}$$

451. Thermal Couples.—When two wires of dissimilar metals are joined together at the ends so as to form a closed circuit

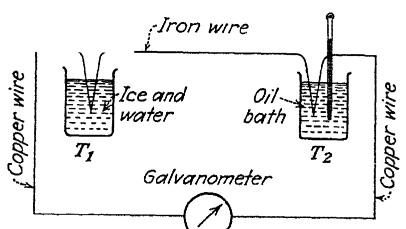


FIG. 409.—Thermoelectromotive force between metals.

(Fig. 409), and one of the junctions thus formed is kept at one temperature while the other is kept at a higher temperature, a current of electricity flows in the circuit. The electromotive force producing this current depends on the nature of the wires and the difference in temperature between the junctions. The rate at which this thermoelectromotive force changes with an

increase or a decrease in the temperature of one of the junctions is called the **thermoelectric height** of one metal in contact with the other.

By increasing the temperature of the hot junction (Fig. 410), the thermoelectromotive force is at first increased at a nearly uniform rate. As the temperature is still further increased in the case of an iron-copper thermal couple, the rate of increase in the thermoelectromotive force becomes less. When the temperature of the hot junction becomes 275°C., a further increase in its temperature causes a decrease in the thermoelectromotive force. This temperature at which the thermoelectromotive force ceases to increase and begins to decrease is called the **neutral temperature.** When the temperature of the hot junction is raised above the neutral temperature, the thermoelectromotive force continues to decrease and finally becomes zero at a temperature of approximately 550°C. If now the temperature of the hot junction is still further increased, the current in the circuit flows in the opposite direction. That temperature at which the thermoelectromotive force becomes zero is called the **temperature of inversion.**

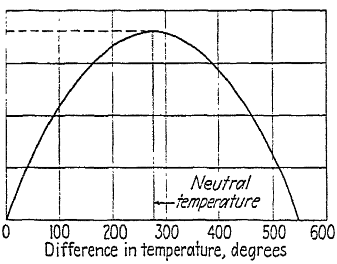


FIG. 410.—Change of thermoelectromotive force with temperature in an iron-copper couple.

By keeping one junction of the thermal couple at constant temperature, the temperature of the other junction can be measured by observing the electromotive force produced in the circuit. Thermal couples are particularly useful at high temperatures. For such purposes one of the wires is often made of platinum and the other of an alloy of 90 per cent platinum and 10 per cent rhodium. With such couples the temperatures of furnaces can be measured, but the thermal couples must be protected from direct contact with furnace gases or molten substances.

452. Thermopiles.—The electromotive force which can be obtained from a single thermal couple is small. To get larger electromotive forces for the same difference of temperature several couples may be connected in series just as the cells in a battery are sometimes connected in series. The total electromotive force is then the sum of the electromotive forces of each couple. Fig-

ure 411 shows a thermopile composed of six thermal couples connected in series. The wires of different materials are connected in alternate or zigzag positions so that every other junction can be heated or cooled. The electromotive forces thus produced are added.

Measurements of radiant heat energy are often made with thermopiles in which the thermal junctions are placed close together. One set of junctions is blackened and exposed to the radiant energy. The other set is shielded and kept at constant temperature. By connecting such a thermopile to a sensitive galvanometer changes of temperature of a hundred-millionth of a degree may be observed. With instruments of this kind, it is possible to measure the heat from distant stars.

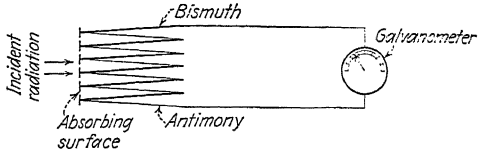


FIG. 411.—Thermopile consisting of a number of thermal junctions connected in series.

453. Peltier Effect.—If a current of electricity flows across the junction of two metals A and B (Fig. 412), for example, bismuth and antimony, there is either an evolution or an absorption of heat at the junction. If the electronic current flows from bismuth to antimony, heat is evolved. If it flows from antimony to bismuth, heat is absorbed. Now this evolution or absorption of heat can arise only because the two metals are at different potentials so that it requires work or heat to carry electricity from one metal to the other, or work is produced when the current flows from one metal to the other. The electron theory gives an explanation of this important phenomenon.

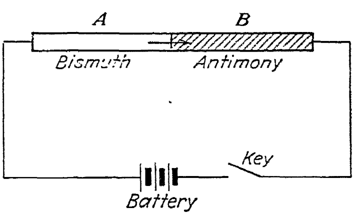


FIG. 412.—Peltier effect between metals.

The number of free electrons in different metals is not the same. When two different metals are placed in contact, there will be a flow of free electrons across the junction from the metal containing the greater number of electrons per unit volume to the metal containing the smaller number per unit volume. As soon as one or more electrons have left one metal (Fig. 413), that metal becomes positively charged and there is an attractive force tending to retard the migration of additional electrons. When these additional electrons have entered the second metal, it becomes charged negatively,

and this negative charge repels other electrons which tend to enter this metal. This repulsion decreases the further migration of electrons from the first to the second metal. As more and more electrons go over from the first to the second metal, this retarding force becomes larger and larger until it finally stops the flow of electrons from the first to the second metal. When equilibrium has thus become established, one of the metals is charged positively and the other charged negatively. There is, therefore, a difference of potential between the faces which are in contact. When a current

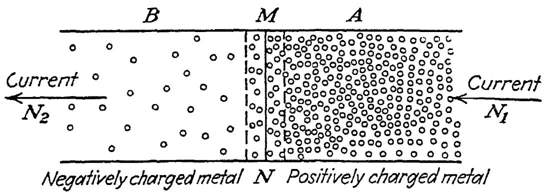


FIG. 413.—Diagrammatic representation of Peltier effect.

of electricity crosses this surface of contact, it flows from a higher to a lower potential or from a lower to a higher potential. In the former case it absorbs heat, and in the latter case it generates heat. In this way, the observed heating or cooling effect at the junction of two dissimilar metals in contact is explained.

454. Thomson Effect.—Let a metal rod AB (Fig. 414) have one end hot and the other end cold so that there is a flow of heat down the rod. To fix our ideas, suppose that the end A is in a mixture of ice and water at 0°C.

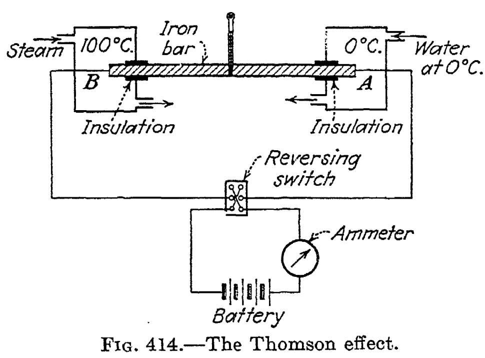


FIG. 414.—The Thomson effect.

and the end B is in steam at 100°C. The temperature of the middle of the rod is then 50°C. Now let the end of the rod be connected to the terminals of a battery so that a current of electricity flows from the hot to the cold end. The temperature indicated by the thermometer at the middle of the rod will be somewhat changed because of the Joulean heating in the rod. If now the current of electricity is reversed so that it flows from the cold to the hot end, the temperature at the middle of the rod will be again changed. This second change in the temperature means that the Joulean heating

is not the same in the rod when the current of electricity and the current of heat flow in the same direction as it is when they flow in opposite directions. The electron theory explains this phenomenon as follows:

In an unequally heated rod, the concentration of electrons will become greater at one end than at the other end. There will result a flow of electrons from, let us say, the hot to the cold end of the rod. In consequence of this flow, the hot end will be charged positively and the cold end charged negatively. This difference in the concentration of the electrons between the two ends of the rod sets up an electric force which finally stops the further movement of electrons along the rod. A state of equilibrium is reached in which the hot end of the rod is at a higher potential than the cold end. When an external electromotive force is applied to drive electrons from hot to cold end, this electromotive force must overcome the electric field in the rod due to the unequal concentration of electrons at the ends. Because of this fact, more work will be necessary to send the current through the rod than would be necessary if the concentration of the electrons were uniform throughout it. If, on the other hand, the current of electricity is caused to flow through the rod in the opposite direction, the heat generated by it will be less than in the former case because the electric field in the rod is now aiding the impressed electromotive force.

Problems

1. An electric toaster carries a current of 6 amp. at 110 volts. How much heat is given off per hour, and what is the cost per hour at the rate of 6 cts. per kilowatt-hour?

2. A 60-watt electric lamp is immersed in a vessel containing 80 l. of water. What fraction of a degree rise in temperature of the water is caused by operating the lamp for 8 min.?

3. An electrocalorimeter contains 300 g. of water. A change from 11 to 19°C. is produced in 12 min. by a certain current flowing through a resistance of 6 ohms immersed in the water. Find the current used.

4. A motor in an electric refrigerator uses energy at the rate of 300 watts. What does it cost to operate the motor for 8 hr., if the energy costs 5 cts. per kilowatt-hour?

5. An electric iron weighing 1.5 kg. has an average specific heat of 0.10. The heating unit takes 5.5 amp. from a 110-volt line. If half of the heat is lost by radiation, how long will it take to bring the iron to a temperature of 160°, if it is at 15°C. originally?

6. An electric flatiron takes a current of 7 amp. when the voltage is 110 volts. If the flatiron has a thermal capacity of 150 g.-cal. per degree centigrade, how long will it take the temperature to rise 10°C.?

7. A heating coil with a resistance of 6 ohms is used to evaporate water at the boiling point at the rate of 1.8 g. per second. What voltage must be applied to the coil?

8. What is the resistance of a coil of wire which when immersed in a liquid having a specific heat of 0.5 cal. per gram, heats 1 l. of the liquid from 25 to 75°C. in 15 min., if the terminals of the wire are connected to a dynamo supplying 110 volts? Specific gravity of liquid = 0.8.

9. A vessel containing 750 g. of water at 25°C. is heated electrically by means of a coil of wire immersed in it. The water begins to boil 4.5 min. after the electric circuit is closed. The difference of potential across the coil is 110 volts. Neglecting the heat capacity of the vessel, what must be the resistance of the coil?

10. How much water will be evaporated in 1 hr. by a current of 15 amp. flowing in a coil of wire having a resistance of 7 ohms? The coil is immersed in the water and 40 per cent of the energy is lost.

11. How much heat is given off per hour by an electric toaster which carries a current of 5.5 amp. at a potential difference of 110 volts?

12. What is the work done in joules in a circuit through which a current of 6 amp. flows for 8 min.? The resistance of the circuit is 16 ohms.

CHAPTER XXXVIII

THE CHEMICAL EFFECT OF AN ELECTRIC CURRENT

455. Electrolysis of Copper Sulphate.—If a beaker is filled with a solution of copper sulphate and two copper plates are inserted in it (Fig. 415) and these plates connected to the terminals of a battery, it will be found that when the plates are removed from the solution, one of them is bright and the other tarnished. By weighing the plates before and after placing them in the solution, it can be determined that the bright plate has gained in weight and that the other plate has lost in weight. There has been a transfer of electricity through the solution and copper has been carried from one plate to the other.

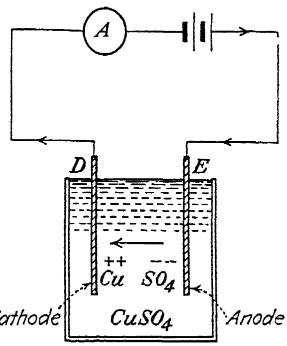


FIG. 415.—Electrolysis of copper sulphate. Copper ions migrate to the cathode and $\overset{--}{SO_4}$ ions to the anode.

The chemical formula for copper sulphate is $CuSO_4$, and when this substance goes into solution, some of it splits up or dissociates into $\overset{++}{Cu}$ and $\overset{--}{SO_4}$ ions. The copper atom losing two electrons carries two elementary charges of positive electricity, and the $\overset{--}{SO_4}$ ion retaining these two electrons carries two elementary charges of negative electricity. When the plates are thus placed in the solution, one connected to the positive terminal of the battery and the other to the negative terminal, each ion moves toward the plate with the opposite charge. Thus the $\overset{++}{Cu}$ ions reach the negative plate and deposit on it as neutral copper atoms by taking two electrons per ion from the plate. The $\overset{--}{SO_4}$ ions are attracted to the positive plate, where each takes up an atom of copper and gives up two electrons. The copper sulphate so formed goes into solution, keeping the amount of copper sulphate constant. Thus, the net effect is a gain of copper by the negative plate and a loss of copper by the positive plate.

The plate or terminal by which the conventional current enters the cell is called the anode, and that by which it leaves is called the cathode. The copper ion, being positive, travels in the cell in the direction of the conventional current, and the cathode thus is the terminal which gains in weight. The gain of copper by the cathode is equal to the loss of copper by the anode.

456. Electrolysis of Water.—Instead of taking a solution of copper sulphate in water, take a dilute solution of sulphuric acid in water. The current enters the solution by means of a platinum or silver strip and leaves by means of another platinum or silver strip. Over each of the metal strips or electrodes by which the current enters or leaves the solution, there is inverted (Fig. 416) a closed tube which is filled with some of the solution. The sulphuric acid in the solution splits up into $\overset{+}{H}$ ions and $\overset{--}{SO_4}$ ions. There are two $\overset{+}{H}$ ions for each $\overset{--}{SO_4}$ ion. Each of the hydrogen ions having lost one electron carries one elementary charge of positive electricity, and each of the $\overset{--}{SO_4}$ ions having an excess of two electrons carries two elementary charges of negative electricity. Under the action of the electric force arising from the cells connected to the electrodes, the hydrogen ions go to the cathode or electrode at which the electronic current enters the solution. The $\overset{--}{SO_4}$ ions go to the other electrode. The hydrogen ions receive electrons from the cathode, two atoms combining to form neutral hydrogen molecules which collect at the top of the tube. The $\overset{--}{SO_4}$ ions give up their charge of excess electrons at the anode and then unite with two atoms of hydrogen to form sulphuric acid. These atoms of hydrogen are taken from a molecule of water, and one atom of oxygen is thus set free. The atom of oxygen thus set free unites with another atom of oxygen to form oxygen gas, and this gas rises to the top of the tube inverted above the anode. The sulphuric acid formed at the anode goes into solution again, and consequently the amount of sulphuric acid in the water does not change. The water, however, is decomposed into its constituents.

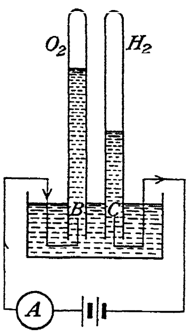


FIG. 416.—Electrolysis of water. The volume of the hydrogen is double the volume of the oxygen.

hydrogen and **oxygen.** The volume of the hydrogen is twice that of the oxygen, and the weight of the oxygen eight times that of the hydrogen. The hydrogen goes in the direction of the positive ion current, because it carries a positive charge of electricity.

457. Copper Plating.—The deposition of metal by means of an electric current is often used in a commercial way to cover one metal with another. Suppose that it is desired to cover a metal with a coating of copper. A solution of copper sulphate is prepared and the metal to be coated is placed in this solution. There is also inserted into the solution a plate of pure copper. The two metals, the copper plate and the metal to be electroplated, are then connected to the terminals of a battery. The copper plate is made the electrode by which the electronic current leaves the solution and the plate to be covered is the electrode by which the electronic current enters the solution. The copper traveling in the solution in the direction of the positive ion current is deposited on the metal to be covered. This metal receives the desired coating and the copper plate loses an equal quantity of copper. The amount of copper sulphate in the solution remains unchanged.

458. Nickel Plating.—If it is desired to nickel-plate a metal, a solution of nickel salt and a plate of nickel are chosen instead of a solution of copper sulphate and a copper plate. Otherwise the arrangement of the apparatus is the same. As before, the solution does not change in strength. The nickel plate loses in weight and the metal to be plated gains an equal weight.

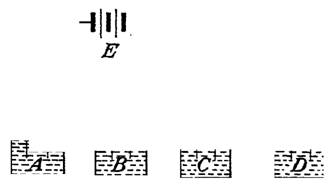


FIG. 417.—Relation of chemical equivalent to mass deposited. The greater the chemical equivalent the greater the mass deposited.

459. Faraday's Laws of Electrolysis.—To understand the quantitative laws of electrolysis, consider a number of electrolytic cells connected in series (Fig. 417). Assume that all of the electrodes are made of platinum so that there are no secondary reactions between the electrodes and the ions in solution. Now let these cells be connected to a battery so that the same current flows through each cell for the same time.

Suppose A contains a solution of silver nitrate, B a solution of hydrochloric acid, C a solution of copper sulphate, and D a solution of nickel chloride. There will be liberated at the respective cathodes silver, hydrogen, copper, and nickel. By determining the amount of substance liberated at each cathode by a given current in a given time, it is possible to establish the two laws of electrolysis.

1. **The mass of any substance liberated is proportional to the current flowing and the time during which it flows.** Hence, the mass liberated is proportional to the product of the current and the time.

Suppose that in cell *B* the current flows until 1 g. of hydrogen is liberated. Then, in cell *A* 108 g. of silver will have been deposited; in cell *C*, 31.5 g. of copper; and in cell *D*, 29 g. of nickel. Whatever current is chosen and whatever the length of time it is allowed to flow, it is found that the masses deposited in these cells always bear the same ratio to each other. **This result may be stated by saying that the masses deposited are always proportional to the quotient obtained by dividing the atomic weight by the valence.** The ratio of the atomic weight to the valence of an element is called the **chemical equivalent, or combining weight.** Where a substance is monovalent, the chemical equivalent is equal to the atomic weight. If the substance is divalent, the chemical equivalent is equal to one-half of the atomic weight.

The second law of electrolysis may be stated as follows:

2. **The masses of different substances liberated by a given current in a given time are proportional to the chemical equivalents or combining weights of the substances.**

Let A = the atomic weight of an element.
v = the valence of the element.
A/v = the combining weight.
m = the mass of element liberated by Q coulombs of electricity.

$$m = KAQ$$

$$K = \frac{1}{96{,}500}.$$

$$m = \frac{AQ}{96{,}500v}$$

$$m = \left(\frac{\text{chemical equivalent}}{96{,}500}\right) \times Q.$$

$$m = ZQ = Z \cdot I \cdot t,$$

where I = the current in amperes.
t = the time in seconds.
Z = the constant known as the electrochemical equivalent.

460. Electrochemical Equivalent.—From Faraday's law of electrolysis it is evident that the mass of a substance liberated by 1 amp. in 1 sec. can be calculated as soon as the mass of some

other element liberated by 1 amp. is known, for this law states that masses of different substances deposited by equal currents in equal times are proportional to the chemical equivalents of the substances. Knowing then that 1 amp. deposits 0.0011180 g. of silver in 1 sec., to find the amount of copper deposited by 1 amp. in 1 sec., divide the amount of silver deposited by the combining weight of silver and multiply the quotient by the combining weight of copper.

$$\left.\begin{array}{l}\text{Amount of copper deposited}\\ \quad\text{by 1 amp. in 1 sec.}\end{array}\right\} = \frac{0.0011180}{107.88} \times 31.5 = 0.0003295 \text{ g.}$$

The amount of substance deposited in 1 sec. by a current of 1 amp. is defined as the electrochemical equivalent of that substance. (For table of electrochemical equivalents, see Appendix C.)

Example.—How much copper will be deposited from a solution of copper sulphate in 12 hr. by a current of 2 amp.?

$$\begin{aligned}\text{Weight deposited} &= \text{electrochemical equivalent} \times \text{current in amperes} \times \\ &\quad\ \text{time} \\ &= 0.0003295 \times 2 \times 12 \times 60 \times 60 \\ &= 28.5 \text{ g.}\end{aligned}$$

461. Effect of Electrolysis on Water Mains.—Electrolysis is an important factor in determining the life of underground pipes which are laid near

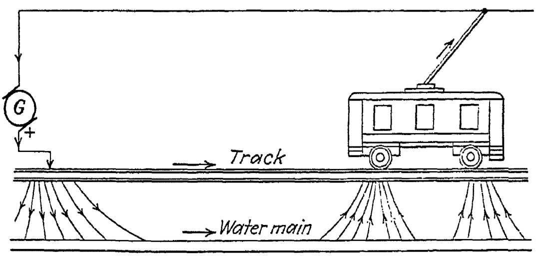


FIG. 418.—Electrolysis of water mains.

electric railways. It is intended that the track should offer a path by which the dynamo is connected to the wheels of the car. In case the joints of the track are not intimately united, the current, instead of traveling from the generator to the car through the track, in part finds its way into the water mains as indicated in Fig. 418 and then returns to the track and the car. Where the current leaves the water main to return to the track and car or to return to the generator, the metal forming the pipe is eaten away as was the anode in the case of the electrolysis of copper sulphate.

The electrochemical action is somewhat complicated, but it is a case of electrolysis in which iron is removed from the positive electrode to make iron sulphate and the weight of the iron thus reduced, the pipe weakened, and its life shortened.

462. Charge on an Ion.—From the data on electrochemical equivalents and from the number of molecules in a gram-molecule of substance, it is possible to calculate the charge on an ion in an electrolytic solution. The important fact comes out of this calculation that the charge on a univalent ion is the same as the charge on an electron as determined directly by Millikan (see Sec. 556).

From the kinetic theory of gases, it is known that the number of molecules in 1 c.c. of a gas at standard pressure and temperature is 2.70×10^{19}. The density of hydrogen at 0°C. and atmospheric pressure is 0.0000898 g. per cubic centimeter. When 96,500 coulombs pass through a solution in which hydrogen is a free ion, 1 g. of hydrogen is liberated. To liberate 1 c.c. of hydrogen, *i.e.*, 0.0000898 g., there must pass through the solution $0.0000898 \times 96{,}500 = 8.67$ coulombs of electricity. There are two atoms of hydrogen in each molecule.

Let N = the number of atoms of hydrogen in 1 c.c. of molecular hydrogen.

$$2 \times 2.70 \times 10^{19} = 5.4 \times 10^{19}.$$

Let e = charge on each hydrogen ion

$$\begin{aligned} Ne &= 8.67 \text{ coulombs.} \\ e &= \left(\frac{8.67}{5.4 \times 10^{19}}\right) \\ &= 1.60 \times 10^{-19} \text{ coulomb} \\ &= 4.80 \times 10^{-10} \text{ e.s.u.} \end{aligned}$$

The charge on the electron as determined by Millikan $= 4.77 \times 10^{-10}$ e.s.u. Hence, the charge on a univalent ion is the same as the charge on an electron.

463. Computing Avogadro's Number.—Now, if we accept the value of the charge on the electron as determined by Millikan, we can calculate the number of atoms in a gram-atom, that is, in as many grams of an element as there are units in its atomic weight. Since the atomic weight of hydrogen is 1.008, it requires 1.008 g. of hydrogen to make 1 g.-atom of hydrogen. Similarly, since the atomic weight of silver is 108, it requires 108 g. of silver to make 1 g.-atom of silver.

Since 1 coulomb of electricity liberates 0.00001045 g. of hydrogen, the number of coulombs necessary to liberate 1.008 g. of hydrogen, which is 1 g.-atom of hydrogen, is $\frac{1.008}{0.00001045} = 96{,}500$ coulombs. This number is known as Faraday's electrolytic constant or as a Faraday. One gram-atom of silver, that is, 108 g. of silver, will also be liberated by 96,500 coulombs.

In each of these cases, the ion carries one positive charge and has lost one electron, and has on it a charge which is just equal to the charge on an electron, except that the charge on the ion is positive and the charge on the

electron is negative. Now, the charge on the electron and, therefore, the charge on each ion of hydrogen or silver is 1.592×10^{-19} coulomb, and it requires 96,500 coulombs to deposit 1 g.-atom of a monovalent element like hydrogen or silver. Hence, the number of atoms in 1 g.-atom is

$$\frac{96{,}500}{1.592 \times 10^{-19}} = 6.06 \times 10^{23} \text{ atoms.}$$

Each molecule of hydrogen contains two atoms. Hence, the number of molecules in a gram-atom of hydrogen = 3.03×10^{23}. According to Avogadro's principle, every gas under the same conditions of temperature and pressure contains the same number of molecules per unit volume.

464. Electrolytic Conduction in Solids.—Certain solids which at ordinary temperatures are thought of as non-conductors of electricity become conductors at higher temperatures. Conduction in these cases is like conduction in ordinary electrolytes, that is, by transfer of ions through the substance. An interesting illustration of electrolytic conduction in solids is shown in Fig. 419. An electric light bulb is immersed in a molten mixture of $NaNO_3$ and $NaNO_2$ in equal proportions. The temperature of the mixture is kept at about 300°C. The filament in the light bulb is heated to incandescence in the usual manner and this heated filament gives off thermions necessary to carry the current from the filament to the inner wall of the glass bulb. One terminal B of the filament is connected through a regulating resistance R and an ammeter A to the negative terminal of the battery. The positive terminal C of the battery is inserted in the molten mixture of $NaNO_3$ and $NaNO_2$. An electric circuit is thus completed through the molten mixture and the evacuated bulb. In the molten mixture, the positive current is carried by the positive sodium ions. These ions also migrate through the glass wall of the bulb and are then deposited on the interior surface of the bulb as a thin layer of sodium. Between the inner surface of the bulb and the filament, the current is carried by thermions or electrons emitted by the filament. In the metallic

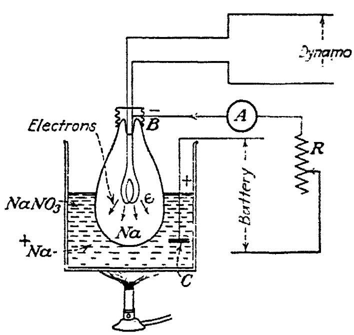


FIG. 419.—Electrolytic conduction through the walls of a glass bulb. Sodium is deposited on the inner surface of the bulb.

wires completing the electric circuit through the battery, the current is carried by electrons moving through these connecting wires. Conduction in the walls of the glass bulb is by means of the transfer of ions through the glass and is, therefore, a case of electrolytic conduction in solid glass. This method of producing a thin layer of sodium on the interior of an evacuated bulb has proved to be of importance in the manufacture of photoelectric cells.

465. Primary Batteries.—In the preceding sections it was seen that an electric current can produce certain chemical actions, like the decomposition of copper sulphate or the decomposition of water into its elements. The reverse effect also takes place. Certain chemical actions when properly arranged can produce currents of electricity. Indeed, this is the first method by which currents of electricity were obtained. These sources of current in which chemical action is directly responsible for the flow of electricity have been called **primary batteries.** Except in one or two cases, they have ceased to be of any practical importance. They have been replaced by storage batteries or by dynamos. Their action will be understood from a study of the Leclanche cell and its modification, the **dry cell.**

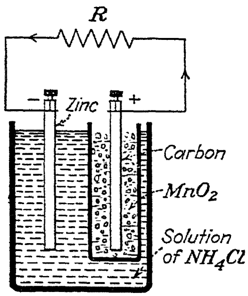


FIG. 420.—Leclanche cell.

466. Leclanche Cell.—The Leclanche cell (Fig. 420) is important because a later modification is now widely used. This cell consists of a zinc rod which dips into a solution of ammonium chloride or sal ammoniac. The other electrode of the cell is a carbon rod which is surrounded by a cup filled with powdered manganese dioxide, called a depolarizer, its purpose being to react chemically with the accumulating hydrogen and produce water vapor. Graphite is also added to increase further the conductivity. The ammonium chloride in the electrolyte dissociates into $\overset{+}{NH_4}$ and $\overset{-}{Cl}$. The $\overset{+}{NH_4}$ carries one positive charge and $\overset{-}{Cl}$ carries one negative charge. While the current is flowing, the $\overset{+}{NH_4}$ goes to the carbon plate and there gives up its charge. It then breaks down into NH_3 and H. The hydrogen collects on the electrode except for the action of the depolarizer. Since the

action of the depolarizer is slow, the cell is adapted to open-circuit work in which it is used for a short time and then allowed to stand for some time.

467. The Dry Cell.—A type of this cell now widely used (Fig. 421) is called the **dry cell.** It differs from the Leclanche cell just described only in the fact that the electrolyte is in the form of a paste instead of the solution of ammonium chloride. The negative electrode is the zinc can which contains the carbon and the paste. The zinc on the inside of the can is covered with several layers of blotting paper, and the space around the carbon rod which forms the positive electrode is filled with a mixture of carbon, manganese dioxide, and sawdust saturated with a solution of ammonium chloride. The top is sealed with wax to prevent the evaporation of the moisture in the paste. This type of cell is now much used in flash lights. It has an electromotive force of about 1.5 volts.

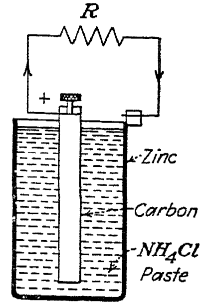


Fig. 421.—Dry cell.

468. Lead Storage Cell.—In the lead storage cell, both the positive and negative plates are made of heavy lead grids full of holes or grooves filled with the active material. The positive plates are made of lead peroxide, and the negative plates are made of spongy lead. A cell is formed of a number of such plates, alternately negative and positive. They are set in a glass jar filled with dilute sulphuric acid. The negative plates are connected together and form one effective plate of the battery, and the positive plates are connected together to form the other plate. There is always one more negative plate than positive plates so that a positive plate always lies between two negative ones. In this way, both sides of the positive plates are charged or discharged. The formation of the oxide which takes place during charging is accompanied by an increase in volume, causing a swelling of the plate. Since this swelling takes place equally on both sides of the positive plate, there is little tendency to buckle or warp the plates.

The nearness of the plates together and the large area obtained by using a number of plates cause the cell to have a small resistance. By using a large number of plates and making the areas as large as possible, the current capacity of the cell is increased.

When a lead storage cell is delivering a current, both the lead peroxide on the positive electrode and the spongy lead on the negative electrode are gradually converted to lead sulphate. Finally, the two electrodes become very much alike and the current ceases to flow. The battery is now said to be discharged.

In order to charge the battery and make it ready for further use it is only necessary to maintain an electric current in it in a direction opposite to that in which the current flows when the cell is in use. This process is known as charging. During this process the sulphate on the negative plate is slowly reduced to spongy lead, while the sulphate on the positive plate is slowly reconverted to lead peroxide. After a sufficient time of charging, the original condition of the battery is restored.

The chemical action taking place during the process of charging and discharging may be represented by the following equations:

Charging:

At positive plate, $PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$.

At negative plate, $PbSO_4 + H_2 = Pb + H_2SO_4$.

Discharging:

At positive plate, $PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O$.

At negative plate, $Pb + SO_4 = PbSO_4$.

It is seen from these equations that, during the process of charging, sulphuric acid is formed. Consequently the density of the electrolyte rises when the batteries are being charged. During discharge, sulphuric acid disappears and water is formed. For this reason, the density of the electrolyte decreases during discharge. By observing the density of the electrolyte, it is possible to find how nearly the battery is discharged. When the battery is fully charged, the density of the acid should be from 1.28 to 1.3. The electromotive force of this cell when fully charged is about 2.2 volts.

469. Edison Storage Cell.—In the Edison storage cell, the positive plate is a nickel-plated steel grid with perforated steel tubes which are filled with alternate layers of nickel hydroxide and flaked nickel. The nickel hydroxide is changed by electrochemical action into nickel peroxide (NiO_2). The flaked nickel is added to reduce the internal resistance of the cell. The negative plate (Fig. 422) is also made of nickel-plated steel grid with

a large number of rectangular pockets filled with powdered iron oxide (FeO). In charging the cell, the iron oxide is changed into metallic iron. The positive plate of the cell thus becomes nickel peroxide (NiO_2) and the negative plate, metallic iron (Fe). The electrolyte is a 21 per cent solution of caustic potash (KOH).

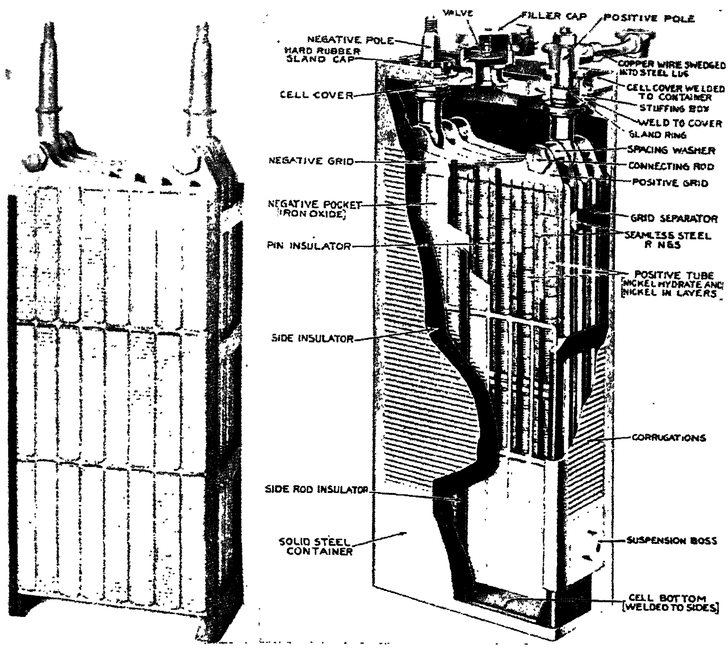


FIG. 422.—Edison storage cell.

When the cell is delivering current, the nickel peroxide is reduced to a lower oxide (Ni_2O_3) and the iron oxidized to form iron oxide (FeO). The reaction is

$$\begin{array}{ccccccc} 2NiO_2 & + & Fe & = & Ni_2O_3 & + & FeO. \\ \text{nickel peroxide} & + & \text{iron} & = & \text{nickel oxide} & + & \text{iron oxide} \end{array}$$

When the cell is being charged by sending a current through it in the direction opposite to that in which the current flows when the cell is discharging, the reaction is reversed and becomes

$$\begin{array}{ccccccc} Ni_2O_3 & + & FeO & = & 2NiO_2 & + & Fe. \\ \text{nickel oxide} & + & \text{iron oxide} & = & \text{nickel peroxide} & + & \text{iron} \end{array}$$

The electrolyte does not enter into either of these reactions. Its density changes only slightly during the reactions. The effect of charging and discharging is to transfer oxygen from one plate of the cell to the other. The normal electromotive force is about 1.2 volts.

470. Weston Standard Cell.—Standard cells are not used to furnish current. They offer a means of obtaining definite and constant electromotive forces. They are the concrete standards in terms of which differences of potential are measured. The most widely used of these standard cells is the **cadmium** or **Weston cell.**

This cell (Fig. 423) consists of an H-shaped, hermetically sealed glass tube containing pure mercury as the positive electrode in

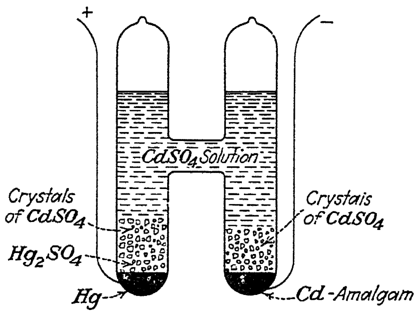


FIG. 423.—Weston cell. A standard for electromotive forces.

one leg and a cadmium amalgam in the other leg as the negative electrode. Platinum wires sealed through the glass connect the electrodes with the circuit. Above the mercury is placed a paste of mercurous sulphate. The electrolyte is a saturated solution of cadmium sulphate. In order to insure that this solution remains saturated at all temperatures, an excess of crystals of cadmium sulphate is added to the solution.

The electromotive force of this cell changes very little with the temperature. For this reason it is considered the best standard available. At 20°C. its electromotive force is 1.0183 volts. For other temperatures, the electromotive force is given by the equation

$$E_t = 1.0183 - 0.00004\ (t - 20)\ \text{volts.}$$

471. Potentiometer.—The fundamental principle of a simple potentiometer is illustrated in Fig. 424, where YZ is wire of

uniform cross section and resistance. This wire is stretched along a scale graduated in centimeters or inches. A battery B with an electromotive force greater than the electromotive force of the battery to be studied is connected through the regulating resistance R to the terminals of the wire YZ, and a constant current is allowed to flow in this wire. The fall of potential per unit length in the wire will be the same at all points along the wire. A standard cell S of known electromotive force is connected to one terminal of the wire and through the galvanometer G to a contact L that can be moved along the wire. By means of a double-throw switch, the standard cell may be replaced by a cell X whose electromotive force is to be determined. The key K allows the circuit through the galvanometer to be opened or closed.

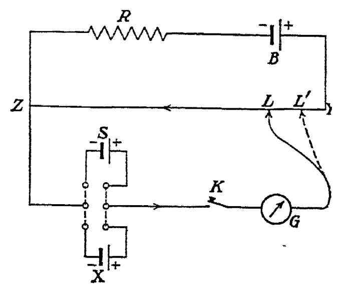


FIG. 424.—Diagrammatic representation of a slide-wire potentiometer.

If the terminals of the battery B and the standard cell S are connected into the circuit so that they oppose each other, the drop of potential along the bridge wire from L to Z tends to send a current through the galvanometer in one direction, and the standard cell tends to produce a current in the galvanometer in the opposite direction. By moving the slider L along the wire, a point can be reached where the two tendencies to send a current through the galvanometer just balance each other. There will, then, be no current in the galvanometer, and the potentiometer is said to be balanced. Since the resistance of the wire is uniform, the fall of potential between Z and L is proportional to the length of the wire between Z and L.

If now the double-throw switch is turned so as to replace the standard cell by the cell X, there will, in general, be a current in the galvanometer G. By moving the slider L along the wire another point, L' may be reached where the electromotive force of the cell X is just balanced by the drop of potential between the point Z and the second balance point L'. The electromotive force of the cell X is proportional to the length of the wire between Z and L'—the balance point on the wire when the cell X is in the circuit.

Let E = the electromotive force of the cell X.
D = the distance from Z to L', the balance point when the cell X is in the circuit.
e = the electromotive force of the standard cell.
d = the distance from Z to L, the balance point when the standard cell is in the circuit.

Then

$$E = kD.$$
$$e = kd.$$
$$\frac{E}{e} = \frac{D}{d}.$$

If a Weston cell with an electromotive force of 1.0183 volts is used as a standard cell,

$$E = 1.0183\frac{D}{d}$$

Hence, by measuring D and d along the potentiometer wire when the potentiometer is balanced, the electromotive force E of the cell X can be calculated. Figure 425 shows a familiar form of accurate potentiometer manufactured by Leeds and Northrup.

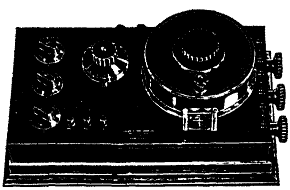


FIG. 425.—Type-K potentiometer. (*Courtesy Leeds and Northrup Company.*)

Example.—In measuring the electromotive force of a cell, it was found that the length of bridge wire to produce a balance when the standard cell was in the circuit was 60 cm., and the corresponding length when the other cell was in the circuit was 80 cm. If a Weston cell with an electromotive force of 1.0183 volts was used as a standard, what was the electromotive force of the other cell?

$$E = 1.0183\,\frac{80}{60} = 1.344 \text{ volts.}$$

Problems

1. A copper plate weighing 109.376 g. is placed in an electroplating bath. A steady current is sent through the bath for 20 min., and the weight of the plate is increased to 110.490 g. What was the current in amperes, assuming copper is divalent?

2. Find the electrochemical equivalent of platinum if a current of 0.080 amp. deposits 0.584 g. in 2 hr.

3. An object which has a surface of 16 sq. cm. is to be plated with silver. What will be the average thickness of the silver when a current of 0.15 amp. flows for 24 hr.?

4. A cylinder of carbon 2 cm. in diameter is immersed to a depth of 8 cm. in a solution of copper sulphate. How long will it take for a current of 2.5 amp. to plate a deposit of copper 0.12 mm. thick on the side of the carbon cylinder?

5. A current of 6 amp. flows for 4 hr. through a series of cells containing nickel nitrate, copper sulphate, and silver nitrate, respectively. Find the quantity of nickel, copper, and silver deposited.

CHAPTER XXXIX

THE MAGNETIC CIRCUIT AND ITS APPLICATIONS

472. Flux and Flux Density.—The introduction of an iron core into a solenoid increases very much the number of lines of force which would cross a small air gap cut across the iron core perpendicular to its axis. The number of lines of force passing through 1 sq. cm. of the solenoid before the iron is introduced is a measure of the field intensity in the solenoid or the **magnetizing force. It is equal to the number of dynes acting on unit magnetic pole placed in the solenoid.** The number of lines passing through each square centimeter after iron or some other substance is introduced into the solenoid is called the **flux density or magnetic induction** and will be denoted by B. The total flux is the flux density times the area of the cross section. If ϕ denotes the total flux and A the area, then

$$\phi = BA.$$

The total flux is measured in maxwells. One line of magnetic force is called a *maxwell.*

473. Permeability.—Since the number of lines of force in the iron core is always much greater than the number in air for the same current in the solenoid, it is convenient to take the ratio of the number in the iron to the number in the air as a measure of the magnetic properties of the iron. This ratio of the flux density B set up in the iron or other magnetic substance to the magnetizing force is called the **permeability.** It is equal to the number of lines of force per square centimeter in the iron divided by the number of lines per square centimeter when the solenoid is filled with air or some other gas.

$$\text{Permeability} \quad \frac{\text{flux density}}{\text{magnetizing force}} = \frac{B}{H}.$$

The permeability is unity for a vacuum and practically unity for air and non-magnetic materials.

Example.—The magnetizing force in a specimen of iron is 3 oersteds and the corresponding flux density is 600 lines per square centimeter. What is the permeability of the iron?

$$\text{Permeability} = \frac{\text{flux density}}{\text{magnetizing force}} = \frac{600}{3} = 200.$$

474. Magnetomotive Force.—Just as hydraulic pressure is necessary to force water through a pipe and an electric pressure to produce a current of electricity in a wire, so also a "magnetic pressure" is necessary to produce magnetic flux in a magnetic circuit. This magnetic pressure is called **the magnetomotive force** and bears the same relation to the magnetic circuit that the electric pressure bears to the electric circuit. **The magnetomotive force can be defined as the work necessary to carry unit magnetic pole around the magnetic circuit.** It is found by multiplying the magnetic field by the length of the magnetic circuit.

$$\text{M.m.f.} = H \times l.$$

For a solenoid

$$H = \frac{4\pi NI}{10l} = \frac{1.26NI}{l},$$

where N is the total number of turns in the solenoid.

$$\text{M.m.f.} = \frac{1.26NIl}{l} = 1.26\ NI.$$

The product NI is known as the **number of ampere turns** on the circuit.

The distinction between the magnetic field inside the solenoid and the magnetomotive force must be kept clearly in mind. The magnetic field is the force on unit pole inside the solenoid. It is determined by the product of the number of turns per centimeter and the electric current.

$$H \quad \frac{4\pi NI}{10l}$$

The magnetomotive force is the work to carry unit pole around the magnetic circuit. The unit of magnetomotive forces in the c.g.s. system is the erg per unit pole and is called the gilbert. It is determined by the product of the current and the total number of turns on the circuit.

$$\text{M.m.f.} = \frac{4\pi NI}{10}$$

Example.—Find the magnetomotive force for a solenoid consisting of 1,500 turns, when the current is 20 amp.

$$\text{M.m.f.} = \frac{4\pi NI}{10} = 1.26NI = 1.26 \times 1{,}500 \times 20 = 3{,}780 \text{ gilberts.}$$

475. Reluctance.—In a magnetic circuit, there is a magnetic reluctance which is analogous to the electric resistance in an electric circuit. **The opposition or resistance which must be overcome when the magnetic flux is established is the reluctance of the circuit.** The reluctance increases with the length of the magnetic circuit and decreases as the area or the cross section of the circuit is increased. The reluctance is inversely proportional to the permeability of the material out of which the circuit is made. To find the resistance of a conductor, the following equation is used:

$$R = \frac{kl}{a},$$

where k = the resistance per unit length of the wire of unit cross section.
l = the length of the wire.
a = the area of the cross section.

The reluctance of a magnetic circuit is given by the equation

$$Rel. = \frac{1}{\mu}\frac{L}{A},$$

where L = the length of the magnetic circuit in centimeters.
A = the area of the cross section in square centimeters.
μ = the permeability of the material of the circuit.

These equations are similar in form, except that the reciprocal of the permeability is used in the equation for the reluctance in place of the specific resistance in the equation for the resistance. A magnetic circuit has a reluctance of 1 c.g.s. unit when a magnetomotive force of 1 gilbert produces a flux of 1 maxwell in it.

Example.—A piece of iron which is 50 cm. long has a cross section of 10 sq. cm. and a permeability of 500. What is the reluctance?

$$\text{Reluctance} \quad \frac{\text{length}}{\text{area} \times \text{permeability}}.$$

$$\frac{50}{10 \times 500} = \frac{1}{100} = 0.01 \text{ c.g.s. unit.}$$

476. The Law of the Magnetic Circuit.—There is a magnetic pressure called the magnetomotive force producing the flux in a magnetic circuit, and there is a magnetic resistance which must be overcome in establishing the flux in the circuit. The relation between the magnetomotive force, the reluctance, and the flux in the circuit is the same as the relation between the electromotive force, the electric resistance, and the current in an electric circuit. Ohm's law states that

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}$$

$$I = \frac{E}{R}.$$

The **law of the magnetic circuit** states that

$$\text{Flux} = \frac{\text{magnetomotive force}}{\text{reluctance}} = \frac{\text{m.m.f.}}{\text{rel.}}$$

$$\phi = \frac{\text{m.m.f.}}{\text{rel.}}$$

where I = current in amperes.
ϕ = magnetic flux lines of force.
e.m.f. = electromotive force in volts.
m.m.f. = magnetomotive force in gilberts.
R = electrical resistance in ohms.
rel. = reluctance in c.g.s. units.

Example.—Find the number of lines of force in magnetic circuit in which the reluctance is 0.05 c.g.s. unit and the magnetomotive force is 45 gilberts.

$$\text{Flux} \quad \frac{\text{magnetomotive force}}{\text{reluctance}} = \frac{45}{0.05} \quad \text{900 lines of force.}$$

477. Magnetization of Iron.—To study the magnetic properties of iron it is best to have the iron in the form of a ring surrounded by a solenoid. The iron fills the whole space in which there are lines of magnetic force, and no lines pass out into the air. As the magnetizing force is slowly increased from zero to larger values, the flux density or induction increases slowly at first but soon rises rapidly. The rate of increase of the induction falls off again at higher values of the magnetizing force and for large values of the magnetizing force the induction increases very slowly. If the induction is plotted against the magnetizing force,

the resulting curve is called the **magnetization curve** (Figs. 426 and 427). From the inspection of such a curve it is seen that sometimes a small increase in the magnetizing force causes a large increase in the flux density, and that at other places on the curve a large increase in the magnetizing force is necessary to produce even a small change in the flux density. This means that the permeability of iron or steel depends on the number of lines of force which it already contains. Hence, the permeability is not a constant but varies with the flux density. The dotted curve of Fig. 426 shows the way in which the permeability of a specimen of iron changes with the number of lines of force in it.

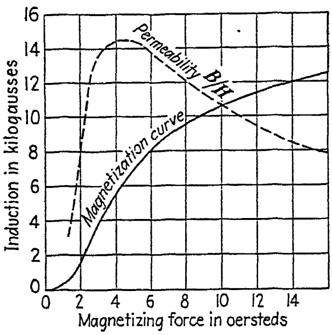


FIG. 426.

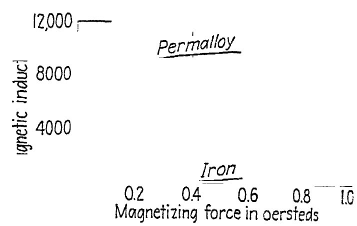


FIG. 427.

FIG. 426.—Magnetization and permeability of iron.

FIG. 427.—Magnetization in permalloy compared to that in iron. For low magnetic fields the permeability of permalloy is very high.

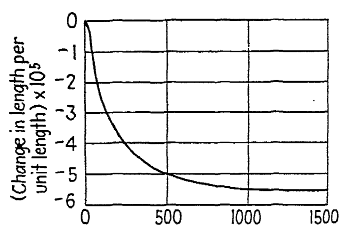


FIG. 428.—Change of length in nickel in a longitudinal magnetic field.

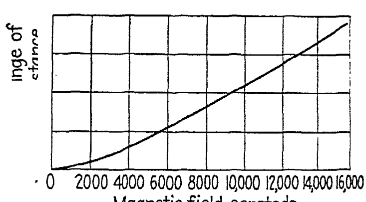


FIG. 429.—Change of resistance of bismuth in a transverse magnetic field.

A comparison of the magnetic induction in iron and permalloy (Fig. 427) shows that the magnetic induction in permalloy for low fields is much greater than it is in iron.

Since for large magnetic fields the flux density increases very slowly with the magnetizing force, it is impracticable to mag-

netize a piece of iron beyond a certain flux density. By this is meant that a point has been reached where a large increase in the magnetizing force is necessary to produce a small increase in the flux density. This point is marked by the knee of the curve (Fig. 426). When iron is used in a motor or generator, it is not profitable to use a flux density which is greater than the flux density corresponding to knee of the magnetization curve. It requires too many ampere turns to produce the added flux. Magnetization changes other properties of substances. For example, the length of a nickel wire is decreased by a longitudinal magnetic field (Fig. 428) and the resistance of a bismuth wire is increased by a magnetic field (Fig. 429).

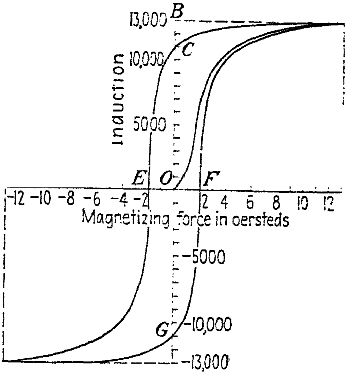


FIG. 430.—Hysteresis cycle. The area of the figure shows the energy spent in the cycle.

478. Hysteresis.—If a specimen of iron has been magnetized by subjecting it to constantly increasing field intensities, and if then the field intensity is decreased, the flux does not decrease along the same curve by which it has increased, but it follows a curve which lies above the magnetization curve. The flux lags behind the magnetizing force. When the magnetizing force has become zero, there still remains a considerable amount of flux in the iron. The flux which remains when the magnetizing force has been reduced to zero is called the **residual magnetism.** If the direction of the magnetizing force be now reversed, the flux quickly becomes zero. That magnetizing force which is necessary to reduce the flux to zero is termed the **coercive force.** It is represented in Fig. 430 by the line *OE*. When the magnetizing force is still further increased, the iron becomes magnetized in the opposite direction. If the magnetizing force is again gradually decreased, the flux again gradually lags behind the magnetizing force, giving a curve which in this case lies below the original curve. When the magnetizing force has been made zero again, there is a residual magnetism in the iron in the opposite direction to that in the former case. This residual magnetism is represented by the line

OG. The reversal of the magnetizing force again causes the residual magnetism to disappear. The coercive force necessary to reduce the residual magnetism to zero is in this case represented by the line *OF*. By increasing the magnetizing force sufficiently, the cycle closes and the iron is in the condition in which it was at the beginning of the cycle.

During the complete cycle of the field intensities, the induction has described a loop called a **hysteresis cycle.** This arises out of the fact that the induction lags behind the magnetizing force. The area included in this loop is a measure of the loss of energy in the iron during the cycle. The energy thus lost cuts down the efficiency of the machine. It is consequently of importance to use soft iron in an electric machine, for soft iron has a smaller loss due to hysteresis than is found in cast iron.

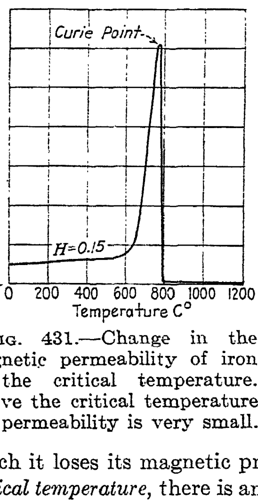


FIG. 431.—Change in the magnetic permeability of iron at the critical temperature. Above the critical temperature the permeability is very small.

479. Influence of Temperature on Magnetic Phenomena.—For convenience consider separately the three groups of substances; that is, *ferromagnetic*, *paramagnetic*, and *diamagnetic substances.*

a. Ferromagnetic Substances.—When a piece of iron, nickel, or cobalt is heated to higher and higher temperatures, it by-and-by reaches a certain definite temperature at which it loses its magnetic properties. At this temperature, known as the *critical temperature*, there is an abrupt change from the ferromagnetic to the paramagnetic state. The behavior of iron near its critical temperature is shown in Fig. 431, where the permeability for a given magnetic field has been plotted against the temperature. It is seen from this curve that at the critical temperature, which for iron is about 790°C., the permeability decreases to a small fraction of its original value. Above the critical temperature, the permeability corresponds to that of a paramagnetic substance.

b. Paramagnetic Substances.—In the case of paramagnetic sugstances such as oxygen, the magnetic susceptibility is independent of the magnetic field but is inversely proportional to the absolute temperature. This relation is expressed in Curie's law which states that the magnetic susceptibility varies inversely as the absolute temperature.

Let χ = the magnetic susceptibility.
T = the absolute temperature.
C = a constant depending on the nature of the substance.

$$\chi = \frac{C}{T}.$$

Weiss has shown that this law is only approximately true.

c. Diamagnetic Substances.—The effect of temperature on the diamagnetic susceptibility of bismuth is shown in Fig. 432. It is seen from these curves that the diamagnetic susceptibility of bismuth changes abruptly when bismuth passes from the solid to the liquid state. The magnetic susceptibilities of water and quartz are nearly independent of the temperature.

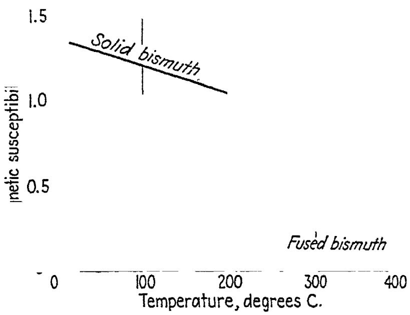


FIG. 432.—Change in diamagnetic susceptibility of bismuth at the melting point. Above the melting point the susceptibility is small.

480. Electromagnet.—When a piece of soft iron is placed inside of a solenoid, it becomes magnetized and thus produces an **electromagnet.** Such magnets are much stronger than permanent magnets. The iron core loses much of its magnetism as soon as the current is removed from the coil. Under the action of the

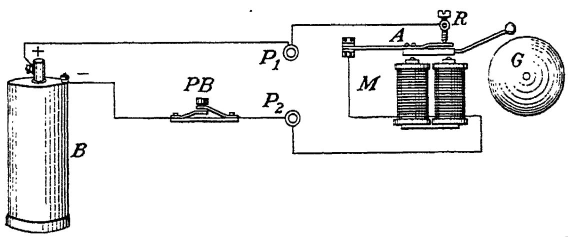


FIG. 433.—Electromagnet in a door bell. The electric circuit is alternately opened and closed.

current, the minute molecular magnets are more or less oriented in one direction. When the current ceases to flow, this directive force is removed and these molecular magnets again point in various directions.

There are many applications of electromagnets. In the horseshoe type the windings are carried around the two legs of the magnet so as to make the windings continuous if the bar were straightened out. In this form of magnet the opposite poles are brought closer together, and both poles are thus made available for lifting or holding. An illustration of this kind of magnet is found in the ordinary door bell. A hammer is pressed against a metallic

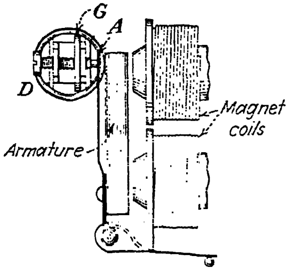


FIG. 434.—Vibrating mechanism of a door bell.

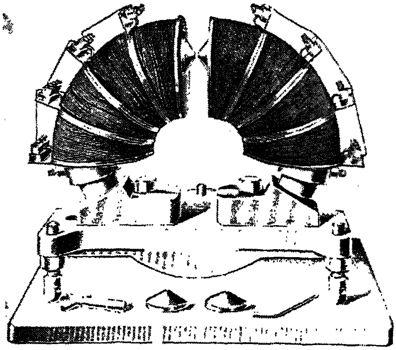


FIG. 435.—Large electromagnet. The pole pieces are made in the form of cones to concentrate the magnetic field.

point by means of a spring A (Figs. 433 and 434). It thus closes the electric circuit containing the battery B. A soft-iron armature attached to the hammer is pulled over to the electromagnet when the circuit is closed through a push button. The electric circuit is thus opened. The hammer is allowed to fly back and the circuit is again closed. The circuit is thus alternately opened and closed.

Large electromagnets have great lifting power. They are frequently used for lifting heavy loads in foundries. They hold the heavy load in the air

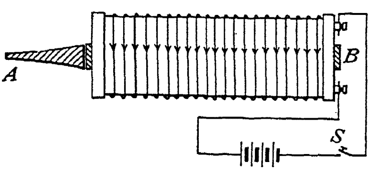


FIG. 436.—Electromagnet for medical work. Used for extracting particles of iron from the eye, etc.

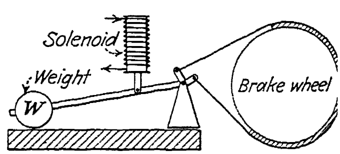


FIG. 437.—Magnetic brake. The brake is operated by magnetic forces arising from the current in the solenoid.

as it is carried from one part of the building to another. Smaller magnets are used in medical work, especially in extracting foreign magnetizable materials from the eye. A magnet used for this purpose is represented in Fig. 436.

481. Magnetic Brake.—The magnetic brake is an application of the forces to be obtained from the magnetic field inside a solenoid. It consists

of a solenoid (Fig. 437) by means of which the weight W on the lever arm of the brake can be lifted. When current is supplied to the solenoid, this weight is lifted and the break released. If the current is shut off, the weight W is released, the brake is applied to the brake wheel, and the machinery is stopped.

482. Magnetic Separator.—A magnetic separator (Fig. 438) consists of two pulleys over which passes a belt that carries the material to be sepa-

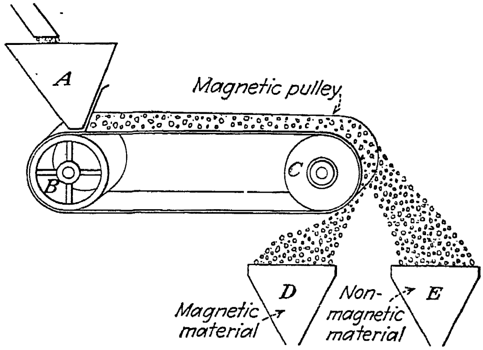


FIG. 438.—Magnetic separator, used to separate magnetic from non-magnetic materials.

rated from the hopper. Some of this material is magnetic and some non-magnetic. This belt with its load passes over the pulley C which is magnetized. The magnetic material is carried around farther than the non-magnetic material so that the magnetic material naturally separates from the non-magnetic. The magnetic material is caught in the hopper D and the non-magnetic in the hopper E.

483. The Electric Horn.—The electric horn as used on automobiles is an application of an electromagnet. When the electric current is closed, the

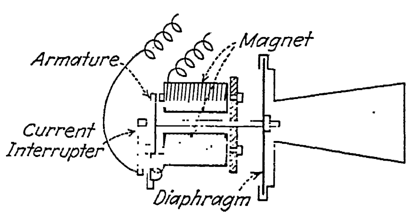


FIG. 439.—Electric horn. The electric circuit is alternately closed and opened.

electromagnet (Fig. 439) attracts the armature which slightly displaces the diaphragm in the horn. This displacement of the armature also causes the electric circuit to be broken at the point in the figure which is marked "current interrupter." The armature is then released by the electromagnet, and the electric circuit is again closed. The action is then repeated with the result that there is a continuous vibration of the diaphragm in the horn. The electric horn behaves essentially like an electric door bell.

Problems

1. A flux density of 210 lines per sq. cm. is produced in a wire of permalloy placed in the earth's field where the latter has an intensity of 0.8 oersted. What is the permeability of the alloy?

2. A magnet has a winding 4 cm. long, with 80 turns to the centimeter. How many ampere turns are there, if the current is 4 amp.? What is the magnetomotive force?

3. An electromagnet has a total flux of 140,000 lines. It is wound with a coil containing 800 turns, carrying a current of 0.06 amp. What is the reluctance of the magnetic circuit?

4. An electromagnet requires a magnetomotive force of 800 gilberts. The magnetizing coil is of 320 turns and 60 ohms resistance. What voltage must be applied to the coil?

5. The reluctance of a large magnet is 0.032 c.g.s. unit and the winding has 1,600 turns. What current is necessary in order to produce a flux of 10^6 lines?

6. A magnetic circuit consists of: (1) 12 cm. of iron with a cross section of 6 sq. cm. and permeability of 200; (2) 80 cm. of wrought iron with a cross section of 18 sq. cm. and a permeability of 1,600; (3) 1.4 cm. of air with a cross section of 10 sq. cm. Calculate the reluctance of this circuit.

7. How many ampere turns are required to produce a flux of 9,000 lines in the air gap of a magnetic circuit, the gap having a width of 0.8 cm. and an area of 22 sq. cm.; the remainder of the circuit consisting of 120 cm. of iron with a cross section of 18 sq. cm. and a permeability of 750?

8. What is the reluctance of an iron rod bent in the form of a circle having a radius of 10 cm.? The diameter of the cross section of the rod is 1 cm., and the permeability of the iron is 800.

9. A solenoid is 30 cm. long and 2.5 cm. in diameter. It is wound with 5,000 turns of copper wire. What current must be sent through the windings to produce a magnetic field of 5 oersteds at the center of the solenoid?

10. An iron anchor ring is wound with a solenoid having 800 turns. If the current in the solenoid is 10 amp. and the permeability of the iron core is 250, what is the magnetic induction, assuming the anchor ring has a cross section of 12 sq. cm. and a mean diameter of 40 cm.?

CHAPTER XL

INDUCED CURRENTS

484. Currents Induced by Moving Magnets.—It was discovered by Faraday that if the ends of a coil of wire of many turns are connected to the terminals of a galvanometer (Fig. 440) and if the pole of a magnet is pushed into the coil, the pointer of the galvanometer moves, showing the presence of a current in the circuit while the magnet is in motion. As soon as the magnet ceases to move, the current ceases in the galvanometer. When the pole of the magnet is removed from the coil, the current flows in the direction opposite to that in which it flowed when the magnetic pole was inserted into the coil. If the opposite pole is now thrust into the coil, the galvanometer deflects in the opposite direction, showing that the current is in the opposite direction to that in which it flowed in the former case.

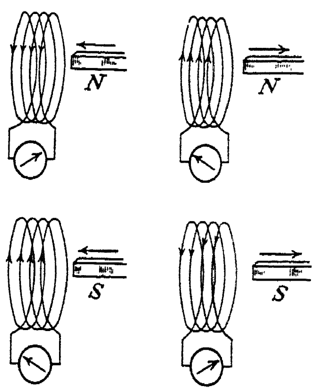


FIG. 440.—Currents induced by a moving magnet. The current flows in opposite directions when the motion of the magnet is reversed.

It is immaterial whether the coil is moved with respect to the magnet or the magnet is moved with respect to the coil. The experiment may then be performed in the following way instead of in the way already described. Over one pole of a magnet is thrust a coil containing many turns of wire. As the coil is moved over the pole, a deflection is noted in the galvanometer indicating an induced current. When the coil is removed, the deflection is in the opposite direction. If the coil is thrust over the north pole instead of the south pole, the deflections are reversed.

These experiments show that it is possible to produce a momentary current by moving a magnet with respect to a coil of wire or by moving a coil with respect to a magnet. Currents pro-

duced in this way are known as **induced currents.** This is the most important method of producing electric currents.

485. Currents Induced by Currents.—It has already been seen that a coil of wire in which a current of electricity is flowing behaves in every way like a magnet. If, therefore, a coil of wire carrying a current of electricity is brought near another coil of wire connected to a galvanometer, effects will be expected very similar to those observed when a magnet is moved with respect to such a coil. In Fig. 441 a coil of wire S is connected to the terminals of a galvanometer, and a second coil P is connected to the terminals of a battery. If the coil P which is carrying a current of electricity is brought near the coil S, a deflection is noted in the galvanometer while P is in motion. When P is removed, there is again a deflection in the galvanometer which is in the direction opposite to that noted while the coil P was being brought up. If the coils are wound in the form of solenoids so that one can be inserted into the other, the induced current is larger in each case. The effect observed in the galvanometer is the same, whether P is brought near to the coil S or the coil S is brought near to P. The greater the current in the coil P, the greater is the deflection in the galvanometer caused by the induced current. The larger the current in P, the greater the magnetic field surrounding it. By inserting an iron core in the coil P, the magnetic field about it may be very greatly increased; and when the coil P containing such an iron core is introduced into the coil S, the induced current in S is much increased over that which was obtained when the coil P did not contain an iron core.

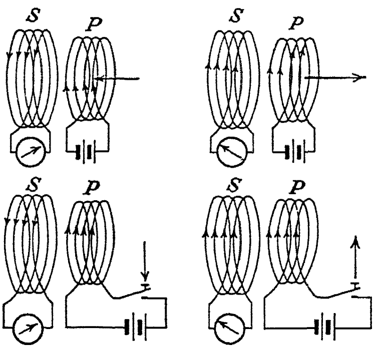


FIG. 441.—Currents induced by changing currents. Induced current in the secondary flows in opposite directions when the flux is increased and when it is decreased.

486. Making and Breaking the Circuit.—When the coil P is placed inside the coil S with the iron core in position, and the current through P is suddenly broken, a large induced current is set up in the coil S. By breaking the circuit of the coil P, the

current in this coil disappears and with it also disappears the magnetic field which surrounded it. The removal of these lines of force from the coil P and the coil S at the same time causes a current in the coil S. This current in the coil S continues only so long as the current in P is changing. If now the circuit of P is again closed allowing the current in P to flow, there is an induced current in S while the current in P is building up to its final value. This current, however, is in the direction opposite to that of the current observed when the circuit was broken.

By comparing the currents obtained in these cases with those obtained by bringing up or taking away the coil P, it is found that the effect of bringing the coil P up to S is the same as closing the circuit in the coil P, and the effect of taking P away from the coil S is the same as opening the circuit of the coil P. The coil P through which the current flows from the battery is known as the **primary**, and the coil S in which the electromotive force is induced is called the **secondary**.

487. Law of Induced Electromotive Forces.—These observations show that there is an induced current set up in the secondary (1) when the current in the primary is started or stopped; (2) when the magnitude of the current in the primary is either increased or decreased; (3) when the primary coil carrying a current of electricity is moved either closer to or farther from the secondary; (4) when a permanent magnet is moved with respect to a coil of wire. All of these facts may be included in the two following laws:

1. **There is an induced electromotive force in any coil of wire in which the number of lines of force is changing, and the magnitude of this electromotive force is proportional to the rate at which the number of lines of magnetic force through the coil is changing.**

2. **There is an induced electromotive force in any conductor which is moving across lines of force, and the magnitude of this electromotive force is proportional to the rate at which the lines of force are being cut by the conductor.**

488. Lenz's Law.—An experimental determination of the direction of the induced currents in these circuits shows that **the induced current always flows in such a direction that its magnetic field opposes the changes of conditions giving rise to the induced current.** This is known as Lenz's law.

To illustrate Lenz's law, consider a north-seeking pole which is being pushed into a coil of wire. A current will flow in the coil, and this current will be in such a direction that its magnetic action will oppose the motion of the magnet. Now the force exerted by a north-seeking pole is the only force which can oppose the approach of a north pole. Hence, the current in the coil of wire will be in such a direction that it will produce a north pole at the end of the coil lying nearest the approaching north pole of the magnet. Consider now that the north pole of the magnet is taken away from the coil. The current in the coil will flow in such a direction that it will oppose the taking away of this north pole. In order to oppose the taking away of the pole, an attraction must be produced. It is, therefore, necessary to make the end of the coil nearest the receding magnetic pole a south pole. Consequently the current in the coil flows in such a direction that a south pole is developed at the end nearest the receding north pole, thus producing an attraction which opposes the motion of a north magnetic pole.

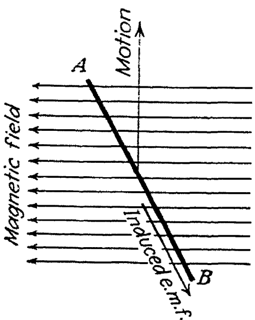


FIG. 442.—The direction of the induced electromotive force depends both on the direction of motion and on the direction of the magnetic field.

489. Direction of Induced Electromotive Force.—It is of much convenience to have a rule by which to remember the direction in which the induced current flows when the direction of the motion and the direction of the magnetic field are known. Suppose that a straight wire is moved across a magnetic field (Fig. 442). If the conductor is carried upward, the current is found to flow from A to B. If the conductor is carried downward, the current flows in the opposite direction. If the direction of the magnetic field is reversed without changing the direction of motion of the conductor, the direction of the current is reversed. In order, then, to know the direction of the current, it is necessary to know the direction of the magnetic field and the direction of motion of the conductor. The following rule has been suggested by Fleming.

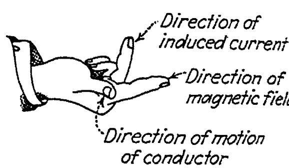


FIG. 443.—Right-hand rule for determining the direction of an induced current in a conductor.

Extend the thumb (Fig. 443), forefinger, and middle finger of the right hand so they are at right angles to each other. Let the thumb point in the direction in which the conductor is moving and the forefinger in the direction of the magnetic field. Then the middle finger will point in the direction of the positive induced current.

490. Electromotive Force Induced in Moving Wire.—The magnitude of the electromotive force induced in a moving conductor depends on three factors: (1) the number of lines of force per unit area in the magnetic field; (2) the speed with which the

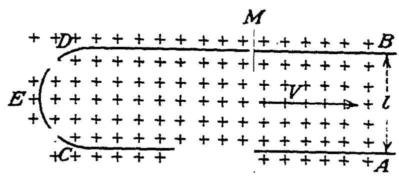


FIG. 444.—The greater the velocity of the wire ML, the greater the magnitude of the induced electromotive force. Reversing the direction of motion of ML reverses the direction of the induced electromotive force.

conductor moves at right angles to the field; (3) the length of the wire which is being moved. In Fig. 444

$$E = \frac{B \times v \times l}{10^8} \text{ volts},$$

where E is the induced electromotive force in volts, v the velocity in centimeters per second, and l the length of the wire in centimeters. The greater the speed with which the conductor moves, and the stronger the magnetic field and the greater the length or number of wires, the greater is the induced electromotive force. These relations may all be expressed by saying that the induced electromotive force is proportional to the number of lines of force cut each second by the moving conductor. If a conductor cuts 100,000,000 lines of force per second, there is a difference of potential of 1 volt between its ends.

Example.—A wire which is 50 cm. long is carried through a magnetic field of 15,000 lines per square centimeter at the rate of 100 cm. per second. What is the electromotive force between its ends?

The number of lines cut = 15,000 × 50 × 100 = 75,000,000 per second.

$$\text{E.m.f.} = \frac{\text{number lines cut}}{10^8} = \frac{75{,}000{,}000}{10^8} = 0.75 \text{ volt.}$$

491. Electromotive Force from Change of Flux.—It has just been seen that the electromotive force in abvolts set up in any moving conductor equals the number of lines of force which it cuts per second. The effect is the same whether the conductor is moved across the lines of force or the lines of force are made to cut across the conductor. Hence it follows that in any circuit or coil the electromotive force produced by induction is equal to the change per second in the number of lines of force included by the circuit. This change may arise out of the fact that the

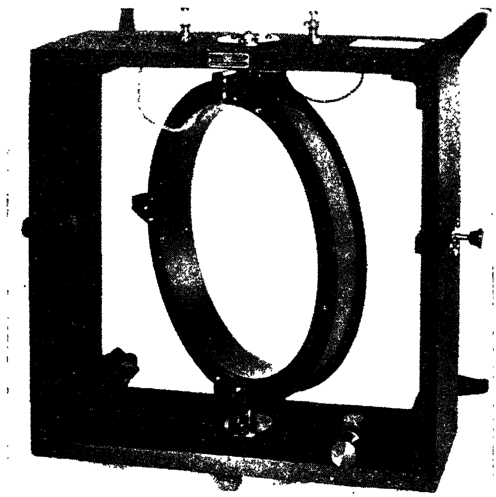


FIG. 445.—Earth inductor, used to measure the magnitude and direction of the earth's magnetic field. (*Courtesy Central Scientific Company.*)

number of lines of force in a circuit of given dimensions either increases or decreases. On the other hand, it may arise from the fact that the dimensions of the circuit either increase or decrease, and because of this fact the circuit includes a greater or less number of lines of force.

If E = the average electromotive force in volts during the time interval t;
N_1 = the number of lines of force through the circuit at the beginning of the interval;
N_2 = the number of lines of force through the circuit at the end of the interval;

then

$$E = \frac{N_2 - N_1}{t \times 10^8} \text{ volts. (Appendix E-15.)}$$

By taking the time interval t very short, the average value of the electromotive force over a very short interval becomes nearly equal to the instantaneous value at that instant.

When several turns of wire are in the coil, each turn of wire cuts all the lines of force. To obtain the total induced electromotive

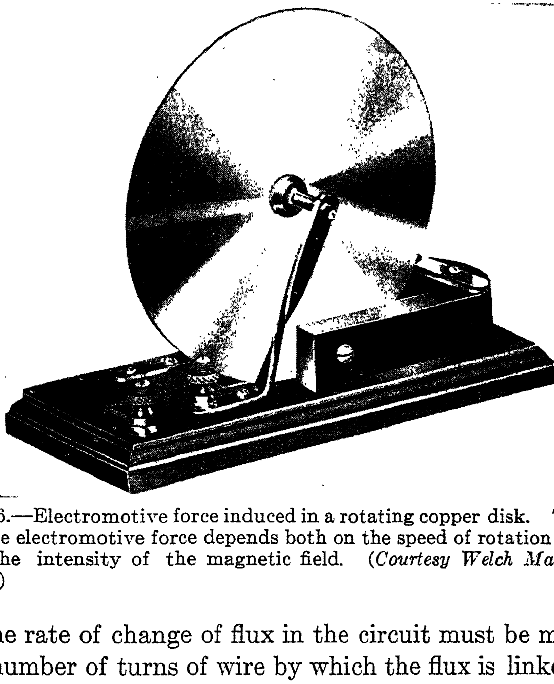


FIG. 446.—Electromotive force induced in a rotating copper disk. The magnitude of the electromotive force depends both on the speed of rotation of the disk and on the intensity of the magnetic field. (*Courtesy Welch Manufacturing Company.*)

force, the rate of change of flux in the circuit must be multiplied by the number of turns of wire by which the flux is linked. The expression for the induced electromotive force then becomes

$$E = \frac{N_2 - N_1}{t \times 10^8} n \text{ volts,}$$

where n is the number of turns in the coil.

When a coil of wire is rotated in the magnetic field of the earth (Fig. 445), an induced electromotive force is produced in it. If a copper disk (Fig. 446) is rotated between the poles of a magnet an electromotive force is produced along the radius of the disk.

Example.—A circuit which consists of a single turn of wire encloses 10,000 lines of force. If these lines of force are removed from the circuit in 0.01 sec., find the induced electromotive force which is set up in the circuit.

$$\text{Induced e.m.f.} = \frac{\text{rate of change of flux}}{10^8} \text{ volts.}$$

$$E = \frac{N_2 - N_1}{t \times 10^8} \text{ volts.}$$

$$E = \frac{10{,}000}{0.01 \times 10^8} = 0.01 \text{ volt.}$$

Example.—A circuit consists of 25 turns of wire, and the magnetic flux in it is changing at the rate of 1,000,000 lines per second. What electromotive force in volts is produced in the circuit?

$$\text{E.m.f.} = \frac{\text{time rate of change of flux} \times \text{number of turns}}{10^8}.$$

$$E = n\frac{N_2 - N_1}{t \times 10^8} \text{ volts.}$$

$$E = \frac{25 \times 1{,}000{,}000}{10^8} = 0.25 \text{ volt.}$$

492. Quantity of Electricity from Induced Electromotive Forces.—The induced current in the circuit at any instant is obtained according to Ohm's law by dividing the instantaneous electromotive force by the resistance of the circuit. Hence,

$$i = \frac{e}{R},$$

and since $e = \frac{N_2 - N_1}{t}$ e.m.u.,

$$i = \frac{N_2 - N_1}{Rt} \text{ e.m.u.}$$

From this equation it is seen that since the resistance of the circuit is constant, the instantaneous value of the current is greatest when the induced electromotive force is greatest; that is, the induced current is greatest when the number of lines of force in the circuit is changing most rapidly.

The quantity of electricity which flows through the circuit is equal to the current in the circuit times the time during which this current flows. Hence,

$$i\,t = Q = \frac{N_2 - N_1}{R \times 10^8} \text{ coulombs.}$$

From this equation it is seen that the total quantity of electricity in coulombs which flows past a given point in the circuit because

of induction is equal to the total change in the number of lines of force through the circuit divided by the resistance of the circuit in ohms and by 10^8. The quantity of electricity is independent of the time during which the lines of force are being cut and is the same whether the lines of force are cut rapidly or whether they are cut slowly. The current, however, is not the same in the two cases. When the lines of force are cut slowly, a smaller current flows for a longer time. When the lines of force are cut rapidly, a larger current flows for a shorter time, so that the quantity of electricity is the same in the two cases.

Example.—Find the quantity of electricity in coulombs which flows past any point in a closed circuit, consisting of 20 turns of wire with a resistance of 10 ohms, when the total flux in it is changed from 10,000,000 to 1,000,000 lines.

$$Q = \frac{10{,}000{,}000 - 1{,}000{,}000}{10 \times 10^8} \times 20 \text{ coulombs}$$
$$= \frac{9{,}000{,}000 \times 20}{10 \times 10^8} = 0.18 \text{ coulomb.}$$

493. The Induction Coil.—An important illustration of induced electromotive forces is found in the induction coil. This coil

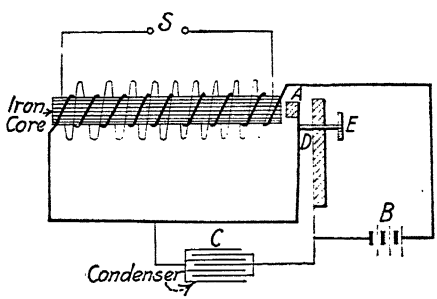


FIG. 447.—Wiring diagram of an induction coil. The current in the primary is interrupted by opening and closing the circuit at D.

consists essentially of a core made of fine soft-iron wire (Figs. 447 and 448). Around this iron core is wound the primary coil made of a few turns of heavy copper wire. Insulated carefully from this primary coil is the secondary which is wound on the outside of the primary. The secondary contains a large number of turns of fine wire which is silk covered for better insulation. By making or breaking the current rapidly in the primary, an induced electromotive force is set up in the secondary. In order to break

the current rapidly in the primary, a hammer interrupter is connected in the primary circuit. This interrupter consists of a heavy spring which is fastened to a piece of iron. This piece of iron is near the iron core of the primary. As the iron core is magnetized by the primary current, it pulls over the spring with this piece of iron. The primary current flows through the spring to a point D in contact with the spring and then out to the battery B. When the spring is pulled away from the contact D, the primary current is broken. The current in the primary dies away and the magnetism of the iron core disappears. The spring

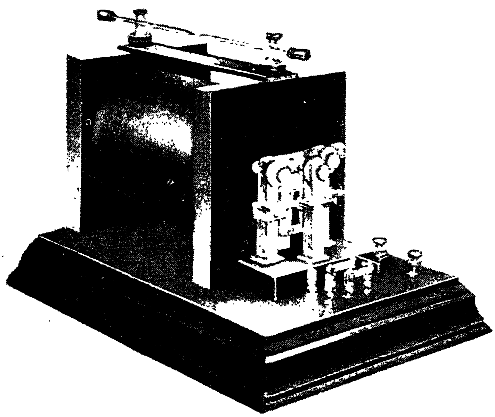


FIG. 448.—Induction coil. *(Courtesy Welch Manufacturing Company.)*

with the piece of iron flies back into position and the primary circuit is again closed. This is the same action which takes place in the electric door bell.

When the primary circuit is broken, an electromotive force is induced in the secondary. This electromotive force is large since the number of turns in the secondary is much larger than the number of turns on the primary. If the terminals of the wire forming the secondary are not joined together, a spark tends to jump from one end of the wire to the other.

To get the greatest induced electromotive force the primary current must be stopped as quickly as possible. To effect this, a condenser C is connected across the gap in the primary. This condenser is made of sheets of tinfoil which are separated by paraffin paper. It acts as a storage place into which the current

can surge when the circuit is broken. The rapidity with which the current in the primary is eliminated is thus increased and the magnitude of the induced electromotive force in the secondary is correspondingly increased. The condenser also helps to prevent sparking at the contact when the primary current is broken.

494. Make-and-break Ignition.—The principle of the induction coil is made use of in the make-and-break system of ignition (Fig. 449). A coil of wire consisting of many turns is wound on a soft-iron core. This coil is connected through a battery E to two points inside the cylinder of the gas engine. One of these points is stationary, and the other moves. When the points separate, the current is broken; but the tendency of the current to keep on going is so great that a spark occurs at the time at which the points are separated. This spark which jumps

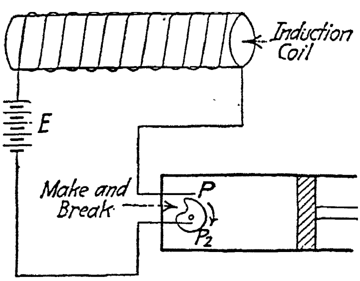


FIG. 449.—Make-and-break ignition.

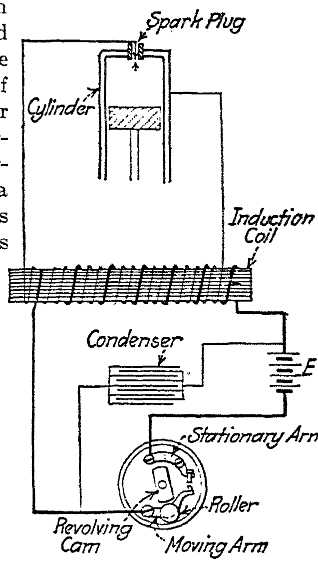


FIG. 450.—High-tension ignition.

across the gap between the points ignites the gases in the cylinder of the engine.

495. High-tension Ignition.—The electrical connections of a high-tension ignition system are represented in Fig. 450. Such a system consists essentially of an induction coil with terminals of the secondary connected to the spark plug. The terminals of the primary are connected through the battery E to the stationary and revolving arms of the contact maker. In order to insure quick cessation of the primary current and prevent destructive arcing at the vibrator contacts, a condenser is connected across the battery and the vibrator contacts. When the piston of the engine is near the end of its forward stroke, the revolving of the contact maker opens the primary circuit and causes a spark in the secondary at the spark plug. This produces an explosion of the gases in the cylinder.

496. The Transformer.—If an iron ring (Fig. 451) of any form be wound with two separate coils which are insulated from each

other and an alternating current be maintained in one of them, an alternating current of the same frequency will flow in the other. Let one of these coils be called the primary and the other the secondary. Energy has been handed from the primary to the secondary through the medium of the iron core. If the small losses in the iron core can be neglected, the energy in the primary circuit must be equal to the energy in the secondary.

Let E_p = the voltage in the primary.
I_p = current in the primary.
E_s = voltage in the secondary.
I_s = current in the secondary.

Then

$$E_pI_p = E_sI_s.$$

The ratio N_p to N_s, of the number of turns in the primary to the number of turns in the secondary determines the ratio of E_p to E_s. If there are a large number of turns on the secondary and few turns on the primary, the electromotive force of the secondary will be large in comparison with that of the primary. It can be shown that

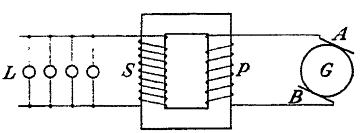


FIG. 451.—A transformer showing primary and secondary connected by an iron core.

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}.$$

It is seen from this equation that in that coil, or winding, of a transformer where the electromotive force is large, the current is small; and that in that coil, where the electromotive force is small, the current is large. Hence, by means of such a transformer, a small electromotive force and a large current may be transformed into a large electromotive force and a small current. There are, then, two types of transformers: **step-up transformers,** which increase the voltage and decrease the current; and **step-down transformers,** which decrease the voltage and increase the current. In the former, the secondary has a large number of turns in comparison with the primary. In the latter, the reverse is the case. In practice, the current is generated at high voltage, then transmitted to the place where it is to be used, and there transformed into a lower voltage. There is

less loss of energy in transmitting small currents at high voltages than there is in transmitting large currents at low voltage.

A transformer for ringing door bells has been represented in Figs. 452 and 453. It takes a small current at 110 volts and transforms it into a larger current at a potential of a few volts.

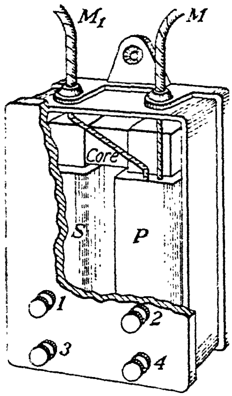


FIG. 452.—Door-bell transformer.

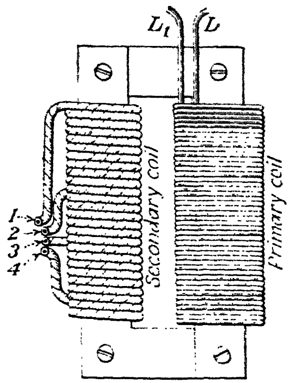


FIG. 453.—Details of door-bell transformer.

497. The Autotransformer.—For small transformers where the ratio of transformation is not large, economy of construction and efficiency of operation are obtained by using the same coil for both primary and secondary. Such a transformer is known as an **autotransformer.** The arrangement and connections of the coils are shown in Fig. 454. The entire coil AC is the primary of the transformer, and the part between B and C is the secondary. If N_1 is the total number of turns on the primary AC; N_2 the number of turns on the secondary between B and C; E_1 the electromotive force applied to the primary; E_2 the electromotive force set up in the secondary; I_1 the current in the primary; and I_2 the current in the secondary, then

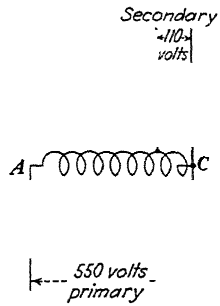


FIG. 454.—An autotransformer to produce low voltages in the secondary.

$$\frac{E_1}{E_2} \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}.$$

Since N_2 is less than N_1, E_2 is also less than E_1, and the transformer steps down the voltage.

Such step-down autotransformers are used for reducing the electromotive force applied to alternating-current motors while

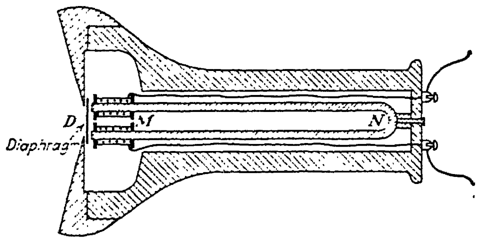


FIG. 455.—Telephone receiver. Change in the current in the coils about the poles of the magnet causes a movement of the diaphragm D.

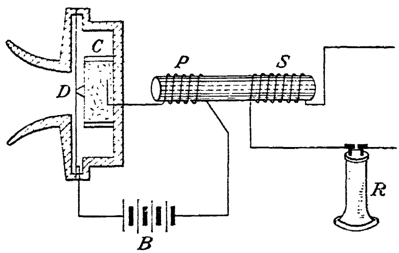


FIG. 456a.—Diagrammatic representation of a telephone transmitter. Change of pressure on the carbon granules causes a change of current in the primary which induces a current in the secondary.

the motor is being started. In such cases they take the place of a starting box.

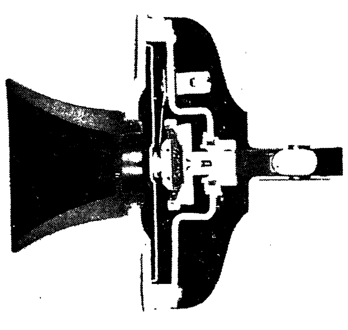


FIG. 456b.—Telephone transmitter. (*Courtesy Bell Telephone Laboratories.*)

498. Telephone.—The receiver of a telephone (Fig. 455) consists of a hard-rubber handle containing a hard-steel horseshoe magnet, around the end of which are wound coils of fine wire. The ends of this wire are brought to binding screws on the end of the receiver. Close to the end of the magnet MN which carries the coil of wire, there is placed a disk D of thin sheet iron. This disk is supported at its edges so that it is free to vibrate in the middle which is placed near the end of the magnet. It is clamped at the edges to the receiver by means of a hard-rubber cap with a hole in its center.

The transmitter (Fig. 456a) consists of a cell C which contains carbon granules between two plates of polished carbon. In front of this cell is

mounted a metal diaphragm D. The terminals of the battery B are connected to metal plates at the front and back of this carbon cell so that the current from the battery goes through the carbon resistance. The battery B is also connected in series with the primary P of a small induction coil. The secondary S of this induction coil is connected to the line wires leading to the receiving station. As the diaphragm D vibrates in response to sound waves, it changes the pressure on the carbon granules in the cell C. This causes a change in the resistance of the cell, since the resistance of loose, carbon contacts varies rapidly with the pressure. This variation in resistance produces a fluctuation in the current from the battery B. This fluctuation of current in the primary produces induced currents in the secondary. These induced currents in the secondary pass along the line

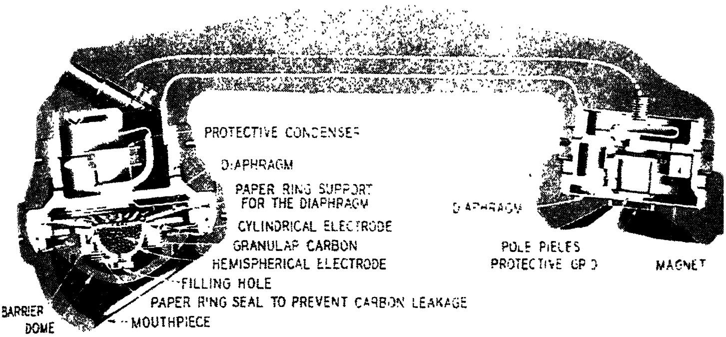


Fig. 456c.—Combined telephone receiver and transmitter. (*Courtesy Bell Telephone Laboratories.*)

wires to the receiver at the other end of the line. They there change the strength of the magnet which is acting on the diaphragm in the receiver. Vibrations are thus set up in the diaphragm of the receiver. These vibrations correspond to the sounds which produced the vibrations of the diaphragm in the transmitter. Hence, the sounds imparted to the diaphragm of the transmitter are reproduced by the diaphragm of the receiver.

Problems

1. A wire 250 cm. long moves across a magnetic field with a velocity of 8 m. per second. Find the voltage induced in the wire, if the field intensity is 4,000 lines per square centimeter.

2. An electromotive force of 4 volts is obtained by moving a wire 160 cm. long at a rate of 15 m. per second across a uniform magnetic field. What is the intensity of the field?

3. An exploring coil has an area of 1.5 sq. cm. and consists of 14 turns. When it is jerked out of a magnetic field with its plane perpendicular to the field, it gives a throw in the galvanometer of 6 cm. A magnetic standard having 12 turns is connected to the same galvanometer, and the deflection

is found to be 18 cm. when a total flux of 25,000 lines is cut by each of the turns of the magnetic standard. If the resistance of the circuits is the same in the two cases, what is the intensity of the field cut by the exploring coil?

4. A coil of 80 turns with a radius of 6 mm. and a resistance of 20 ohms is placed between the poles of an electromagnet and suddenly removed. A charge of 9×10^{-6} coulomb is sent through a galvanometer connected to the coil. The resistance of the galvanometer is 2,400 ohms. What is the intensity of the field of the magnet?

5. A galvanometer with a resistance of 800 ohms gives a full-scale deflection for 8×10^{-5} coulomb of electricity. A coil of 120 turns and 80 ohms is to be constructed to study fields up to 9,000 oersteds by observing deflections produced when the coil is suddenly removed from the field. What is the maximum radius allowable for the coil?

6. A coil of 300 turns with an area of 380 sq. cm. is placed with its plane perpendicular to the earth's field and rotated in $\frac{1}{60}$ sec. through a quarter turn, so that its plane is parallel to the earth's field. What is the average electromotive force induced, if the earth's field has an intensity of 0.75 oersted?

7. A toy transformer has a 1,600-turn primary to be connected to a 110-volt circuit. How many turns of secondary must be used to get 6, 9, 15, 22, and 32 volts, respectively?

8. Calculate the electromotive force which is induced in the axle of a car which is moving with a speed of 30 m. per second, where the vertical component of the earth's magnetic field is 0.55 oersted. Assume that the length of the axle is 110 cm.

9. Find the electromotive force induced in a coil having 400 turns when it is removed from the air gap between the poles of a magnet which produces 12,000 lines of force per square centimeter. It is assumed that the plane of the coil is originally perpendicular to the lines of force and that it is removed from the magnetic field in 0.025 sec. The area of the coil is 1 sq. cm.

10. The efficiency of a transformer is 95 per cent. The secondary has 200 times as many turns as the primary. It is used on a 110-volt circuit. What is the voltage across the secondary?

CHAPTER XLI

THE DYNAMO

499. The Current in a Revolving Loop.—If we rotate a rectangular coil of wire between the poles of a large electromagnet, we have a picture of what goes on in a dynamo in the simplest case. Let the loop of wire be revolving as shown in Fig. 457. Suppose we start with the coil in the vertical position and turn it clockwise. The side AB of the loop will be moving down across the magnetic field. Applying the right-hand rule to the wire AB the induced electromotive force is found to be in the direction B to A. Applying the same rule to DC the electromotive force is found to be from D to C. Hence, a current flows around the loop in the direction indicated.

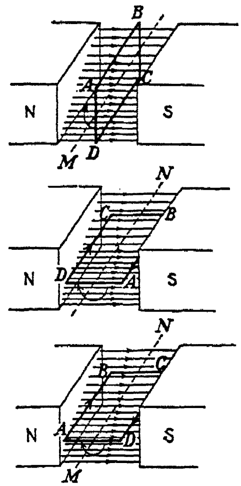


FIG. 457.—Current in a revolving loop of wire in a magnetic field. The direction of the current in the wire reverses twice in each revolution.

When the loop has moved one-half revolution from its vertical position, the side AB begins to move up and the side DC to move down. The current in the wire will now be reversed, since the direction of motion has been reversed. During this half of the revolution, the current will go around the loop in the opposite direction to that in which it flowed in the first half of the revolution. When such a loop is revolving in a magnetic field, it has in it an alternating current which reverses its direction twice during each revolution. This current will have its least value when the coil is in the vertical position. In this position the wires in the coil are moving parallel to the magnetic field and are therefore not cutting any lines of magnetic force. When the coil is in the horizontal position, each of the wires is cutting the lines of force most rapidly because it is moving at right angles to these lines of force. At the instant the coil is in the horizontal position,

the current has its maximum value. The relation between the position of the coil and the electromotive force is evident from Fig. 458.

500. Sine Curve of Electromotive Force.—If we measure the angle through which the coil has rotated from its vertical position, the horizontal position would be 90 deg. from this posi-

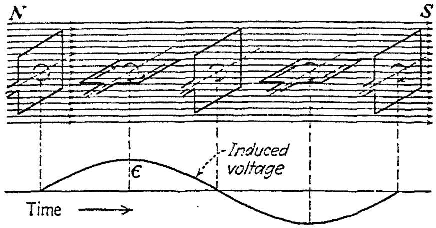


FIG. 458.—Rate of cutting lines of force is greatest when coil is in horizontal plane.

tion. When it is again vertical, it would be 180 deg. from this position; and when it is again horizontal, it is 270 deg. from its initial position. Now plot a curve between the position of the coil and the electromotive force generated in it. The position of the coil will be measured from its first vertical position and will be plotted on the horizontal axis. The electromotive force generated in the coil will be plotted on the vertical axis. Electro-

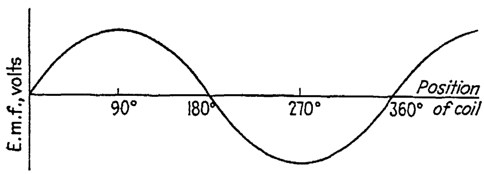


FIG. 459.—Sine curve of electromotive forces. In commercial circuits the frequency is 60 cycles per second.

motive forces in one direction are arbitrarily called positive, and those in the opposite direction are called negative. By this method, a curve like that shown in Fig. 459 is obtained. This curve shows that the voltage generated in the coil rises rapidly to a maximum value when the coil has turned through 90 deg. and then decreases to zero again when the coil is vertical. Here its direction is reversed. It then rises to a maximum value in the opposite direction. From this position it decreases to zero a second time and is again reversed when the coil has made one

complete revolution. This cycle of events takes place during every complete revolution of the coil.

501. Collecting Rings.—If an outside circuit is connected continuously to the two ends of the revolving coil, the electromotive force in the outside circuit behaves just like the voltage in the revolving coil. This is an electromotive force which reverses its direction twice for each revolution of the coil. In order to make continuous connection between the revolving coil and the outside circuit, the ends of the wire forming the coil are fastened to two rings (Fig. 460) which are mounted on the axis of the revolving coil. On each of these rings presses a spring called a **brush,** and to these brushes are connected the terminals of the outside cir-

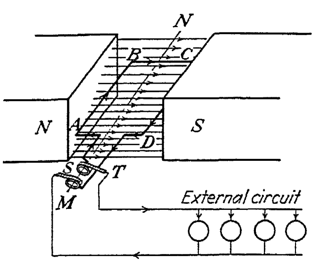


FIG. 460.—Collecting rings on a dynamo. There is an alternating current in the external circuit.

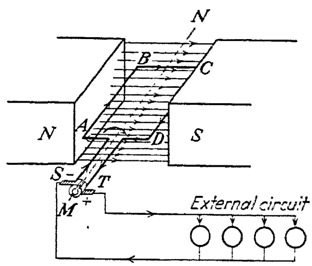


FIG. 461.—Direct-current dynamo. Because of the commutator the current flows in only one direction in the external circuit.

cuit. As the coil revolves, these rings revolve with it, moving under the brushes and always making contact with the wires forming the external circuit. In Fig. 460 the external circuit is represented as a number of lamps. These lamps form a continuous circuit with the revolving coil, and the electromotive force generated in the coil maintains an alternating current in these lamps.

502. The Commutator.—To get a direct current, that is, one which always flows in the same direction through the circuit from a loop of wire revolving between the poles of a magnet, the terminals of the wire instead of being joined to two rings are joined to a divided ring called a **commutator,** shown in Fig. 461. On this divided ring press two brushes which are connected to the external circuit and so placed that they slip from one segment to the other at the time at which the electromotive force in the revolving coil is zero. To understand better how this commuta-

tor works, consider a simple case. If the revolving loop is rotating clockwise, the wire *CD* which is moving downward will give a current in the direction of the arrow, and the current in the wire *AB* will be in the direction of the arrow on that wire. Hence, the current will leave the brush *T* and circulate through the external circuit in the direction of the arrow from *T* to *S*. When the loop has made one-half revolution, the segment of the commutator which was in contact with the brush *T* makes contact with the brush *S*, and the other segment makes contact with the brush *T*. The wire *CD* is now moving up and the current in it is reversed. The wire *AB* is moving down and the current in it is also reversed. The segments of the commutator have, however, reversed their positions. Hence, the current will still leave by the brush *T* and flow through the external circuit in the same

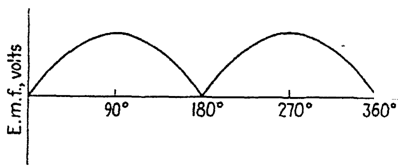


FIG. 462.—With a commutator the current in the external circuit pulsates but always flows in one direction.

direction in which it flowed in the former case. By this means the current in the external circuit is always kept flowing in the same direction. In the revolving coil the current flows first in one direction and then in the other.

The voltage across the brushes may now be represented by a curve of the form shown in Fig. 462. When the coil is in the vertical position between the poles, it will be moving parallel to the lines of force and no electromotive force will be generated in it. When it has revolved 90 deg. from this position, it will be cutting lines of force most rapidly and the electromotive force will have its greatest value. When the coil has again turned into the vertical position, the electromotive force at the brushes is again zero, and, one-quarter of a turn later, the electromotive force will be again a maximum. Since the position of the commutator has been reversed with respect to the brushes, this electromotive force will have the same direction at the brushes as in the former case. A machine provided with a commutator for keeping the current always in the same direction in the external circuit is a direct-current dynamo.

503. Ring and Drum Armatures.—There are two general ways in which armatures are wound: (1) the drum type and (2) the ring type.

In the drum type a cylinder of iron is built up out of thin sheets of iron. The use of such sheets of iron for the armature decreases the losses which would occur if the cylinder were made of a solid piece of iron, as will be

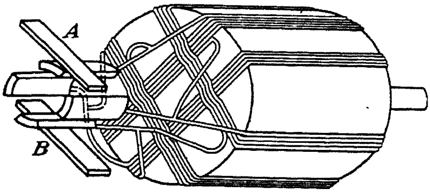


FIG. 463.—Drum armature.

explained later. Around the outside of this cylinder are wound the wires in which the electric currents are induced. The surface of such a core is usually slotted so that the loops of wire lie between projections as in Figs. 463 and 464.

In the ring type of winding, the cylinder is built up as in the drum type except that it is hollow and the wires are wound in and out in such a way

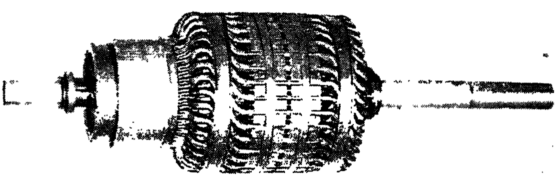


FIG. 464.—Drum armature and commutator.

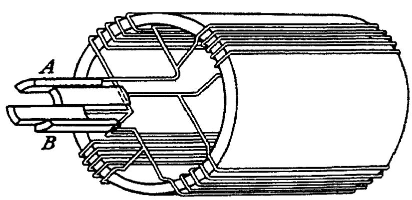


FIG. 465.—Ring armature.

that they do not pass continuously around the outside of the cylinder (Fig. 465). In the ring armature, only one side of the wires cuts the lines of force. The other wires are on the inside of the ring where there are few lines of force. On the other hand, in the drum armature all the wires are on the outside, and each loop cuts all the lines of force. The drum type is the one now more commonly used because it is stronger and simpler.

504. Excitation of the Fields of Generators.—Generators and motors are often classified with regard to the manner in which the electromagnets

which produce their field are excited. From this point of view generators may be classified as **separately excited** or **self-excited.** The self-excited generators may again be divided into **series-**, **shunt-**, and **compound-wound** generators according to the way in which the field windings are connected to the external circuit. In a separately excited generator or motor, the current for the field coils is taken from an outside source of current. In the self-excited generators, the current for the field coils is supplied by the generator itself. Where it is desirable to keep the field strength in the machine independent of the current which the machine is taking or delivering, separately excited generators or motors are used. Self-excited generators may change their polarity in starting and stopping, but this is not true of separately excited generators.

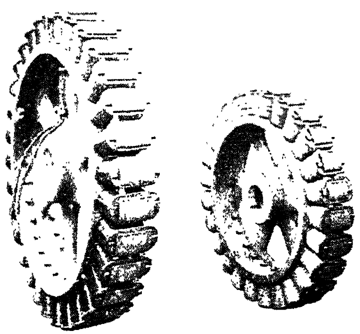


FIG. 466.—Revolving magnetic fields of a generator.

505. Series-wound Dynamo.—In a series-wound dynamo the current which is in the main line also goes through the wires which form the field coils. The coils producing the magnetic field are thus in series with the external circuit (Fig. 467). The field coils consist of a few turns of coarse wire and have, therefore, a low resistance. When the generator is not delivering current, there is no magnetic field except that due to residual magnetism.

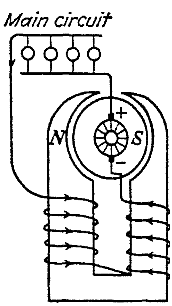


FIG. 467.—Series-wound dynamo. All generated current flows through the field coils.

The greater the current taken by the line, the greater is the current in the field coils and the greater is the magnetic field in which the armature rotates and, consequently, the greater the electromotive force generated in the armature. In order to keep the magnetic field constant and therefore the electromotive force constant, it is necessary always to use the same current in the external circuit. Such a dynamo is satisfactory for operating a number of arc lights in series, for in that case the current is always the same. It is not satisfactory for operating a number of incandescent lamps in parallel, for every time one or more lamps are turned off or on, the voltage of the dynamo changes.

506. Shunt-wound Dynamo.—In this type of winding the field coils are in parallel with, *i.e.*, shunted across, the main circuit as in Fig. 468. Many turns of fine copper wire are wound on the field coils, giving them a high resistance. It is not desirable to have a large current in the field coils as this decreases the efficiency of the machine.

When the current in the external circuit is increased, the voltage at the terminals of this kind of generator decreases somewhat. This drop in voltage tends to decrease still further the current in the field coils and cut down the magnetic field and the voltage of the machine. A shunt-wound generator does not, therefore, supply constant voltage for all loads.

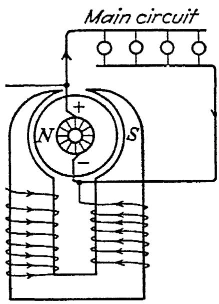


FIG. 468.

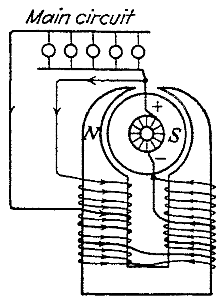


FIG. 469.

FIG. 468.—Shunt-wound dynamo. Only part of the generated current flows through the field coils.
FIG. 469.—Compound-wound dynamo.

507. Compound-wound Dynamo.—On increasing the current supplied by a series-wound dynamo, the voltage also increases. On the other hand, increasing the current supplied by a shunt-wound generator causes a drop in the voltage. It should then be possible to combine these two types of windings and produce a type of winding which gives constant voltages for all loads. This is what is done in the compound-wound dynamo (Fig. 469). The field coils are wound first with a large number of turns of fine wire just as in the shunt-wound type. On top of these windings is then wound a series coil in series with the main circuit. When the current in the external circuit is increased, these series windings tend to increase the voltage which the machine is making. At the same time the effect of this increase of current in the external circuit is to decrease the magnetic field produced

by the shunt windings. These two effects may be made to neutralize each other and the voltage thus kept constant for different loads.

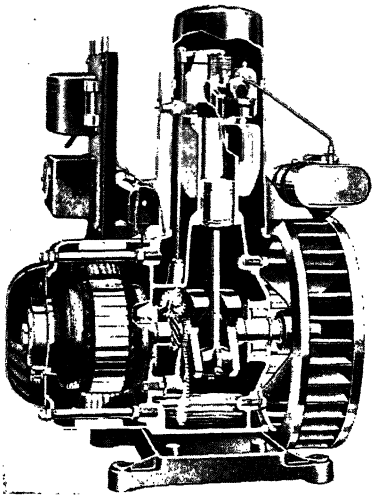


FIG. 470.—A farm light plant.

508. Losses in a Generator.—The losses which occur in a generator may be divided into three classes: (1) mechanical losses such as friction; (2) the copper losses due to ohmic heating in the field and armature; (3) the iron losses due to eddy currents. To find the mechanical losses, it is necessary to determine the mechanical forces necessary to overcome such forces as friction. The copper losses can be computed by multiplying the square of the current in the armature by the resistance of the armature and adding to this the product obtained by multiplying the square of the current in the field coils by the resistance of the field coils. Thus

$$W_a = i_a{}^2R_a.$$
$$W_f = i_f{}^2R_f.$$
$$\text{Copper loss} = W_a + W_f.$$

Whenever a conductor cuts lines of force, there are set up in it induced electromotive forces which tend to send electric currents through it. The

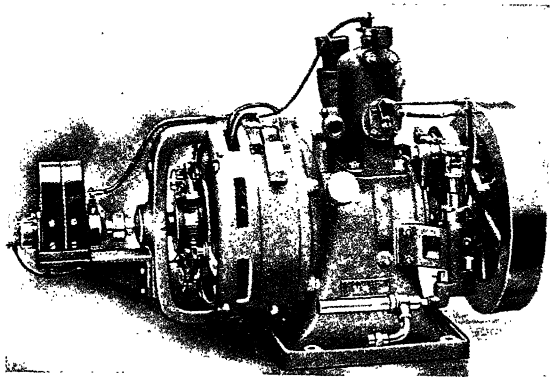


FIG. 471.—A 1-kw. gasoline electric generating set. (*Courtesy General Electric Company.*)

copper wires of an armature are wound on an iron core, and this iron core is an electrical conductor which is revolving in a magnetic field and cutting

lines of force. There will be set up in this iron core induced electromotive forces which will tend to send currents of electricity through it. These **eddy currents**, as they are called, will heat the iron of the armature. Their presence also makes it more difficult to rotate the armature in the magnetic field and thus requires that more energy be supplied for the operation of the

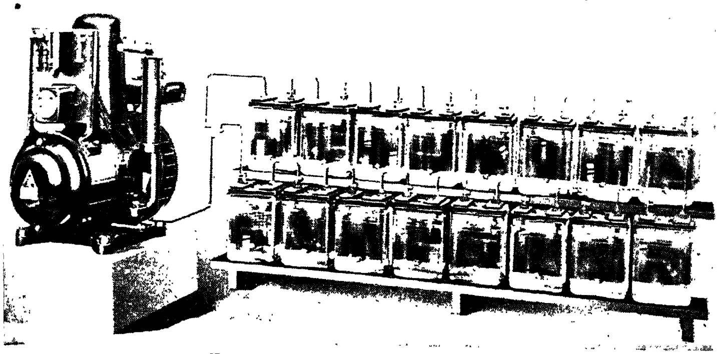


FIG. 472.—A farm light plant.

generator. In order to reduce these eddy currents, and the losses to which they give rise, to a minimum, the armature is built up of thin sheets of iron which are insulated from each other. These sheets of iron are so placed that their planes are perpendicular to the direction in which the currents tend to flow. This cuts down the induced electric currents without changing

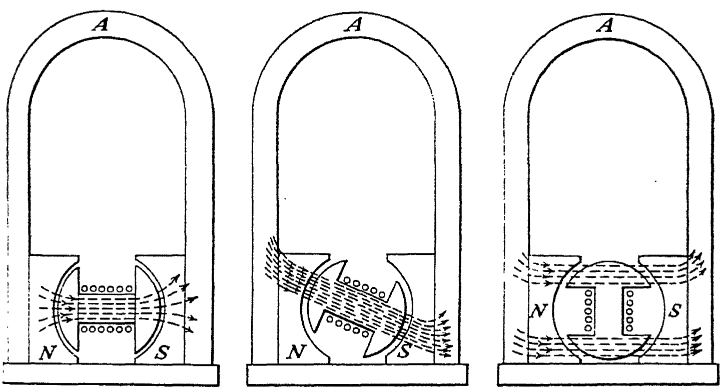


FIG. 473.—Distribution of flux in a magneto.

the magnetic flux. The thinner these sheets of iron, the less the eddy-current losses.

509. Magneto.—The operation and construction of a magneto are the same as those of the simple dynamo except that the magnetic field in which the armature rotates is produced by permanent magnets in the magneto and by electromagnets in the dynamo. The cycle of operation through

which the magneto goes will be exactly like that described for the dynamo. The distribution of flux in a magneto in different positions is shown in Fig. 473. The construction of a magneto is shown in Fig. 474.

If the external circuit from such a magneto is carried to the spark plug of an engine, the armature must be driven so that it is making its maximum electromotive force when the spark is needed in the engine. This is provided for in practice by properly timing the spark and controlling the rate at which the armature rotates. The armature is driven from the flywheel of the engine through a reducing gear, so that the armature makes one revolution to two revolutions of the flywheel. It would be possible to reduce the rate of revolution of the armature still further, but this is rarely done.

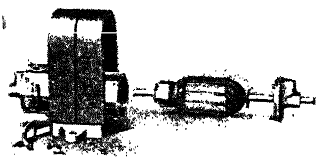


FIG. 474.—Disassembled magneto.

510. Efficiency of Generators.—The efficiency of an engine has been defined as the output divided by the input.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

In a generator it is easy to measure the output by measuring the current and the voltage. The input is obtained by adding the losses to the output. In this case the efficiency may be written as

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

Example.—In a shunt dynamo the stray-power loss is 415 watts and the copper loss is 220 watts. What is the efficiency of the dynamo when it is delivering 65 amp. at 110 volts?

$$\begin{aligned}
\text{Efficiency} &= \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}. \\
\text{Total loss} &= \text{stray-power loss} + \text{copper loss} \\
&= 415 + 220 = 635. \\
\text{Output} &= 65 \times 110 = 7{,}150 \text{ watts.} \\
\text{Input} &= 7{,}150 + 635 = 7{,}785 \text{ watts.} \\
\text{Efficiency} &\ \frac{7{,}150}{7{,}785} \ 91.9 \text{ per cent.}
\end{aligned}$$

Example.—The armature of a shunt-wound generator has a resistance of 0.5 ohm and the field coils a resistance of 50 ohms. What is the copper loss when the generator is delivering 23 amp. at 100 volts?

$$\text{Current in field coils} = \frac{100}{50} = 2 \text{ amp.}$$

$$\text{Joules generated per second in field coils} = 2^2 \times 50 = 200 \text{ joules per second.}$$

$$\text{Joules generated per second in armature} = (25)^2 \times 0.5 = 312 \text{ joules per second.}$$

$$\text{Copper loss} = 200 + 312 = 512 \text{ joules per second.}$$

Problems

1. A generator which develops 120 volts at no load furnishes only 114 volts when a current of 40 amp. is drawn. What is the resistance of the armature?

2. The armature of a dynamo has a resistance of 0.12 ohm. When run at its rated speed, it yields 115 volts on open circuit, and 112 volts on full load. What is the current in the dynamo when it is running on full load? How much power is delivered to the external circuit?

3. A shunt generator has field coils with a resistance of 180 ohms. At full load, the brush potential is 117 volts. What is the current in the field coils, and how much power is consumed in them?

4. The armature of a shunt generator has a resistance of 0.09 ohm. When the current through the armature is 9 amp., the brush potential is 119 volts. What will be the brush potential when the current is 85 amp., assuming that the field strength and the speed remain unchanged?

5. A shunt generator furnishes a potential of 122 volts at the terminals when an external current of 19 amp. is drawn, in addition to 1.33 amp. flowing through the field coil. What is the electromotive force being generated? The armature has a resistance of 0.16 ohm.

6. A shunt generator delivers 120 amp. at a brush potential of 120 volts. If the stray-power loss is 1,400 watts, what is the efficiency of the generator? The field coils have a resistance of 40 ohms, and the armature has a resistance of 0.15 ohm.

7. The brush potential of a generator when delivering 10 amp. is 540 volts. When the generator delivers 45 amp., the difference of potential across the brushes falls to 535 volts. What is the resistance of the armature of the generator?

CHAPTER XLII

MOTORS

511. The Dynamo Reversed.—When a dynamo is driven by some source of power as by a steam engine, it can produce a flow of electricity. If this same dynamo is supplied with current from some outside source as from a storage battery, it is capable of doing work, and it may be now used to operate almost any kind of machinery. This machine being driven backward by a current is called a **motor.** It is essentially like a dynamo, consisting of an armature, a commutator, and field magnets. In order to understand its operation, it is necessary to consider the effect which is produced by placing a wire carrying a current of electricity in a magnetic field.

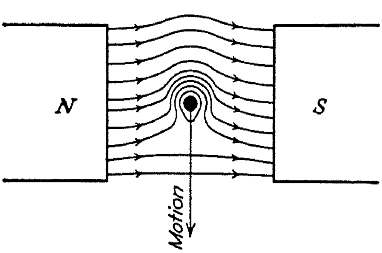


FIG. 475.—Force on a conductor in a magnetic field. Magnetic field below the conductor is less than above it.

512. Force on a Wire in a Magnetic Field.—When a wire carrying a current of electricity is placed in a magnetic field so that the lines of force are perpendicular to the wire carrying the current downward into the paper, as indicated in Fig. 475, the conductor experiences a force directed across the magnetic field. To understand this fact, recall that every current produces about itself a magnetic field of its own. This magnetic field is represented by concentric circles with the wire carrying the current as center. If a wire carrying a current is brought into a uniform magnetic field directed from left to right, the lines of force from the magnetic field of the wire will add to those in the uniform field above the wire and subtract from those below the wire. In consequence of this fact, the number of lines of force per square centimeter above the wire will be greater than the number per square centimeter below it (Fig. 435). Experiment shows that a resulting force then acts on the wire in such a way as to cause it to move from the region where the magnetic field is stronger to where it is weaker. In this case the

wire will move downward. If the current in the wire had been in the opposite direction, the magnetic lines of force would have been denser below the wire and the wire would have moved upward.

Figure 476 shows the cross section of a single loop of wire which, carrying a current of electricity, is introduced into a uniform magnetic field. It is assumed that the current goes in at A and out at B. The flux is weakened below A and strengthened below B. There results a force tending to push A downward and B upward. These two forces cause the loop to rotate. The stronger the current in the wire, the greater the force tending to rotate it. The greater the length of the wire and the greater the magnetic field in which it is placed, the greater the force tending to rotate the loop. The force acting on one of the wires is (Appendix E-11)

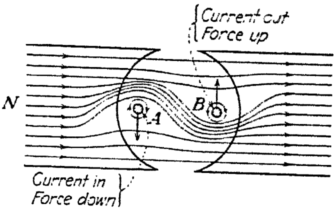


FIG. 476.—Torque on a coil in a magnetic field. The difference between magnetic field above and below the conductor produces the torque.

$$F = \frac{IBl}{10} \text{ dynes.}$$

where F = the force in dynes.
I = the current in amperes.
B = the magnetic field in gausses.
l = the length of wire in centimeters.

The torque acting on the loop is Fd, where d is the perpendicular distance between the conductors.

Example.—A wire 40 cm. long is lying in a magnetic field of 40,000 gauss. If the current in the wire is 50 amp., what is the force acting on it?

$$= \frac{I \times B \times l}{10 \times 980}$$
$$= \frac{50 \times 40{,}000 \times 40}{10 \times 980} \quad 8{,}163 \text{ g.}$$
$$= 8.16 \text{ kg.}$$

513. Hand Rule for Motor.—For finding easily the direction in which a conductor carrying a current in a magnetic field will move, there is a hand rule which is very similar to that described for telling the direction in which a current is induced in a moving

conductor. In this case, the left hand is used instead of the right hand as in the other case. **Using the left hand, place the thumb, the forefinger, and the middle finger so that they are at right angles to each other. Point the forefinger in the direction in which the magnetic field is pointing and the middle finger in the direction in which the positive current is flowing in the wire. Then the thumb will point in the direction in which the conductor will move under the action of the magnetic field.**

514. Types of Motors and Back Electromotive Force in a Motor.—Dynamos and motors for direct currents are often made in exactly the same way. There will, therefore, be three types of motors for direct currents just as there were three types of generators. These three types of motors are series, shunt, and compound. The type of motor chosen will depend on the uses for which it is intended.

In the study of the dynamo it was seen that as the armature consisting of a large number of turns of wire revolves in a magnetic field, there is generated in it an electromotive force. Now since in the motor the construction is precisely the same as in the dynamo, we again have an armature revolving in a magnetic field. There should therefore be set up in this armature an electromotive force. No matter what the machine, when a coil of wire revolves in a magnetic field it will have an electromotive force produced in it. This means that every motor is at the same time a dynamo. There is only one difference. In the case of the dynamo, the armature is made to revolve by the action of some external force whereas in the case of the motor the current flowing into the armature from some outside source causes the armature to revolve. In other words, the cause for the rotation of the armature is different in the two cases.

By applying the right-hand rule to the armature revolving in the magnetic field, it is found that the direction of the electromotive force generated in the armature is such that it opposes the current flowing in the armature. For this reason this electromotive force is called *a back electromotive force or a counter electromotive force.*

When the armature revolves more rapidly, the back electromotive force is increased Hence, the opposition to the flow of the current from the outside is increased and the current in the armature is decreased; that is, the faster the armature revolves,

the less current will flow through it, if the voltage applied to the motor is constant.

If an incandescent lamp is connected in series with a small motor while the armature is held stationary, the lamp glows as if the motor were absent since its resistance is small. When, however, the armature is allowed to revolve, the lamp becomes dim. This is because the electromotive force generated in the armature opposes the impressed electromotive force and thereby reduces the current in the circuit.

515. Starting Box.—Since the current in the armature depends on the difference between the impressed electromotive force and the back electromotive force, this current will be largest when the armature is at rest, for in that case the back electromotive force is zero. In order to keep the current from being excessive at the start, it is customary to introduce in series with the motor a resistance which reduces the current flowing in the circuit. As the speed of the motor increases and the back electromotive force becomes appreciable, this resistance is reduced. Such a resistance is called a starting resistance or a starting box (Fig. 477).

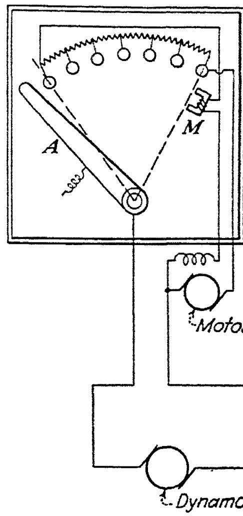


Fig. 477.—Starting box. The voltage applied to the motor is reduced during starting.

Example.—The armature of a shunt motor contains 0.3 ohm resistance. A difference of potential of 110 volts is applied to it. What is the current in the armature before the motor starts?

$$I = \frac{110 \text{ volts}}{0.3 \text{ ohm}} = 366 \text{ amp.}$$

If the back electromotive force is 108 volts when the motor has speeded up, the current becomes

$$I = \frac{110 - 108}{0.3} = \frac{2 \text{ volts}}{0.3 \text{ ohm}} = 6.66 \text{ amp.}$$

516. Efficiency of Motors.—The efficiency of a motor, like the efficiency of a generator, is obtained by dividing the output of the motor by the input:

$$\text{Efficiency} \quad \frac{\text{output}}{\text{input}}.$$

The input is obtained by multiplying current supplied to the motor by the voltage. To get the output of the motor, the

losses in the motor must be subtracted from the input. The efficiency becomes

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

Example.—A motor running at full load takes 75 amp. at 110 volts. The stray-power loss is 800 watts, and the copper loss is 450 watts. What is the efficiency of the motor?

$$\begin{aligned}
\text{Efficiency} &= \frac{\text{input} - \text{losses}}{\text{input}}.\\
\text{Input} &= 75 \times 110 = 8{,}250 \text{ watts.}\\
\text{Losses} &= \text{Stray-power loss} + \text{copper loss.}\\
&= 800 + 450 = 1{,}250 \text{ watts.}\\
\text{Output} &= \text{input} - \text{losses}\\
&= 8{,}250 - 1{,}250 = 7{,}000.\\
\text{Efficiency} &= \frac{7{,}000}{8{,}250} \quad 84.8 \text{ per cent.}
\end{aligned}$$

517. Recording Wattmeter.—The Thomson form of recording wattmeter (Fig. 478) consists of a little shunt motor whose armature turns at a speed

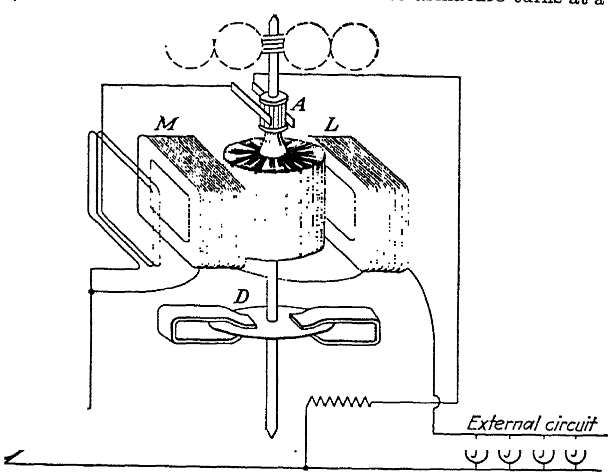


FIG. 478.—Watt hour meter. The electromagnetic brake stops the motor when the current ceases to flow or keeps it moving at a speed proportional to power supplied to the load.

which is proportional to the energy that is supplied to it. The armature is geared to dials which record the energy in kilowatt-hours. The stationary field coils L and M are connected in series with the generator supplying the current to the external circuit. The field strength of the motor is therefore proportional to the current supplied to the lamps. The armature is connected across the line, as a voltmeter would be connected. The voltage across the armature is thus the voltage supplied to the lamps, and the current

in the armature is proportional to the voltage. The torque which turns the motor is proportional to the product of the current in the armature and the magnetic field in which the armature turns. Hence, the torque which turns the armature is proportional to the product of current supplied to the lamps and the voltage at which it is supplied, *i.e.*, to the watts in the line.

In order that the motor may not run too fast and in order also that it may stop as soon as the current ceases to flow, an electromagnetic brake *D* is attached to it. This brake consists of an aluminum or copper disk rotating between the poles of permanent magnets. The eddy currents induced in the disk by its rotation between the poles of the magnet retard its motion and cause it to stop as soon as the current ceases to flow.

This type of wattmeter can be used with either direct or alternating currents.

Problems

1. A wire 25 cm. long lies at right angles to the lines of magnetic force. A current of 18 amp. in the wire produces a force of 15 gm. on the wire. Find the strength of the magnetic field in which the wire is placed.

2. The flux density between the poles of a certain motor is 15,000 oersteds. A conductor carrying 5 amp. lies in this magnetic field so that it is perpendicular to the field. Find the force on it per centimeter of length.

3. Find the torque produced on a rectangular loop of wire of one turn when its plane is parallel to the magnetic lines of a field with a strength of 6,000 oersteds, when a current of 8 amp. is sent through the loop. The dimensions of the loop are 18 cm. parallel to the lines, and 24 cm. right angles to them (see Fig. 476).

4. The armature of a motor connected to a 220-volt line draws 50 amp. at the instant the connection is made when the motor is at rest, through a starting resistance of 3 ohms. What is the resistance of the armature? What is the counter electromotive force when the current drawn is 35 amp. (with the starting resistance disconnected)?

5. A shunt motor takes a total current of 40 amp. from 110-volt mains. The resistance of the armature is 1.6 ohms, and that of the field is 80 ohms. Find the current in the field coils and in the armature, and the power used in heating the armature and the field coils.

6. A series-wound motor has an armature with a resistance of 0.40 ohm and field coils with a resistance of 5 ohms. The motor draws 8 amp. at 120 volts. Find the number of watts supplied to the motor, and the number of watts transformed into heat.

7. A motor running at full load on a 115-volt line develops a back electromotive force of 107 volts and draws a current of 6 amp. through the armature. What is the mechanical power output of the motor, disregarding frictional losses?

8. A motor draws 25 amp. at 120 volts. If the back electromotive force in its armature is 108 volts, what is the resistance of the armature?

9. The current in the armature of a motor which supplies 2 hp. is 16 amp. The mechanical losses in the motor amount to 65 watts. Find the counter electromotive force which is developed in the armature.

CHAPTER XLIII

MUTUAL INDUCTANCE AND SELF-INDUCTANCE

518. Mutual Inductance.—Consider two neighboring circuits (Fig. 479), one with and the other without a battery in it. If the current in coil A be started, magnetic lines of force due to this current will link coil B. As these lines of force cut the coil B, an induced electromotive force and an induced current are set up in the coil B. The current in the coil B lasts only while the current in the coil A and the magnetic field around it are changing. As soon as the current in the coil A ceases to increase or

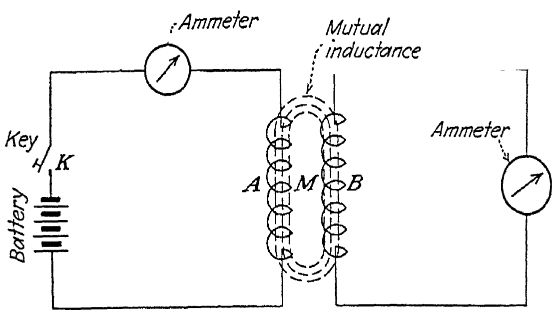


FIG. 479.—Mutual inductance between two circuits. The greater the flux linkages, the greater the inductance.

decrease, there is no longer a current in the coil B. When the current in the coil A ceases to flow, the magnetic lines of force which link the coil B disappear, and during their disappearance there are a momentary electromotive force and current in the coil B. Coil A is called the **primary circuit,** and coil B the **secondary circuit.** While the current in the primary circuit is increasing, the induced current in the secondary produces a magnetic effect opposite in direction to that produced by the current in the primary. When the current in the primary is decreasing, the induced current and electromotive force in the secondary reverse their direction. The electromagnetic effect of one circuit on another is called **mutual inductance.**

519. Coefficient of Mutual Induction.—When two circuits are so situated that a change of 1 amp. per second in one of the circuits induces an electromotive force of 1 volt in the other circuit, the mutual inductance between the two circuits is said to be 1 henry.

If M = the coefficient of mutual induction between the two circuits in henrys,
I_1 = the first value of the current in the primary circuit,
I_2 = the second value of the current in the primary circuit,
t = the time during which the current in the primary circuit changes from I_1 to I_2,
E_s = the electromotive force induced in the secondary by the current changing in the primary,

then (Appendix D-13)

$$E_s = M\left(\frac{I_1 - I_2}{t}\right).$$

Example.—The coefficient of mutual induction between two circuits is 0.55 henry. The current in the primary changes from 5 to 3 amp. in 0.10 sec. What is the electromotive force induced in the secondary?

$$\binom{\text{E.m.f. induced}}{\text{in the secondary}} \quad \binom{\text{coefficient of}}{\text{mutual induction}} \times \binom{\text{rate of change of}}{\text{current in primary}}.$$

$$E_s = M\left(\frac{I_1 - I_2}{t}\right).$$

$$E_s = 0.55 \times \frac{-3}{0.10} \text{ volts}$$

$$= 11 \text{ volts.}$$

520. Self-inductance.—Instead of two neighboring circuits, consider a single circuit (Fig. 480) consisting of a battery, a resistance, an ammeter, and a large coil of wire. When a current is flowing in this circuit, it has around it a field of force of its own. The field of force inside the coil of wire is linked once for each turn of wire on the coil. When the magnetic flux in the coil is changed, each of these turns of wire will have an electromotive force induced in it, just as an electromotive force was induced in the secondary circuit of Fig. 479 by a change of current in the primary. Since all of these coils are connected in series and since the flux passes through them in the same direction, the electromotive forces induced in each of these turns of wire will be added together and give the net electromotive force

induced in the entire coil. The electromagnetic action in this case is fundamentally like the electromagnetic action in the case of mutual inductance. There is only one important difference. In the case of mutual inductance, the change of a current in one circuit, called the primary, sets up an induced electromotive force in a neighboring circuit, called the secondary. In the case now under consideration there is only a single circuit, and the change of current in this circuit sets up an induced electromotive force in this same circuit. Since the change of current and the induced electromotive force take place in the same circuit, this kind of inductance is called **self-inductance** to distinguish it from mutual inductance.

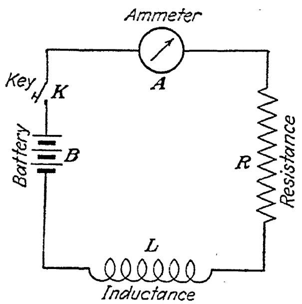


FIG. 480.—Circuit containing both resistance and inductance.

The influence of the self-inductance on the current in the circuit can be understood from the following experiments. When the key K of the circuit in Fig. 480 is closed, the ammeter in

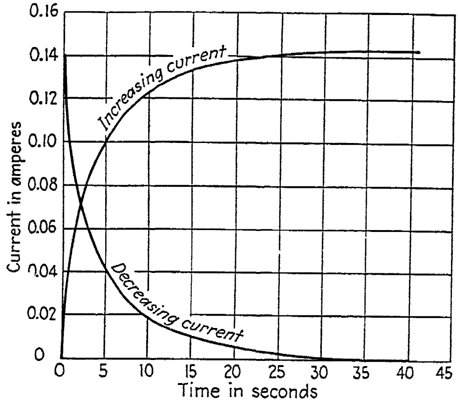


FIG. 481.—Rise and decay of current in a circuit with both resistance and inductance.

series with the coil does not register its full value at once. It requires considerable time for the current to reach its final steady value. When the current in the ammeter is plotted against the time since the switch was closed, a curve like the upper curve shown in Fig. 481 is obtained. From this curve it is seen that,

for a certain circuit, it required about 30 sec. for the current to rise practically to its full value. After the current reaches its full value, it remains constant. If, instead of connecting the battery to the terminals of the inductance, it had been connected to the terminals of a straight wire having the same resistance as the wire in the inductance, the current in the ammeter would have risen instantly to its final value. Figure 482 shows how the currents would behave in these two cases.

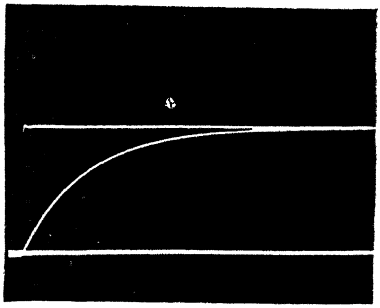


FIG. 482.—Rise of current in an inductive and in a non-inductive circuit.

If, on the other hand, the battery is disconnected from the terminals of the large inductance and at the same instant a short wire is substituted for it (Fig. 483) so that the electric circuit remains closed, the current which was flowing in the circuit tends to keep on flowing, and the reading of the ammeter does not at once become zero. If readings of the current in the ammeter are taken at regular intervals after the battery is short-circuited and the readings are plotted, a curve similar to the lower curve, shown in Fig. 481, will be obtained. This curve shows that considerable time elapses after the removal of the battery before the current in the circuit becomes zero. If, as before, a straight wire of resistance equal to that in the inductance has been used, the current would have dropped immediately to zero.

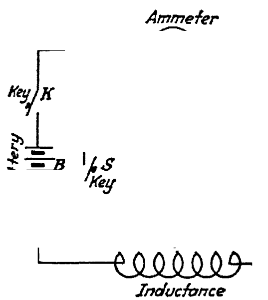


FIG. 483.—Short-circuiting a battery in an inductive circuit.

That property of the circuit which opposes the building up or the dying down of the current in it is called the **self-inductance of the circuit.** The momentary electromotive force which is set up in the circuit when the current is either increasing or decreasing is the induced electromotive force due to the self-inductance of the circuit. This induced electromotive force prevents the current from rising immediately to its final

value and also prevents it from becoming zero immediately after the electromotive force is removed from the circuit. When the circuit is in the form of a straight wire, it has no self-inductance and there is no appreciable induced electromotive force. When a circuit contains a large self-inductance, the induced electromotive force is great and the current builds up slowly and also dies down slowly. The effective electromotive force in the circuit is equal to the difference between the impressed electromotive force and this induced electromotive force. The current at any instant is given by the equation

$$i = \frac{E - e}{R}$$

where E = the impressed voltage.
i = the instantaneous value of the current.
s = the instantaneous value of the induced electromotive force.
R = the resistance of the circuit.

When the circuit is first closed, the induced electromotive force is large. After the circuit has been closed for some time, the induced electromotive force is nearly zero and the effective electromotive force is nearly equal to the impressed electromotive force. The current is then approaching its largest value.

521. Coefficient of Self-induction.—The unit of inductance is the **henry. A circuit is said to have a self-inductance of 1 henry when an induced electromotive force of 1 volt is produced in the circuit by changing the current at the rate of 1 amp. per second.**

If E = the average induced electromotive force in volts,
I_1 = the first value of the current in amperes,
I_2 = the second value of the current in amperes,
t = the time in seconds during which the current changes from I_1 to I_2,
L = the coefficient of self-induction in henrys,

then

$$\frac{I_1 - I_2}{t} = \text{rate of change of the current,}$$

and

$$E = L\left(\frac{I_1 - I_2}{t}\right). \quad \text{(Appendix D-14.)}$$

Example.—A circuit has an inductance of 2.25 henrys. What voltage will be induced in it when the current changes from 12 to 2 amp. in 0.25 sec.?

Induced electromotive force = coefficient of self-induction × rate of change of current.

$$E = L\left(\frac{I_1 - I_2}{t}\right)$$

$$= 2.25 \times \frac{12 - 2}{0.25} \quad 90 \text{ volts.}$$

A variable standard of inductance (Fig. 484) is obtained by rotating one coil with respect to another.

522. Energy Stored Up about a Current.—When a circuit carrying an electric current is broken, the energy which is stored

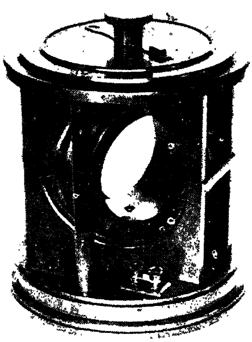


FIG. 484.—Variable standard of inductance. The rotation of the coils with respect to each other varies the inductance. (*Courtesy Leeds and Northrup Company.*)

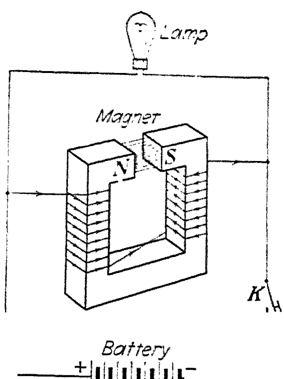


FIG. 485.—Energy stored up in a magnetic field. This energy lights the lamp when the current in the battery is interrupted.

up in the magnetic field associated with the current is given up as heat in the conductors in the neighborhood. Induced currents, set up in these conductors while the magnetic field is changing, heat in the ordinary way the conductors through which they flow. The amount of this energy is independent of the rate at which the current is built up. It depends only on the final value of the current and the coefficient of self-induction of the circuit. When the key K (Fig. 485) is released, the lamp glows more brilliantly than before this key was released. This additional energy supplied to the lamp comes from the energy stored up in the magnetic field of the magnet.

To find an expression for this energy W, suppose for simplicity that the current falls from its initial value I to its final value zero at a constant rate. If t is the time required for the current to decrease from I to zero, its rate of decrease is I/t. The induced electromotive force which arises because of this change of current is $(LI)/t$. Since the average current in the circuit during the time t is $\frac{1}{2}I$, the quantity of electricity which flowed around the circuit while the current was decreasing to zero is $\frac{1}{2}It$. The work done in this process is the electromotive force times the quantity of electricity; that is,

$$L\frac{I}{t} \times \frac{1}{2}It = \frac{1}{2}LI^2 \text{ joules.} \quad \text{(Appendix E-14.)}$$

This is the energy which is released by the circuit when the current in it is annulled, and it is equivalent to the Joulean heat produced in the neighboring conductors.

Example.—In a circuit which has a coefficient of self-induction of 10 henrys, there is flowing a current of 0.1 amp. How much energy is stored up about the circuit?

$$E = \frac{1}{2}LI^2$$

$$= \frac{1}{2} \times 10 \times (0.1)^2 = 0.05 \text{ joule.}$$

Problems

1. Two circuits have a coefficient of mutual induction of 1.75 henrys. What electromotive force is introduced in the secondary by a change from 0 to 35 amp. in 0.008 sec. in the primary?

2. How much energy is stored in a circuit of 16 henrys self-inductance when a current of 0.12 amp. has been built up?

3. A coil has an inductance of 4 henrys and carries a current of 8 amp. What time must be allowed for the current to die out in order that the average induced electromotive force may be 450 volts?

4. The coefficient of self-inductance of a coil is 6 henrys and the current in it is 10 amp. If the current is reduced to zero in 0.003 sec., what is the average voltage set up in the coil?

5. The current in a circuit changes from 20 amp. to zero in 0.006 sec. If the average induced electromotive force is 4,800 volts, what is the coefficient of induction of the circuit?

CHAPTER XLIV

CAPACITANCE OF CONDENSERS

523. The Condenser.—When two parallel plates A and B are connected to the terminals of a battery, as indicated in Fig. 486, electrons flow through the battery away from the plate A to the plate B. The plate B thus becomes charged with negative electricity and the plate A with positive electricity. This flow of electricity will cease as soon as the difference of potential between the plate A and the plate B is just equal to the difference

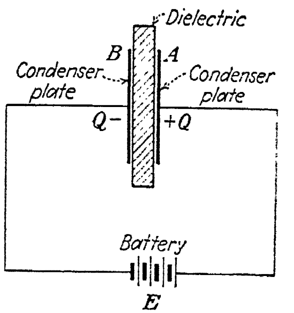


FIG. 486.—A dielectric between plates of a condenser.

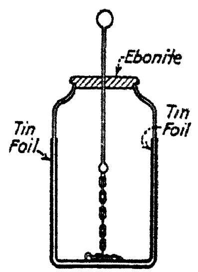


FIG. 487.—A Leyden jar, a simple form of condenser.

of potential across the terminals of the battery. The charges on the plates are opposite in sign but equal in magnitude. If the plates are brought near to each other, the difference of potential between them will be decreased, and more electrons will flow to the plate B until the difference of potential between the two plates is again just equal to the difference of potential across the terminals of the battery. The greater the distance between the two plates, the less is the quantity of electricity which is necessary to produce a given difference of potential between them. By having the distance between the plates small, it is possible to get relatively large charges on the plates without creating a very large difference of potential between them. Such a pair of plates or their equivalent is called a *condenser*. A Leyden jar (Fig. 487), which consists of a glass jar with one coating of tinfoil

on the inside and another on the outside, is a very simple form of such a condenser.

A condenser may consist of a number of metallic plates which are separated from each other by some insulator like glass or mica. In Fig. 488, is represented a condenser consisting of two groups of plates. The plates of each group are connected together, but

FIG. 488.—Condenser. The metallic plates are oppositely charged.

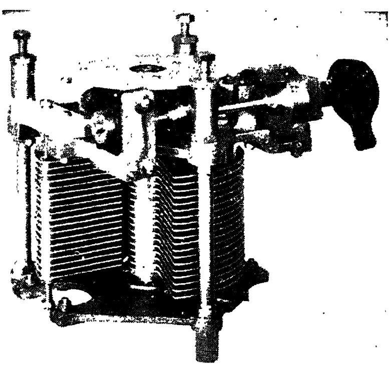


FIG. 489.—Variable capacity. The rotation of the plates with respect to each other varies the capacity. (*Courtesy General Radio Company.*)

each group is carefully insulated from the other group. Figure 489 shows a precision type of variable condenser.

524. Relation between Charge and Voltage.—The difference of potential or voltage between the plates of a condenser is directly proportional to the charge or quantity of electricity on one plate or group of plates in the condenser. This relation is expressed by the equation,

$$Q = CV$$

where Q = the quantity of electricity on the condenser in coulombs.

V = the difference of potential between the terminals of the condenser in volts.

C = the capacity of the condenser in farads—a constant determined by dimensions and materials of the condenser.

This constant C depends on the size and shape of the condenser and on the character of the insulating material which separates the plates from each other. In a simple condenser consisting of two parallel plates, moving the plates closer together decreases the difference of potential between them for a given charge and thus increases the constant C. Increasing the area of the plates also increases the constant C.

525. Capacitance or Capacity.—The factor of proportionality by which the difference of potential across the terminals of a condenser must be multiplied in order to get the quantity of electricity on the condenser is called the **capacity** of the condenser. **Capacitance or capacity may also be defined as the ratio of the charge on the condenser to the difference of potential or voltage across its terminals**; that is,

$$\text{Capacitance} = \frac{\text{quantity of electricity}}{\text{difference of potential}}$$

$$C = \frac{Q}{V}.$$

526. Unit Capacitance.—Since the capacitance of a condenser is unlike any other physical constant it is necessary to agree upon a unit in which it can be measured. **When 1 coulomb of electricity produces a difference of potential of 1 volt between the terminals of a condenser, the condenser is said to have a capacitance or a capacity of 1 farad; that is, a condenser has a capacity of 1 farad when 1 coulomb of electricity will raise the difference of potential between its plates by 1 volt.** This unit is too large for practical purposes and a smaller unit is chosen, the **microfarad,** which is the millionth part of 1 farad.

An electrostatic unit of capacitance called a *statfarad* is also used. It is much smaller than the microfarad.

One microfarad = 900,000 statfarads.

Example.—A condenser has on it a charge of 2 coulombs, when the difference of potential between its terminals is 4 volts. What is its capacitance in farads?

$$\text{Capacitance in farads} = \frac{\text{charge in coulombs}}{\text{difference of potential in volts}}.$$

$$C = \frac{Q}{V}.$$

$$\frac{2 \text{ coulombs}}{4 \text{ volts}} = 0.5 \text{ farad}.$$

527. Dielectric Constant.—If a plate condenser of given area with the plates a fixed distance apart is filled with air and its capacitance determined and if it is then filled with some other medium like paraffin, it is found that the capacitance is about twice as great in the latter case as in the former. If other media are substituted for the paraffin (Fig. 486), still other values will be found for the capacitance. **The ratio of the capacitance of a condenser filled with some medium to the capacitance of the same condenser filled with air is called the dielectric constant or specific inductive capacitance of the medium in comparison with air.**

Example.—In a certain condenser consisting of parallel plates the measured capacitance was 1.3 mf. when the dielectric between the plates was air, and it was found to be 2.65 mf. when the dielectric was kerosene. What is the dielectric constant of kerosene?

$$\text{Dielectric constant} = \frac{\text{capacitance with kerosene}}{\text{capacitance with air}}$$

$$= \frac{2.65 \text{ mf.}}{1.3 \text{ mf.}} = 2.04.$$

528. Capacity of an Isolated Sphere.—If a single sphere of radius R is removed sufficiently far from other bodies so that their influence may be neglected, and it is charged with Q units of electricity, the potential of the sphere is,

$$\text{Potential} = V = \frac{\text{charge}}{\text{radius}};$$

but, also,

$$\frac{\text{charge}}{\text{capacity}} = \frac{Q}{C};$$

hence, in this case,

$$C = R,$$

and the capacity of a sphere is numerically equal to its radius. In this case, the radius must be measured in centimeters; the capacity, the charge, and the difference of potential in electrostatic units or statfarads.

Example.—The radius of a sphere is 10 cm. What charge in electrostatic units is necessary to give it a potential of 3 e.s.u.?

$$V = \frac{\text{charge}}{\text{radius}} = \frac{Q}{R}.$$

$$3 \text{ e.s.u.} = \frac{Q}{10 \text{ cm.}}$$

$$Q = 30 \text{ e.s.u. or statfarads.}$$

529. Capacity of a Spherical Condenser.—Let the condenser consist of two concentric spheres one of which is connected to the earth (Fig. 490).

Let a = the radius of the inner sphere.
b = the radius of the outer sphere.
Q = the charge on the inner sphere.
$-Q$ = the charge on the outer sphere.
V = the difference of potential between the outer and inner spheres.

$$V = \frac{Q}{a} - \frac{Q}{b}$$

The capacity of the two concentric spheres is

$$\text{Capacity} = \frac{\text{charge}}{\text{difference of potential}}.$$

$$C = \frac{Q}{\frac{Q}{a} - \frac{Q}{b}} = \frac{1}{\ } = \frac{ab}{b - a} \text{ e.s.u. or statfarads.}$$

If the radii of the spheres are made more and more nearly equal to each other, the capacity of this spherical condenser increases as $b - a$ decreases. If the distance between the two spheres is kept constant, the capacity of the condenser increases as the radii of the spheres are increased.

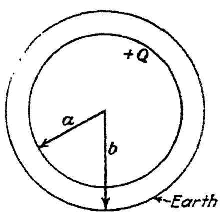


FIG. 490.—Capacity of concentric spheres. The spheres are oppositely charged.

Example.—Find the capacity of a condenser consisting of two concentric spheres, one of radius 20 cm., and the other of radius 21 cm.

$$\text{Capacity} = \frac{ab}{a - b}$$
$$\frac{20 \times 21}{21 - 20} = 420 \text{ statfarads.}$$

530. Capacity of Two Parallel Plates.—If the radii of the two concentric spheres considered in the last section are allowed to increase until they are very large, while the difference, $b - a$, remains constant, the surfaces of the spheres become approximately a plane surface. The product $a \times b$ becomes nearly equal to a^2, since a and b are nearly equal to each other when both a and b are very large.

Let S = the area of the sphere of which a is the radius.
$d = b - a$ = the distance between the spheres.
$S = 4\pi a^2.$
$a^2 = \frac{S}{4\pi}.$

$$C = \frac{ab}{b-a} = \frac{a^2}{d} \text{ approx.}$$
$$\frac{S}{4\pi d} \quad \frac{\text{area}}{4\pi d}.$$

If this relation is true for the entire spherical condenser, it is also true for a small portion of it, provided only the area of that portion instead of the entire area of the sphere is used. If, then, the spheres are very large, and only a small portion of area equal to A is cut out of the spherical surfaces, the condenser, obtained in this artificial way, consists of two parallel plates at a distance d apart, each plate having an area A. The capacity of such a condenser is then,

$$C = \frac{A}{4\pi d} \text{ e.s.u. or statfarads.}$$

where A = the area of each of the plates.
d = the distance between the plates.

If the space between the plates is filled with a substance whose dielectric constant is k, the capacity of the condenser is

$$C = \frac{Ak}{4\pi d} \text{ e.s.u. or statfarads.}$$

Example.—Find the capacity of a condenser consisting of two parallel plates which are 1 cm. apart. Each of the plates has an area of 100 sq. cm. The space between the plates is filled with a medium whose dielectric constant is 3

$$\text{Capacity} \cdot \frac{\text{area} \times \text{dielectric constant}}{4\pi \times \text{distance between plates}}$$
$$\frac{Ak}{4\pi d}$$
$$\frac{100 \text{ sq. cm.} \times 3}{4\pi \times 1 \text{ cm.}} \quad 23.9 \text{ e.s.u. or statfarads.}$$

531. Second Definition of Capacitance.—The capacitance of a condenser may also be defined as the ratio of the current flowing into the condenser to the rate of change of voltage across it.

A condenser is then said to have a capacitance of 1 farad when a change of 1 volt per second across it produces a current of 1 amp.

This relation between the change of voltage and the current in the condenser is given by the equation

$$I_{\text{ave.}} = C\frac{E_2 - E_1}{t}$$

where $I_{\text{ave.}}$ = the average current in amperes.

C = the capacitance in farads.

E_1 = the initial value of the electromotive force in volts.

E_2 = the final value of the electromotive force in volts.

t = the time during which the voltage changed from E_1 to E_2.

$\frac{E_1 - E_2}{t}$ = the average rate of change of voltage.

Example.—In a certain condenser when the voltage changed from 5 to 65 volts in 2 sec., a current of 0.0003 amp. flowed in the condenser circuit. What was the capacitance of the condenser?

$$I_{\text{ave.}} = C\frac{E_2 - E_1}{t}.$$

$$0.0003 = C\frac{65 - 5}{2} = C \times 30.$$

$$C = \frac{0.0003}{30} = 0.00001 \text{ farad.}$$

532. Hydraulic Analogy.—The charging of a condenser is analogous to the action of the water in a closed circuit (Fig. 491) across which there is stretched a rubber diaphragm. If the centrifugal pump is set in operation, the pressure of the water will stretch the diaphragm A until the force exerted by the diaphragm is sufficient to stop the flow of the water. The diaphragm will remain stretched as long as the pump is in operation. If the pump is stopped, there will be a backward flow of water until the diaphragm comes back to its normal position.

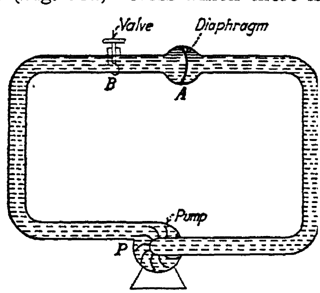


FIG. 491.—Hydraulic analogue of a condenser.

In like manner, when an electric condenser is connected in series with a battery or generator, there will be a momentary flow of current in the circuit. This flow will last for only a short time after the switch is closed. If the voltage after this time remains constant, there will be no further flow. When the battery is removed or the generator is stopped, electricity flows back through the circuit for a short time.

If instead of stopping the generator or short-circuiting the battery, the switch is first opened, the electricity cannot flow back when the generator is stopped. This is equivalent to closing the valve B in Fig. 491 and thus preventing the backward flow of the water when the pump is stopped. The diaphragm is left stretched just as the condenser is left charged.

The quantity of charge on the condenser is obtained as follows: The voltage rises from 0 to E volts in t sec. If this change of voltage is uniform, the rate of change of voltage is E/t and the current through the condenser is

$$i = C \times \frac{E}{t}.$$

The quantity of electricity on the condenser i

$$Q = it = C\frac{E}{t} \times t = C \times E,$$

$$Q = CE$$

where Q = the quantity of electricity in coulombs.
C = the capacitance in farads.
E = the electromotive force in volts.

This equation is consistent with the definition of capacitance given in Sec. 525.

Example.—A condenser has a capacitance of 25 mf., and the difference of potential between its terminals is 250 volts. Find the number of coulombs on the condenser.

Quantity = capacitance × difference of potential.

$$Q = C \times E$$
$$= 0.000025 \times 250$$
$$= 0.00625 \text{ coulomb.}$$

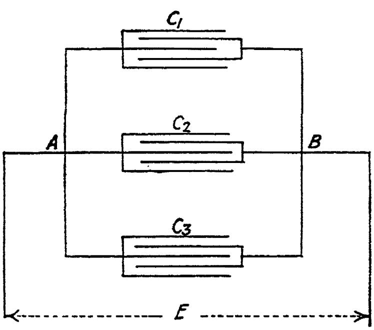


FIG. 492.—Condensers in parallel. The effective capacity is equal to the sum of the separate capacities.

533. Condensers in Parallel. If a number of condensers having capacitances C_1, C_2, C_3, etc. (Fig. 492) are placed in parallel, the whole system will have a capacitance C which is equal to the sum of the separate capacitances. This result can be proved as follows:

The condensers are all charged to the same difference of potential. Let E denote this difference of potential and let Q_1, Q_2, and Q_3 be the charges on the condensers C_1, C_2, and C_3, respectively. Let Q be the total charge on all of the condensers.

Then

$$Q = Q_1 + Q_2 + Q_3, \text{ etc.}$$
$$Q_1 = EC_1, \quad Q_2 = EC_2, \quad Q_3 = EC_3, \text{ etc.,}$$

and

$$Q = EC.$$

Substituting these values,

$$EC = EC_1 + EC_2 + EC_3.$$

Dividing by E,

$$C = C_1 + C_2 + C_3.$$

Hence, to find the equivalent capacitance of a number of condensers connected in parallel, it is only necessary to add together the separate capacitances.

Example.—Three condensers having capacitances of 3.5, 0.25, and 4.5 mf. are connected in parallel. What is their effective capacitance?

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 3.5 + 0.25 + 4.5 = 8.25 \text{ mf.} \end{aligned}$$

534. Condensers in Series.—If condensers are placed in series (Fig. 493), the same quantity of electricity is stored up on each

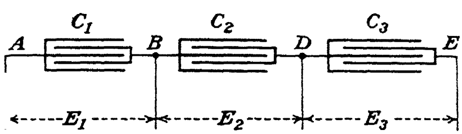


FIG. 493.—Condensers in series. The reciprocal of the effective capacity is equal to the sum of the reciprocals of the separate capacities.

condenser. In general, however, the difference of potential between the terminals of the different condensers will not be the same. The difference of potential over the combination will be equal to the sum of the differences of potential over the separate condensers.

Let Q be the charge on each condenser; C, the equivalent capacitance of the condensers when joined in series; and E_1, E_2, and E_3, the differences of potential between the terminals of C_1, C_2, and C_3, respectively. Since the total difference of potential is equal to the sum of the separate differences of potential,

$$E = E_1 + E_2 + E_3.$$

Since the charge on each condenser is the same,

$$Q = E_1C_1 = E_2C_2 = E_3C_3;$$

and also

$$Q = EC.$$

Then

$$E = \frac{Q}{C},\ E_1 = \frac{Q}{C_1},\ E_2 = \frac{Q}{C_2},\ E_3 = \frac{Q}{C_3}.$$

Substituting these values,

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Dividing by Q,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

This equation gives the equivalent capacitance of the condensers in series in terms of the capacitances of the separate condensers.

Example.—*A capacitance of 4 mf. is connected in series with one of 5 mf.* What is the equivalent capacitance of the combination?

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

$$\frac{1}{C} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}.$$

$$C = \frac{20}{9} = 2.22 \text{ mf.}$$

535. Energy Stored in a Condenser.—In charging a condenser, it is necessary to do work to carry the electricity from one terminal of the condenser to the other. At the beginning, the two coatings of the condenser are at the same potential. As the charging goes on, the difference of potential between the two coatings increases, and more work is required to transfer a given charge from one terminal to the other. Suppose that the final difference of potential between the terminals of the condenser is V volts and that in charging it Q units of electricity were transferred from one terminal to the other, giving a charge of Q on one coating and of $-Q$ on the other. At the beginning of the process of charging, the difference of potential was zero; and at the end of the charging, the difference of potential was V. The average difference of potential during the charging of the con-

denser was $\frac{1}{2}V$. The work done is the average difference of potential times the quantity of electricity transferred from one terminal to the other, *i.e.*, $\frac{1}{2}V \times Q$. This energy is again released when the condenser is discharged. If the condenser is allowed to discharge through a wire, the energy is used in heating the wire.

The charging of a condenser is analogous to the filling of a vertical pipe of uniform cross section with water. At the beginning the pressure in the pipe is zero and at the end the pressure is equal to the weight of a column of water of height H. The average pressure is equal to the weight of a column of water of height $\frac{1}{2}H$. The work to fill the pipe is equal to the weight of the water in the pipe times one-half the height of the pipe. (Appendix E-9.)

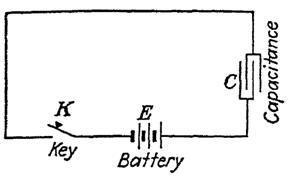


FIG. 494.—Charging a condenser through a circuit without a resistance.

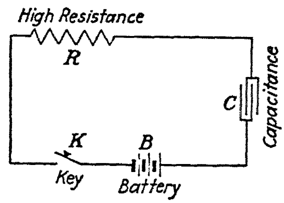


FIG. 495.—Charging a condenser through a high resistance.

Example.—A condenser having a capacitance of 1 mf. is charged with 0.01 coulomb of electricity. How much energy is stored in it?

$$\begin{aligned}\text{Energy} &= \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \\ &= \frac{1}{2} \times \frac{0.01 \times 0.01}{0.000001} \\ &= 50 \text{ joules.}\end{aligned}$$

536. Charging and Discharging a Condenser through a Resistance.—If the terminals of the battery instead of being connected directly to the terminals of the condenser, as in Fig. 494, are connected to them through a large non-inductive resistance as in Fig. 495, it takes some time for the charge on the condenser to build up to its final value. The way in which the charge on the condenser varies with the time is represented in Fig. 496, where the time has been plotted on the horizontal axis and the charge on the condenser on the vertical axis. The final charge on the condenser is the same as if the condenser had been charged

without the resistance R in the circuit. The larger the resistance R, the longer is the time required for the condenser to receive its full charge.

If, after the condenser has been fully charged, the key K is opened and the key S (Fig. 497) is closed, the condenser begins

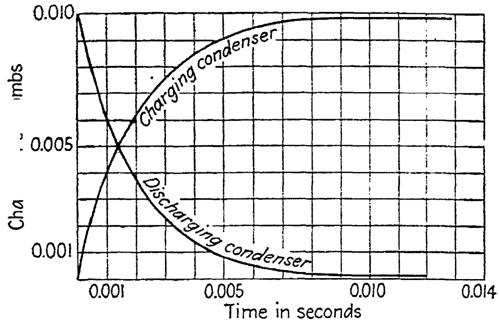


FIG. 496.—Rate of charge and discharge of a condenser through a high resistance.

to discharge. (Appendix E-14.) It requires some time, however, for the condenser to discharge completely. The way in which the charge on the condenser decreases with the time is shown in Fig. 496 where the time after the short-circuiting of the battery has been plotted on the horizontal axis, and the quantity of electricity at that time on the condenser has been plotted on the

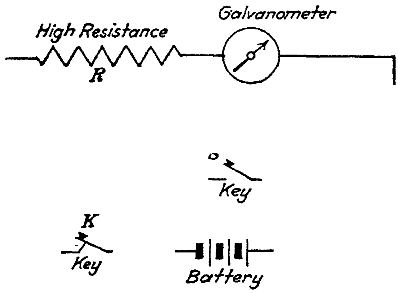


FIG. 497.—Discharging a condenser through a high resistance.

vertical axis. The greater the resistance in the circuit, the greater is the time for the condenser to discharge to a definite fraction of its original charge. (Appendix E-13.)

537. Oscillatory Discharge.—If the circuit contains both inductance and capacitance without any resistance, the condenser does not discharge continuously in one direction, but the

charge oscillates back and forth so that the plates become alternately charged positively and negatively. If, in addition to inductance and capacitance, the circuit also contains ohmic resistance, the charge continues to be oscillatory for certain values of the inductance, capacitance, and resistance. In this case, however, some of the energy stored in the condenser is dissipated in the resistance, so that the quantity of electricity which surges from one side of the condenser to the other becomes less and less. Figure 498 is a record taken with an oscillograph to show the way such a discharge behaves.

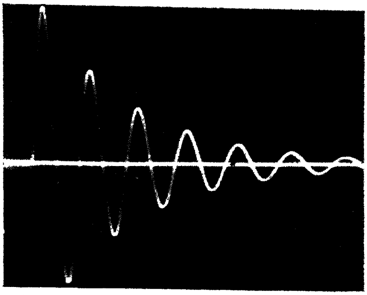


FIG. 498.—Oscillatory discharge of a condenser through resistance and inductance.

Problems

1. Find the charge on a 2-mf. condenser when a potential of 550 volts is applied to it.

2. A charge of 0.006 coulomb is stored in a condenser at a potential of 900 volts. What is the capacitance of the condenser?

3. What is the difference of potential between the terminals of a condenser which has a capacitance of 24 mf. when the charge on the condenser is 0.05 coulomb?

4. What is the capacitance of three condensers of 5, 4, and 0.75 mf., respectively, when they are all joined in series?

5. Condensers of 2, 2, and 1 mf., respectively, are arranged so that they may be connected in series or in parallel. What capacitance is obtained in each case?

6. A condenser whose insulation can withstand an applied potential of 6,500 volts has a capacitance of 8 mf. What is the maximum energy the condenser can have?

7. What capacitance must a condenser have in order to hold energy equivalent to 30 g. lifted 40 cm. when charged to a potential of 8,000 volts?

8. A condenser consists of two parallel plates which are separated by a sheet of mica which is 0.2 cm. thick. The area of each plate is 400 sq. cm. The condenser is charged to 5,000 volts. Find the energy stored up in each cubic centimeter of the mica between the plates of the condenser. Dielectric constant of mica is 6.0.

9. A condenser is made up of 200 sheets of tin foil each 25 by 20 cm. These sheets of tin foil are separated by sheets of paraffined paper which are 0.15 cm. thick and have a dielectric constant of 3.8. What is the capacity of the condenser when alternate sheets of tin foil are joined together?

10. How much energy is stored up in a condenser having a capacity of 2,000 mf. when it is charged to a difference of potential of 7,500 volts?

CHAPTER XLV

ALTERNATING CURRENTS

538. Alternating Currents.—It was seen in Sec. 499 that when a rectangular loop revolves in a magnetic field, the current flows in opposite directions at regular intervals. Such a current is called an *alternating current.* When the current rises from zero to a maximum, returns to zero, then rises to a maximum in the opposite direction, and returns to zero again, it has completed a *cycle.* This cycle is repeated over and over again, and the

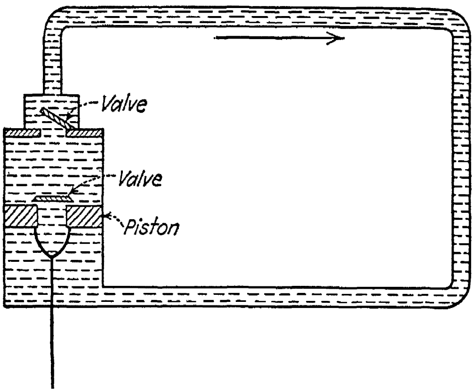


FIG. 499.—Hydraulic analogue of a direct current. The flow is always in one direction.

number of times the cycle is repeated each second is known as the *frequency.* The frequency ordinarily used in electrical power production is 60 cycles per second.

539. Water Analogy.—The flow of an electric current in a circuit may be likened to the flow of water in a closed system. Figure 499 shows a system of pipes containing a pump. The valves are so arranged that the water can flow in only one direction, whatever the direction in which the piston moves. Such an arrangement is the analogue of a direct-current system in which the pipes take the place of the wires and the pump the place of the generator. The valves allow the water to flow only in one direction, and the commutator allows the electricity to flow only in one direction.

In Fig. 500 there are no valves, and the direction of the current in the pipes depends immediately on the direction of the motion of the piston.

Such a pump represents a generator provided with collecting rings instead of a commutator (Fig. 460). In such a system the flow of water is first in one direction and then in the other. In like manner, the flow of electricity in the external circuit (Fig. 460) is alternating.

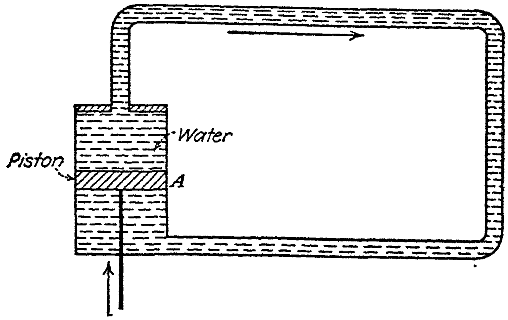


FIG. 500.—Hydraulic analogue of an alternating current. The flow reverses its direction when the piston reverses its direction of motion.

540. Instantaneous Electromotive Force.—In case the armature consists of a single coil of wire revolving in a horizontal magnetic field, the instantaneous voltage induced in it can be represented by a sine curve (Fig. 501), in which the angle through

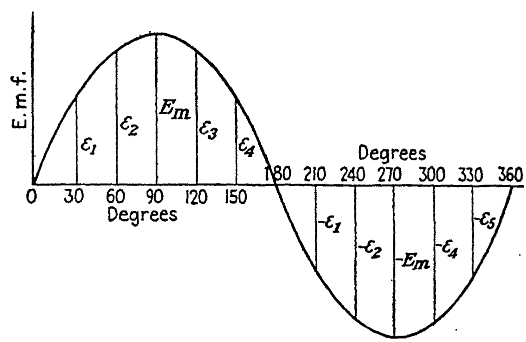


FIG. 501.—Variation of electromotive force with time in an alternating-current circuit.

which the coil has turned from its vertical position is plotted on the horizontal axis and the corresponding induced voltage on the vertical axis. The equation which connects the instantaneous voltage with the angle through which the coil has turned is

$$e = E_{\text{max.}} \sin \varphi,$$

in which $E_{max.}$ = the maximum voltage in the coil.
φ = the angle through which the coil has turned from its vertical position.

Example.—The maximum value of an alternating electromotive force is 250 volts. What is the instantaneous electromotive force when the plane of the coil (Fig. 420) makes an angle of 45 deg. with the vertical position?

$$e = E_{max.} \sin \varphi.$$
$$e = 250 \sin 45 \text{ deg.}$$
$$= 250 \times 0.707 = 177 \text{ volts.}$$

541. Average Value of Alternating Electromotive Force.—Since one-half the instantaneous values of the alternating electromotive forces are negative, and the other half are positive, and since the negative values are just equal to the positive values, the average value of the electromotive force over a complete cycle is zero. The average value can, however, be found by plotting a number of instantaneous values over one-half of the cycle (Fig. 501) and then finding their average value. This average value can be found from the maximum value by means of the calculus (Appendix D-17).

Average value of e.m.f. = 0.637 times maximum value of e.m.f.

$$E_{ave.} = 0.637E_{max.}.$$

This equation makes it possible to calculate the maximum value of the electromotive force from the average value or to calculate the average value from the maximum value.

Example.—In an alternating-current generator one set of coils consists of 200 turns. This coil cuts 10,000,000 lines of force eight times each second. What is the maximum voltage induced in the coil?

$$\text{Average voltage} = \frac{\text{lines cut per second}}{10^8} \text{ volts}$$
$$= \frac{10{,}000{,}000 \times 8 \times 200}{10^8} = 160 \text{ volts.}$$
$$E_{ave.} = 0.637E_{max.}$$
$$E_{max.} = \frac{160}{0.637} = 251 \text{ volts.}$$

542. Effective Value of Alternating Current.—The heating effect of any electric current depends on the square of the current. Hence, an alternating current is said to be equivalent to a certain direct current when it produces the same heating effect as this

direct current. This effective value of an alternating current is found by taking the average of the squares of all the instantaneous values of the current and then extracting the square root of this average value. The effective value of the alternating current obtained in this way is known as *the square root of the mean square of the current.* It is this effective value which is important in most alternating-current problems.

The effective value of the current is (Appendix D-18).

$$I_{eff.} = 0.707 I_{max.}$$
$$= \frac{I_{max.}}{\sqrt{2}},$$

where $I_{eff.}$ = the effective value of the current.
$I_{max.}$ = the maximum value of the current.

In like manner for the voltage,

$$E_{eff.} = 0.707 E_{max.}$$
$$= \frac{E_{max.}}{\sqrt{2}},$$

where $E_{eff.}$ = the effective value of the voltage.
$E_{max.}$ = the maximum value of the voltage.

Example.—The effective value of an alternating current is 50 amp. What is its maximum value?

$$I_{eff.} = \frac{I_{max.}}{\sqrt{2}}.$$
$$50 = \frac{I_{max.}}{\sqrt{2}}.$$
$$I_{max.} = 50\sqrt{2} = 70.7 \text{ amp.}$$

Example.—What is the effective value of an alternating electromotive force for which the maximum value is 400 volts?

$$E_{eff.} = \frac{E_m}{\sqrt{2}}.$$
$$E_{eff.} = \frac{400}{\sqrt{2}} = 400 \times 0.707 = 282.8 \text{ volts.}$$

543. Relation of Current to Voltage in Circuits.—When an alternating electromotive force is applied to a circuit (Fig. 502) which contains an ohmic resistance R without any inductance, the curve representing the variation of the current with the time

has the same form as the corresponding curve for the impressed voltage. The current and the voltage each reach their maximum value at the same instant (Fig. 503), and their zero values at the same instant. In such a case the current is said to be *in phase* with the electromotive force. The current and the electromotive force are then given by the following equations:

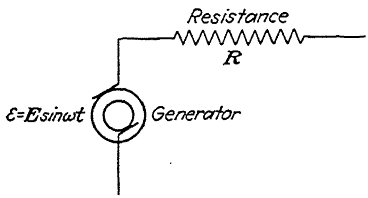


FIG. 502.—Circuit containing an alternating electromotive force and a non-inductive resistance.

$$i = I_m \sin\varphi = I_m \sin\omega t.$$
$$e = E_{max.} \sin\varphi = E_m \sin\omega t.$$

Ohm's law in its simplest form applies in this case so that

$$i = \frac{e}{R}$$
$$I_{max.} = \frac{E_m}{R}$$

If the circuit contains an inductance L (Fig. 504) in addition to the resistance R, the current in the circuit will lag behind the

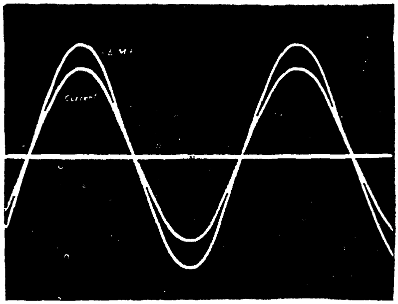


FIG. 503.—Relation of current to electromotive force in a non-inductive circuit. Electromotive force and current are in phase.

impressed electromotive force and reach its maximum value later than the time at which the impressed voltage reaches its maximum value. The relation between the current and the impressed voltage will be as shown in Fig. 505, in which the curve representing the current is seen to be displaced with respect to the curve representing the voltage. This displacement arises

out of the fact that the current lags behind the voltage. In this case the current is given by the equations,

$$i = I_{\text{max.}} \sin (\varphi - \theta) = I_{\text{max.}} \sin (\omega t - \theta)$$

and

$$e = E_{\text{max.}} \sin \varphi = E_{\text{max.}} \sin \omega t$$

where θ is the angle of lag of the current behind the electromotive force.

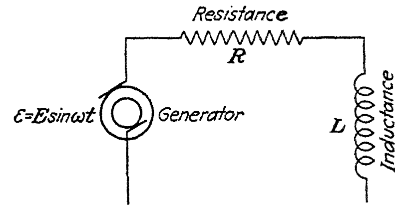


FIG. 504.—Circuit with inductance, resistance, and an alternating e.m.f.

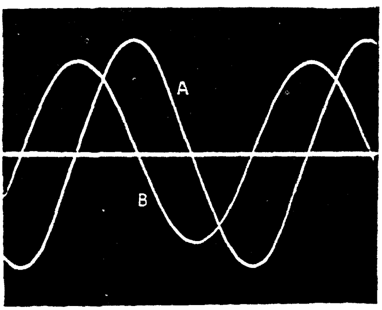


FIG. 505.—Lag of current behind electromotive force in an inductive circuit. A is current; B is electromotive force.

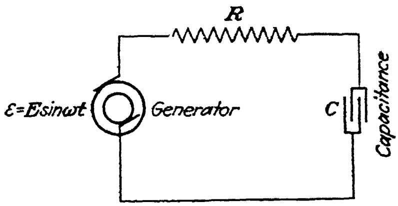


FIG. 506.—Circuit with capacitance and resistance.

If the circuit contains a capacitance C (Fig. 506) instead of an inductance, the current in the circuit is ahead of the impressed electromotive force and the relation between the current and the impressed voltage is as represented in Fig. 507. The current reaches its maximum value before the voltage has become a

maximum. In such a case the current is said to be *ahead of the voltage* or *to lead the voltage*. The greater the capacitance, the greater is the angle by which the current leads the impressed voltage. The equations expressing the current and the voltage at different instants are

$$i = I_{\text{max.}} \sin (\varphi + \theta)$$

and

$$e = E_{\text{max.}} \sin \varphi$$

where θ is the angle by which the current leads the voltage.

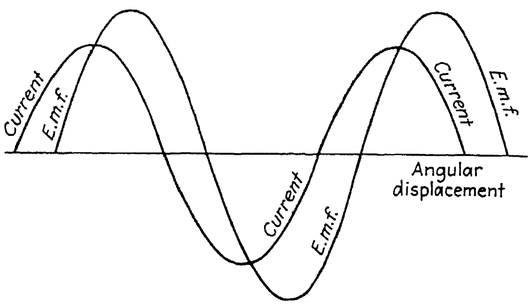


FIG. 507.—Lead of current ahead of electromotive force in a circuit with capacitance and resistance.

Example.—In an alternating-current circuit the current is 30 deg. ahead of the voltage. The effective current is 50 amp. What is the instantaneous current when the voltage is zero?

$$i_{\text{max.}} = \frac{I_{\text{eff.}}}{0.707} \quad \frac{50}{0.707} = 70.5 \text{ amp.}$$
$$i = I_{\text{max.}} \sin (\varphi + \theta)$$
$$= 70.5 \sin (0 + 30) = 70.5 \sin 30 = 35.2 \text{ amp.}$$

544. Resonance in Series Circuits.—If a circuit contains both capacitance and inductance in series, in addition to ohmic resistance (Fig. 508), the effect of the inductance is partially or wholly neutralized by the capacitance. The inductance causes the current to lag behind the electromotive force, but the capacitance alone would cause it to be ahead of the electromotive force. Hence, when both inductance and capacitance are present in the circuit, the current and the electromotive force are more nearly in phase than they would be with either inductance or capacitance alone in the circuit. By suitably choosing the inductance and the capacitance, the effect of one may be made

exactly to neutralize the effect of the other. The circuit then behaves as if it contained neither inductance nor capacitance but only resistance. When this condition is realized, the current has its largest value, and the circuit is said to be *in resonance*.

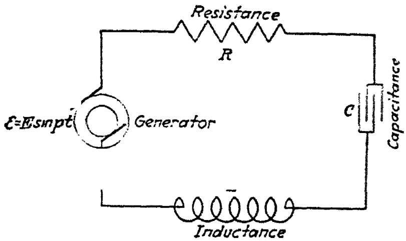


FIG. 508.—Circuit with inductance, capacitance, and resistance.

The current is then

$$i = \frac{e}{R}$$

as in the case of a circuit containing neither inductance nor capacitance.

545. Power in Alternating-current Circuits.—The power at any instant in an alternating-current circuit is obtained by multiplying the current at that instant by the voltage at the same instant.

Power = instantaneous current times instantaneous voltage.

$$p = ei$$

where p = the instantaneous power in watts.
i = the instantaneous current in amperes.
e = the instantaneous electromotive force in volts.

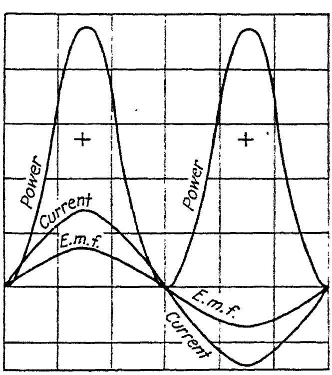


FIG. 509.—Power in a circuit when electromotive force and current are in phase.

The power will vary from instant to instant, just as the current and the voltage vary from instant to instant. The curve representing the power at each instant is obtained by multiplying the current by the corresponding voltage. Such a power curve is shown in Fig. 509 for the case when the current and voltage are

in phase with each other. The power is always positive, although the current and voltage are negative in one-half the cycle. The current and voltage change signs at the same instant so that their product is always positive. This means that when the current and voltage are in phase, there is never a time at which energy is flowing backward and being returned from the circuit to the

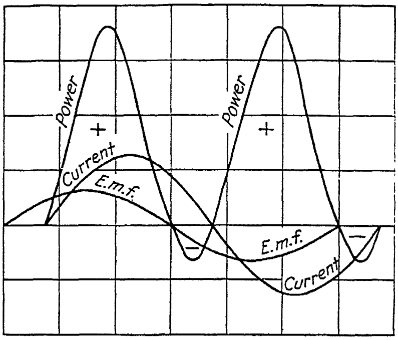


FIG. 510.—Power in a circuit when electromotive force and current are not in

generator. The effective power in such a circuit is equal to the effective current times the effective voltage.

$$P = EI.$$

The power in this circuit is all used in the resistance R, and the instantaneous power is, therefore,

$$p = i^2R.$$

The average power is

$$P = \text{average } i^2R.$$

Since, by definition, average $i^2 = I^2$,

$$P = I^2R;$$

and, since $E = IR$, by Ohm's law,

$$P = IE.$$

When the current and voltage are out of phase because of the presence of inductance and capacitance in the circuit, the power curve has the form shown in Fig. 510. In this case, the power is negative in part of the cycle, and that part of the power curve lies

below the horizontal axis. This part of the power curve is obtained by multiplying a positive value of the current by a negative value of the voltage, or by multiplying a positive value of the voltage by a negative value of the current. When the power is negative, energy is flowing back to the generator from the circuit during that part of the cycle.

Problems

1. The effective value of an alternating electromotive force applied to a condenser is 6,000 volts. What is the maximum value?

2. An alternating current flowing through a resistance of 18 ohms produces heat at the rate of 660 watts. What is the effective value of the current? of the voltage?

3. The average value of the electromotive force in the armature of an alternator is 180 volts. What is the maximum value? the effective value?

4. An alternating electromotive force has a frequency of 60 cycles and an effective value of 220 volts. What is the instantaneous value of the electromotive force at an instant $\frac{1}{360}$ sec. after it passes the zero value?

PART V.—ELECTRONICS

CHAPTER XLVI

FREE ELECTRONS

546. Conductivity of Gases in the Normal State.—A gas in its normal state is a poor conductor of electricity. For a long time it was very doubtful whether gases conducted electricity at all. This question was, however, settled by the following experiment. The gas to be studied is enclosed in a large spherical vessel (Fig. 511) containing a charged system which takes the form of a pair of gold leaves suspended from a brass rod. The end of the rod carrying the gold leaves is insulated by means of a piece of sulphur, *S*. This piece of sulphur is in turn supported by a brass rod *E* which passes through the ebonite stopper *C*. The rod *E* is charged to a suitable potential by connecting it to a battery of about 500 volts. The other terminal of this battery is connected to the silver coating on the vessel. Contact between the brass rod *E* and the gold leaves is made whenever necessary by means of the fine iron wire *W* which can be moved from outside by a magnet in such a way as to make contact with the rod carrying the gold leaves. The wire is then allowed to move back into its original position, leaving the leaves charged to the potential of the battery.

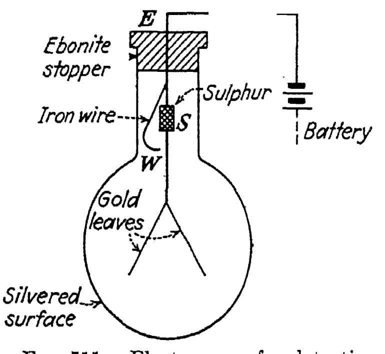


FIG. 511.—Electroscope for detecting the conductivity of the air. The leaves of the electroscope slowly collapse owing to the residual ionization of the air.

Since the gold leaves and the brass rod *E* are at the same potential, there is no tendency for the electricity on the gold leaves to leak across the sulphur plug *S*. Any loss of charge from the gold leaves takes place through the air surrounding them. Even

under these conditions the leaves very gradually lose their charge and slowly collapse. Hence, a gas in its normal state allows a very small current of electricity to pass through it. This leak is so small that for most purposes a gas may be considered a perfect insulator, but under certain circumstances the gas may acquire a conductivity which is very many times larger than the conductivity it has in the normal state.

547. Ionization of Gases.—There are many ways in which gases may be put into the conducting state. If, for example, a quantity of radium is brought near the gold leaves of the electroscope (Fig. 511), these leaves collapse with considerable rapidity, showing that the gas surrounding the leaves has acquired the power to conduct electricity. Similar results are observed if an X-ray tube is set in operation in the neighborhood of the charged

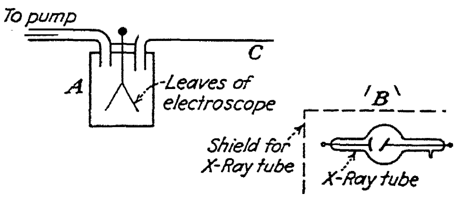


FIG. 512.—Apparatus for showing the persistence of the conductivity in a gas.

gold leaves. Gases which have been drawn from the region around a Bunsen flame (Fig. 512) have the power to conduct electricity temporarily. The capacity of the Bunsen flame to make the gas conducting is enormously increased if the flame is fed with some volatile salt such as sodium chloride. Light of short wave length also has this power of rendering a gas conducting.

In such cases, the gas retains its conductivity for some little time after the agent which made it a conductor has ceased to act. Its conductivity, however, always diminishes after the agent is removed and finally disappears. This persistence of the conductivity may be shown by placing an electroscope in a glass vessel A (Fig. 512), in which there are two tubes. One of these tubes leads to a pump by means of which air can be drawn through the vessel containing the electroscope. The other tube extends from the vessel A to the region where the X-ray tube is in action. The X-ray tube is enclosed in a lead box except for the

window B through which the X-rays from the tube pass into the air around the mouth of the tube leading to the vessel A. The lead box shields the electroscope from the direct action of the X-rays. If, by means of the pump, air is drawn past the leaves of the electroscope while the X-ray tube is in action, the leaves will gradually collapse. When the pump is stopped and the flow of air ceases, the discharge from the leaves of the electroscope also ceases. If a plug of cotton wool is inserted in the tube at C and the air then drawn through the tube as before, it is found that the electroscope will retain its charge even when the X-ray tube is in action. The conductivity has thus been removed by filtering the air through the cotton wool.

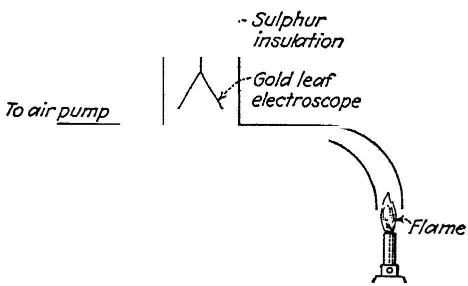


FIG. 513.—Gases rendered conducting by a Bunsen flame.

The conductivity may also be removed from the gas by making it traverse a strong electric field in such a way that a current of electricity passes through it. From this fact it must be inferred that the gas in the conducting state must contain electrified particles. To these electrified particles is due the power of the gas to conduct electricity. These electrified particles are either **electrons** or **ions.** The process by which a gas is made into a conductor of electricity is called the **ionization of the gas.**

The conductivity of the gas is then explained by assuming that charged particles are produced in the gas by the action of various agencies and that these charged particles or gaseous ions, moving under the action of the electrostatic field, constitute the current through the gas. The transfer of electricity through a gas then becomes very similar to the transfer of electricity through an electrolyte by the motion of positively and negatively charged electrolytic ions.

548. Ionization by Flames.—If two platinum wires (Fig. 514) are placed in the flame of a Bunsen burner in such a way that they are not in contact, a current will flow from one to the other when they are connected to the terminals of a battery. The current, however, is quite small. If now some kind of salt, for example sodium chloride, is introduced into the flame its conductivity is greatly increased. At the temperature of the flame, ions are liberated from the salt and these ions make the flame a better conductor of electricity. The salt may also be supplied to the flame by spraying a solution of it into the illuminating gas by which the flame is being fed. The curve showing the relation between the difference of potential between the wires A and C and the current in the galvanometer has the form represented in Fig. 515. The value of the current must be corrected for the corresponding current when there is no salt in the flame. The current for a given concentration of salt in the flame rises rapidly with an increase in the difference

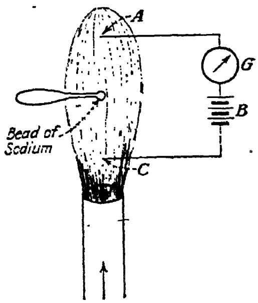


FIG. 514.—Ionization of salt in a Bunsen burner.

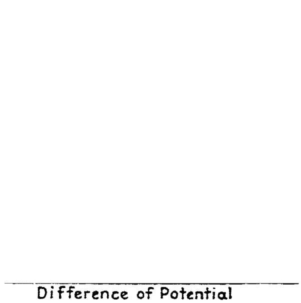


FIG. 515.—Relation between the difference of potential and the current in a flame.

of potential between the wires A and C. It then becomes nearly parallel to the horizontal axis, showing that a further increase in the difference of potential only slightly increases the current.

549. Velocity of Electrons.—The velocity of electrons, moving in an electric field, depends on the voltage which is applied across the electrodes between which they are moving. The following table shows the velocities of electrons for a few selected electric fields. When the voltage on the tube

Kilovolts	Velocity in Centimeters per Second
25	9.0×10^9
75	14.7×10^9
100	16.5×10^9
200	20.9×10^9

is not uniform, *i.e.*, not the same over every possible path of an electron, the velocities will not be the same for different electrons.

Example.—Find the speed of an electron which has passed through a difference of potential of 4,000 volts.

e = the charge on the electron
$= 1.59 \times 10^{-20}$ e.m.u.
m = the mass of the electron
$= 9.0 \times 10^{-28}$ gm.
V = the difference of potential in electromagnetic units.

$$Ve = \frac{1}{2}mv^2$$

$$v = 3.7 \times 10^9 \text{ cm. per second.}$$

550. Variation of Current with Potential.—The current of electricity through a conducting gas is not proportional to the impressed electromotive force except when this electromotive force is small. Consider two metal plates A and B (Fig. 516) immersed in a gas, and let the gas between the plates be exposed to the ionizing action of an X-ray tube. If the plates are connected through a sensitive galvanometer to the terminals of a battery, a current of electricity passes from one plate to the other. As the potential of the battery is increased, the current through the galvanometer increases. For the lower values of the potential, there is an approximate proportionality between the potential and the current; that is, Ohm's law holds. As the difference of potential (Fig. 517) is raised above a certain value which depends on the nature and pressure of the gas, the distance between the plates, and the intensity of ionization, a stage is reached where the current increases less rapidly than the difference of potential. If the difference of potential is still further increased, a point is reached at which further increase in the difference of potential causes no further increase in the current through the gas. At this point, the current becomes independent of the applied voltage. This maximum current through a gas is called the **saturation current.** The voltage to produce this saturation current depends on the distance between the plates and on the intensity of ionization of the gas. Except for cases of very intense ionization, a field of from 20 to 30 volts per centimeter between the plates is sufficient to produce saturation.

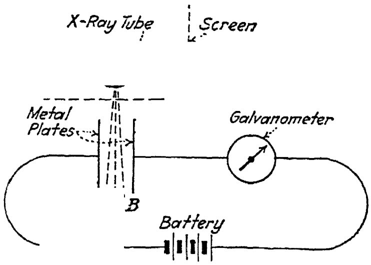


FIG. 516.—Apparatus for showing the variation of current in a gas with potential.

551. Ionization by Impact.—When the saturation value of the electric current has been reached, a moderate further increase in the applied voltage does not produce an appreciable increase in the current through the gas. If, however, the electric field is

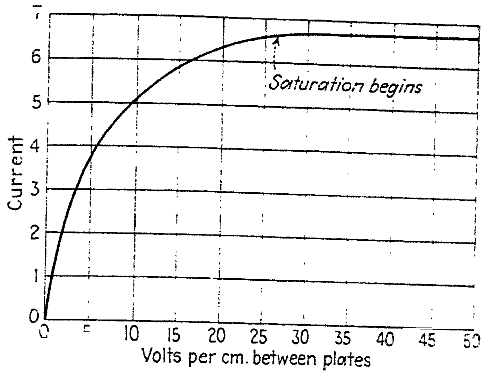


FIG. 517.—Saturation current in an ionized gas.

increased above a certain value, the current again begins to increase (Fig. 518). This increase is at first slow. It soon becomes more rapid and finally a spark passes between the plates. This progressive effect is to be ascribed to the collisions between

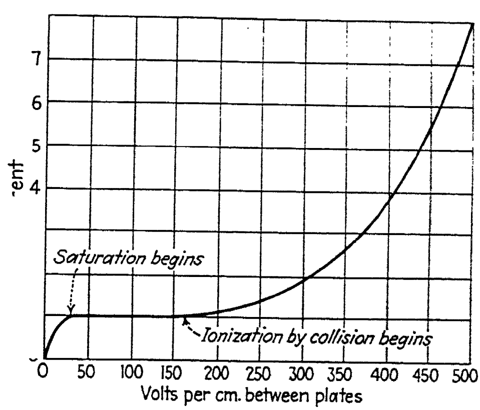


FIG. 518.—Ionization by collision in a gas.

the ions in the gas and the molecules of the gas. When one of the ions already present in the gas has acquired a sufficient velocity under the action of the impressed voltage and then strikes a neutral molecule, one or more electrons may be dislodged from

the molecule or the molecule may be split into positively charged and negatively charged ions. In this way fresh ions are produced in the gas, and these act as carriers of the current of electricity and thus make it possible for a greater current to traverse the gas. The rapid rise in the curve which begins at about 200 volts per centimeter indicates that when the electric field has reached that value, the ions already present in the gas move with sufficient speed to produce new ions. This production of additional ions by the collision of rapidly moving ions is called **ionization by impact,** or **ionization by collision.**

The minimum speed necessary to produce ions in this manner depends on the nature of the projected ion and upon the characteristics of the molecules which it strikes. It is more than a thousand times as great as the speed of the swiftest rifle bullet. The distance an ion travels before striking a molecule in a gas is only a fraction of a millimeter at ordinary pressures.

A good illustration of ionization by collision is found in what is called a "brush discharge" from the terminals of a high-potential transformer or an induction machine. Jagged filaments of light extend from these terminals. These filaments are minute sparks terminating in the air surrounding the terminals. They are due to ionization by impact of negative ions on molecules of air. There are always a few residual ions in the space about the terminals. These negative ions are accelerated by the electric fields in which they are located and produce other ions by impact. If the terminals are brought close together, the electric force on the ions is increased until a spark leaps across from one terminal to the other. When this disruptive discharge takes place, there is a supply of ions produced by the collision of ions already in the gap with molecules of air surrounding the terminals.

552. Discharge in Gases at Low Pressures.—If a discharge of electricity takes place in a gas at low pressure between electrodes enclosed in a glass tube, some beautiful and interesting effects are observed. If the difference of potential between the electrodes is not much greater than that necessary to maintain the current, the luminosity is confined to the region around the electrodes and the remainder of the tube is dark. As the pressure is reduced to less than 1 mm. of mercury, the glow extends outward from the two electrodes and occupies the greater part of the tube. The potential necessary to maintain a current across the tube is also decreased until a certain degree of exhaustion is reached. Beyond this point the higher the exhaustion, the greater the difference of potential required to produce a discharge.

At this stage, the chief characteristics of the discharge are as follows (Fig. 519): A faint velvety glow, known as the cathode

glow, covers the surface of the negative electrode, or cathode. Outside of this is a space called the Crookes dark space. Beyond the Crookes dark space is a luminous region known as the negative glow and then a second dark space (the Faraday dark space). The Faraday dark space is followed by a second luminous region, reaching nearly to the positive electrode, or the anode, as it is called. This column, which is known as the

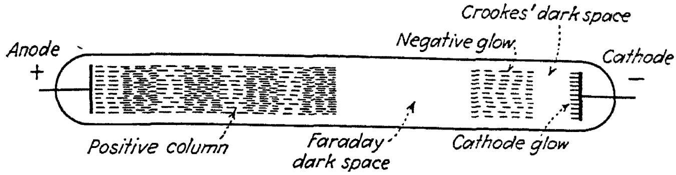


FIG. 519.—Discharge of electricity in a gas at low pressure.

positive column, is not continuous but shows alternate bright and dark layers across the path of the discharge. If the distance between the electrodes is increased, the appearance at the cathode or negative electrode is not much changed. The positive column is, however, increased in length and reaches nearly to the negative glow.

553. Cathode Rays.—If the pressure in the tube is sufficiently low, invisible streams of particles known as **cathode rays,** proceeding normally from the cathode, may reach the farther boundary of the tube and produce a vivid fluorescence where they strike the glass. The following evidence shows that these cathode rays are charged particles of some very attenuated form of matter.

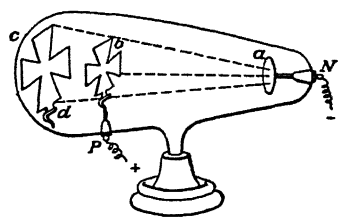


FIG. 520.—Cathode rays travel straight lines.

1. *The Rays Travel in Straight Lines.*—This fact is made evident by inserting some obstacle in the path of the rays (Fig. 520) and noting that its geometric shadow on the boundary of the tube no longer shows fluorescence. The obstacle has prevented the rays from striking the wall of the tube behind the obstacle.

2. *The Rays Emerge Normally from the Surface of the Cathode.*—If the cathode is made with a concave surface, the cathode rays come to a focus which is near the geometric center of the

concave surface. Except for the fact that the rays somewhat repel each other, the focus would be exactly at the geometric center of the concave surface.

3. *The Rays Penetrate Small Thicknesses of Matter.*—If a small window of thin aluminum leaf is inserted in the end of the tube (Fig. 521) where the rays can strike it without passing through

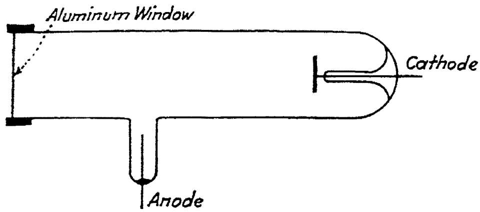


FIG. 521.—Passage of cathode ray through thin aluminum foil.

the glass wall, the rays will pass through this window and make themselves evident by the luminous streamers which they produce in the air on the far side of the window.

4. *The Cathode Rays Are Deflected by a Magnetic Field.*—If a bar magnet is held near the discharge tube, the magnetic field causes a deflection of the rays from their original path. This deflection is evident from the displacement of the fluorescent spot on the

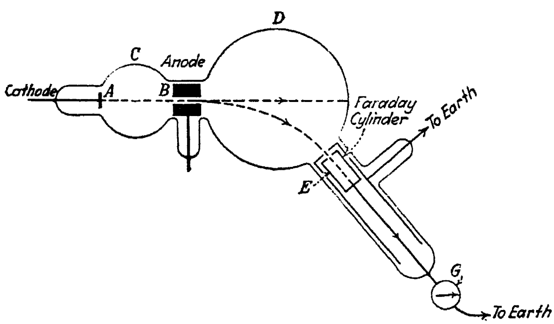


FIG. 522.—Cathode rays deflected by a magnetic field.

farther boundary of the tube. The rays are deflected at right angles to the lines of magnetic force which come from the end of the bar magnet. The rays behave as a flexible wire carrying a current of electricity would behave under similar conditions.

5. *The Cathode Rays Carry a Negative Charge of Electricity.*—.e large discharge tube (Fig. 522) is connected to a smaller

spherical tube C which contains the cathode A and the anode B. The cathode rays starting from the cathode A pass through a

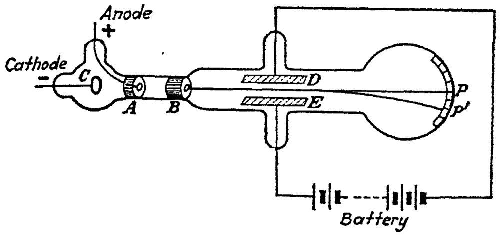
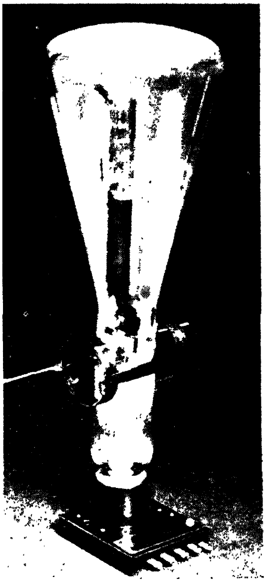


FIG. 523.—Cathode rays deflected by an electric field.

small opening in the anode B and then traverse the larger bulb D. Where they strike the wall of the tube, they produce a fluorescent spot. At E is a small cylindrical vessel with a small opening facing the center of the tube. This vessel is insulated from the glass bulb but is connected to a galvanometer or electrometer, G. When the discharge passes through the bulb D, the hollow cylinder E receives very little charge; but when the cathode rays are deflected by a magnet so as to make them enter the cylinder E, a large negative charge is indicated by the galvanometer or electrometer. This experiment shows that the stream of cathode rays carries with it charges of negative electricity.



ode-ray oscillograph. Its action depends on the simultaneous deflection of electrons by an electric and a magnetic field. (*Courtesy Bell Laboratories.*)

6. *The Cathode Rays Are Deflected by an Electrostatic Field.*—If an electric discharge is caused to pass from C to A in the discharge tube (Fig. 523), the stream of cathode rays will pass through the perforated anode A and the second perforated ring B. In this way, a pencil of rays will be isolated. This pencil will traverse the tube and, after passing between the two plates D and E, will strike the end of the discharge tube at P and produce a bright fluorescent spot at that point. If now an electric

field is applied between the plates D and E by connecting them to a high potential battery, the stream of cathode rays will be deflected downward or upward according to the direction of the electric field. This deflection arises out of the fact that the charged particles while passing between the plates are attracted to one of them and thus deflected from their original course. By noting the direction of this deflection, it will be found that the cathode stream has been pulled toward the plate which is charged with positive electricity. This deflection again shows that a stream of cathode rays consists of negatively charged particles.

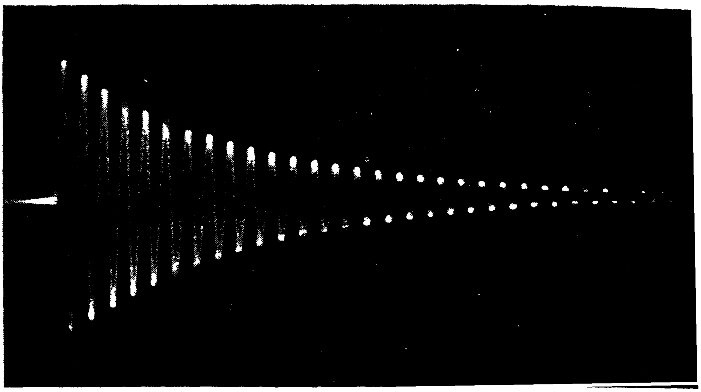


FIG. 525.—Oscillating discharge photographed by means of a cathode-ray oscillograph. (*Courtesy Bell Laboratories.*)

The simultaneous deflection of a stream of electrons by an electric and a magnetic field is made use of in an oscillograph tube (Fig. 524) which indicates the change of rapidly varying electromotive forces or rapidly varying magnetic fields. Figure 525 gives a photographic record of a damped electric oscillation taken with an oscillograph.

554. Determination of the Ratio of the Charge to the Mass.—The simplest method of determining the ratio of the charge to the mass of a cathode particle is based on the simultaneous deflection of these cathode particles by an electric and a magnetic field. If an electric field is applied between the plates D and E (Fig. 523) so as to deflect the cathode particles downward, a magnetic field perpendicular to the plane of the figure may be so directed as to deflect them upward. By suitably adjusting the intensities of the electric and magnetic fields, the magnitude

of the upward and downward deflections may be made just equal to each other. In such a case, under the simultaneous action of both the electric and magnetic field, the stream of cathode particles will not be deflected. As soon as the intensities of both the electric and the magnetic fields are known, and the deflection produced by the magnetic field alone has been observed, it becomes possible to calculate the ratio of the charge on the cathode particle to its mass. This ratio comes out to be

$$\frac{e}{m} = 1.77 \times 10^7 \text{ e.m.u. per gram.}$$

if the charge is measured in electromagnetic units and the mass in grams.

555. Condensation of Water Vapor on Ions.—It was found by C. T. R. Wilson that gaseous ions, whether positive or negative, act as nuclei for the condensation of water vapor. If there is a gas free from dust in a closed vessel and if the gas is saturated with water vapor and contains free positive and negative ions, a cloud will be produced by a sudden expansion of the gas when the ratio of the volume of the gas after expansion to its volume before expansion is greater than 1.25. An expansion of this amount produces only a slight condensation in the gas if it does not contain ions. The water vapor condenses around the ions and, if the ions are not too numerous, each ion becomes the nucleus of a drop of water. These drops are visible, and their rate of fall can be observed. From this rate of fall the radius of the drop can be calculated, and by weighing the amount of water which falls after a single expansion, the number of drops in the cloud can be determined. Then by measuring the total charge carried down by the cloud, the charge on each drop and therefore the charge on each ion can be calculated. It is, however, nearly impossible to be sure that each drop has a single ion at its center. Hence, the results may be in error because there are fewer drops than there are ions.

Figure 526 shows the way in which this experiment is carried out. To produce the expansion, a very light movable piston P is suddenly pulled down so as to increase the volume of the space above it. This movement of P causes a sudden expansion and consequent cooling of the air in A. By means of an X-ray tube at C the air in A has been previously ionized. The

condensation of the water vapor on the ions forms a cloud. From the rate at which this cloud falls, the average radius of the drops is calculated. The amount of the expansion is so chosen that drops form only about the negative ions. The quantity of electricity carried down by the cloud is measured by means of an electrometer or galvanometer connected to the plate on which the water falls.

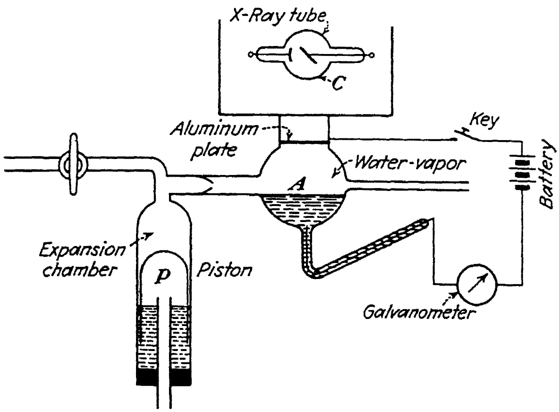


FIG. 526.—Apparatus for producing the condensation of water vapor on ions.

The mass of n drops is given by the equation,

$$M = \frac{4}{3}\pi\rho r^3 n$$

where ρ = the density of the water.
M = the mass of the water deposited from the cloud.
r = the radius of each drop.
n = the number of drops in the cloud.

If Q is the total charge of electricity brought down by the cloud, the charge on each ion is

$$e = \frac{Q}{n}.$$

Figure 527 shows the condensation of water vapor on electrons which have been set free by X-rays passing through a condensation chamber.

556. The Charge on an Electron.—Since the preceding evidence shows that an electron carries a charge of negative elec-

tricity, the next important step is the exact determination of the amount of this charge. The measurement of this charge has been carried out in a number of ways. The most exact of them is the method perfected by Millikan. In this method, two horizontal plates B and C (Fig. 528) are placed a few centimeters apart. By means of an atomizer A, a small quantity of oil is sprayed into the space above these plates. This spray of oil is in the form of drops which are so small that they do not settle for a long time after the air has become quiet. After a time, one of these small drops finds its way through a small opening O in the upper plate. A telescope M is focused on this drop so that its movements can be observed over a long period of time. The rate at which the drop falls can thus be measured by means of the telescope with a micrometer eyepiece. A beam of X-rays is next sent through the air between the horizontal plates. By means of this beam of X-rays, electrons are detached from the atoms of air or other gas, leaving the residue of an atom charged with positive electricity. This residue is called an **ion.** A battery of high potential is connected across the horizontal plates.

FIG. 527.—Photograph of water vapor condensed on ions.

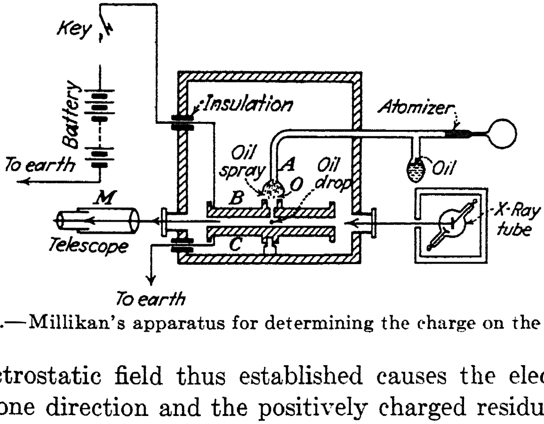


FIG. 528.—Millikan's apparatus for determining the charge on the electron.

The electrostatic field thus established causes the electrons to drift in one direction and the positively charged residues of the atoms, *i.e.*, the ions, in the opposite direction. These ions and electrons occasionally strike the small drop of oil and adhere to it.

The drop thus becomes charged with either positive or negative electricity and is pulled one way or the other by the electrostatic field. The potential of the battery is so adjusted that the drop hangs in the air for a long time without appreciable motion. It is pulled upward by the electrostatic field and downward by gravity.

In order to get the charge on the drop, it is only necessary to know the difference of potential between the plates, their distance apart, and the weight of the drop. If e is the charge on the drop, V the difference of potential between the plates, d the distance between the plates, and W the weight of the drop.

$$W = \frac{eV}{d}.$$

If a single electron adheres to the drop, it would be possible in this way to calculate its charge. But ordinarily a number of electrons or ions may be attached to the drop, and the charge will be more than that on a single electron. Very careful observations showed that whatever charges were present on the drops, their magnitudes were always even multiples of a single elementary charge. This elementary charge is $e = 1.592 \times 10^{-19}$ coulomb.

Whatever the character of the gas in the chamber and whatever the nature of the drop on which the charge collects, the result is always the same. It is safe to conclude from these experiments that the electron always carries the same charge of negative electricity and that this charge is the smallest known unit of electricity. This fundamental unit of electricity is

$$\begin{aligned} e &= 1.592 \times 10^{-19} \text{ coulomb} \\ &= 4.77 \times 10^{-10} \text{ e.s.u.} \end{aligned}$$

Example.—What must be the charge on a drop of oil weighing 1 mg. to keep it at rest in an electrostatic field of 10 e.s.u. of intensity?

$$\begin{aligned} mg &= VQ. \\ V &= 10 \text{ e.s.u.} \\ g &= 980 \text{ cm. per second per second.} \\ m &= 0.001 \text{ g.} \\ Q &= \frac{0.001 \times 980}{10} \quad 0.98 \times 10^{-1} \text{ e.s.u.} \\ & \quad\quad 3.27 \times 10^{-11} \text{ coulomb.} \end{aligned}$$

557. Mass of an Electron.—By means of observations on the magnetic and electrostatic deflections of a beam of cathode rays,

it is possible to determine the ratio of the mass to the charge of an electron. This ratio comes out to be

$$\frac{e}{m} = 1.77 \times 10^7 \text{ e.m.u. per gram.}$$

Since the absolute value of the elementary charge has been found in the way described in the preceding experiment, it is now possible to find the actual mass of an electron. This mass is

$$m = 8.96 \times 10^{-28} \text{ g.}$$

This is a very small mass, the smallest mass known. It is about 1/1,845 the mass of a hydrogen atom.

Example.—Find the mass of the electrons transported by a current of 100 amp. flowing for 2 hr.

$$\begin{aligned} \text{Quantity} &= \text{current} \times \text{time} = \text{number of electrons} \times \text{charge on an electron} \\ &= \frac{100 \times 3{,}600 \times 2}{10} \\ &= 72{,}000 = N \times 1.59 \times 10^{-20}. \\ N &= \frac{72{,}000}{1.59 \times 10^{-20}} = 4.5 \times 10^{24}. \\ \text{Mass in grams} &= 9 \times 10^{-28} \times 4.5 \times 10^{-24}. \\ &= 40.5 \times 10^{-4} \text{ g.} \end{aligned}$$

558. The Mass of an Electron as a Function of Its Velocity.—In the case where an electron moves with a speed which approaches the velocity of light it has been shown by careful experiments that the so-called "mass" of the electron increases with the speed so that at these high speeds the mass is much larger than it is for electrons moving with low speeds.

Let m_0 = the mass of the electron at rest.
m = the mass of the electron moving with a velocity v.
v = the speed of the electron.
c = the velocity of light.

Then,

$$\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From this equation it is seen that, if $v = c$, the mass of the electron becomes infinite. It will be seen later that beta particles are rapidly moving electrons ejected from the nucleus of atoms which are radioactive. Such beta particles move with velocities which are as great as 180,000 miles per second. Consequently, the mass of these rapidly moving particles is greatly in excess of the mass of an electron at rest. The following table shows the mass of some beta particles or electrons as a function of the velocity.

MASS OF A BETA PARTICLE AS A FUNCTION OF ITS VELOCITY

Ratio of velocity to the velocity of light	0.01	0.50	0.70	0.90	0.98
Ratio of mass to mass of electron at rest	1.00	1.115	1.40	2.94	5.03

Example.—What is the relative mass of an electron at rest and in motion with a velocity of 1.6×10^{10} cm. per second?

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - (1.6/3)^2}} \quad 1.18.$$

559. Passage of Electrons through Matter.—When a stream of electrons falls on a thin shell of metal foil, a small fraction of the electrons is reflected. A fraction of them is completely stopped by the metal foil and never gets through it. The remainder of the electrons pass through the foil and emerge with diminished velocities. The fraction of the electrons that emerge from the metal foil depends upon the velocity with which they strike it. This fraction also depends on the thickness of the foil and the characteristics of the metal out of which it is made.

Let N_0 = the number of electrons of a given velocity incident on the foil.
N = the number of electrons of a given velocity emerging from it.
d = the thickness of the foil.
e = the natural logarithmic base.
a = a constant depending on the nature of the metal and the velocity of the electrons.

$$N = N_0 \cdot e^{-ad}$$

560. Computing the Weight of Atoms.—By means of the oil-drop experiment, Millikan determined the elementary charge on the electron. The value of this charge is 15.9×10^{-19} coulomb. Hence, the number of electrons which would have on them a charge of 1 coulomb is

$$N = \frac{1}{15.9} \times 10^{20} = 6.29 \times 10^{18}.$$

Each univalent ion, like $\overline{\text{Cl}}$, has on it one excess electron, and this electron is liberated at the anode in electrolysis. The total number of electrons liberated at the anode when 1 coulomb passes through the cell is then

$$6.29 \times 10^{18}.$$

Each of these electrons has associated with it one atom since chlorine is univalent. Hence, the number of atoms liberated at the anode by a coulomb of electricity is 6.29×10^{18}. Now in the case of chlorine, the mass of

chlorine liberated by 1 coulomb is 0.0003675 gm. This mass is known from the electrochemical equivalent of the elements. Hence, the mass of each atom of chlorine is

$$\begin{aligned}\text{Mass of atom of chlorine} &= \frac{\text{mass of Cl deposited by 1 coulomb}}{\text{number of electrons in 1 coulomb}}\\ &= \frac{0.0003675}{6.29 \times 10^{18}}\\ &= 5.84 \times 10^{-23} \text{ g.}\end{aligned}$$

$$\begin{aligned}\text{Mass of atom of hydrogen} &= \frac{\text{mass of hydrogen liberated by 1 coulomb}}{\text{number of electrons in 1 coulomb}}\\ &= \frac{0.00001045}{6.29 \times 10^{18}}\\ &= 1.66 \times 10^{-24} \text{ g.}\end{aligned}$$

Since oxygen is divalent and each oxygen atom has two extra electrons, the number of ions to carry 1 coulomb is,

$$\frac{6.29 \times 10^{18}}{2} = 3.14 \times 10^{18}.$$

$$\begin{aligned}\text{Mass of atom of oxygen} &= \frac{\text{mass of oxygen liberated by 1 coulomb}}{\text{number of electrons in 1 coulomb}}\\ &= \frac{0.0000829}{3.14 \times 10^{18}}\\ &= 2.64 \times 10^{-23} \text{ g.}\end{aligned}$$

561. Positive Rays.—In the preceding sections, the mass and charge of an electron obtained from a neutral atom or molecule have been considered. Since the electron is charged with negative electricity and the atom or molecule was originally neutral, the residue after the removal of the electron must carry a positive charge of electricity. As the mass of the electron is small in comparison with the mass of the atom, this positively charged residue has a large mass in comparison with that of the electron. The discharge tube must contain, besides the free electrons, a large number of positively charged atoms or molecules. When an electromotive force is applied between the electrodes of the discharge tube, the electrons drift from the cathode to the anode. On the other hand, the positively charged atoms or molecules drift from the anode to the cathode. If a hole is made in the cathode normal to its surface, these positively charged particles stream through it and cause a fluorescence in the residual gas behind the cathode. Such a stream of positively charged molecules or atoms is known as a stream of **positive rays.** This stream of positive rays moves in the direction opposite to that of the cathode rays and consists of rapidly

moving, positively charged molecules or atoms. These positive rays can be deflected by both an electric and a magnetic field. In this way their mass and charge have been studied.

The discharge tube (Fig. 529) used for this purpose consists of a large bulb C in which the cathode A is pierced by a very

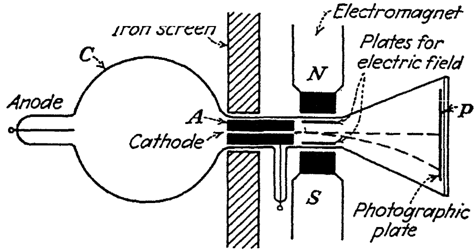


FIG. 529.—Positive rays passing through the hole in the cathode are deflected by either an electric or a magnetic field.

small hole. Through this hole passes a very fine pencil of positive rays which pass between the poles N and S of the electromagnet and then fall on the photographic plate P where they produce a blackening of the plate. By means of an electrostatic field which is in the same direction as the magnetic field, a second deflection which is at right angles to that arising from the magnetic field can be produced. In this way the positive rays of different masses and charges are deflected to different positions on the photographic plate, as shown in Fig. 530. The masses and charges can be determined from the location of the spots on the photographic plate. In this way it has been found that the positive rays consist of positively charged residues of atoms and molecules. They are really what is left of the atoms or molecules when they have lost one or more electrons. They differ from the electrons in the fact that they carry positive charges of electricity and in the further fact that they have masses which are about equal to the mass of the atom or molecule of gas in the discharge tube.

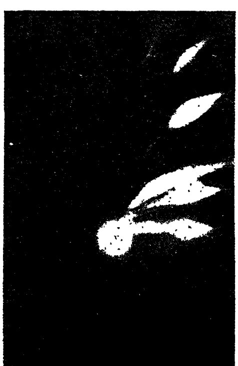


FIG. 530.—Photograph of positive rays after deflection by a magnetic and an electric field.

A modification of this experiment by Aston has given a very precise method for determining the masses of the chemical

elements and from his results is obtained the most precise information about the masses of the isotopes of the elements. When it was thus possible to measure the masses of individual atoms, it was found that we cannot assume that the masses of all atoms of the same element are identical. On the contrary, elements having the same chemical and physical properties may differ with respect to their atomic weights. Such elements occupy the same position in the periodic table and for that reason are called **isotopes.** The chemical method of determining atomic weights gives only the average value of the atomic weights of the isotopes. An element like lithium which has an atomic weight of 6.96 consists in reality of a mixture of two kinds of lithium. One of these types of lithium has an atomic weight of 7 and the other an atomic weight of 6. Since the lithium with an atomic weight of 7 is present in the mixture in much the larger proportion, the observed atomic weight is nearly equal to that of the heavier type of lithium.

The following table shows some of the results obtained by Aston on isotopes:

Element	Atomic number	Atomic weight	Minimum number of isotopes	Masses of isotopes
H..	1	1.008	1	1.008
He.	2	4.00	1	4
Li..	3	6.94	2	7, 6
B..	5	10.9	2	11, 10
Ne.	10	20.2	2	20, 22
Mg	12	24.32	3	24, 25, 26
Cl..	17	35.46	2	35, 37
	80	200.6	(6)	(197–200) 202, 204

Problems

1. A device for measuring very small currents has been designed which measures a current of 63 electrons per second. How many amperes is that?

2. Two small spheres, equally charged with negative electricity, at a distance of 2 cm. repel each other with a force of 10 dynes. How many excess electrons does each sphere carry? What is the mass of these electrons?

3. A 10-watt lamp is turned on for a period of 6 hr. How many electrons pass through in that time? (The source of current is a 110 volt line.)

4. In Millikan's oil-drop experiment, the charge on the oil drop is equal to the charge on three electrons. The oil drop is in an electric field arising from

a difference of potential of 5,000 volts between two plates which are at a distance of 0.75 cm. from each other. Find the force acting on the oil drop.

5. A current of 0.0001 amp. flows through a vacuum tube. How long must the current flow in order that the electrons which pass through the tube may have a mass 0.01 g.?

6. An electron is moving through a uniform magnetic field with one-eighth the speed of light. If the intensity of the magnetic field is 300 oersteds, what force, in dynes, acts on the electron?

CHAPTER XLVII

DISCHARGE OF ELECTRONS FROM HOT WIRES AND APPLICATIONS

562. Discharge from Hot Wires.—If a platinum wire *DE* (Fig. 531) is placed inside of a metal cylinder *C* which is inserted in a very perfectly evacuated vessel, it is found that when the wire is heated to incandescence, there is an evaporation of electrons from its surface. If now the cylinder *C* is connected to the earth through a galvanometer *G* and a battery *B*, there will be a flow of electrons from the wire to the cylinder, through the galvanometer and then to the earth. When the wire is at room temperature, this flow of electrons does not take place. As the temperature of the wire is increased, the number of electrons escaping from the surface of the wire increases rapidly, and the current through the galvanometer *G* also increases. The relation between the current through the galvanometer and the temperature is shown in Fig. 532. Since the current in the galvanometer is proportional to the number of electrons evaporating each second, this curve also shows how the rate of evaporation of electrons increases with the temperature.

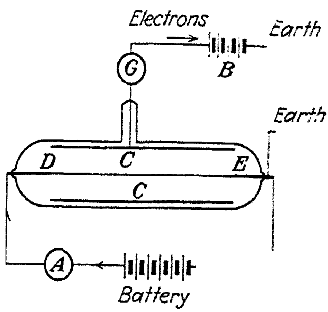


FIG. 531.—Discharge from hot wire.

This escape of electrons from the surface of a hot wire is quite analogous to the evaporation of the molecules from the surface of a liquid. The curve in Fig. 532 showing how the number of electrons increase with rise of temperature is similar to the vapor-pressure curve of a liquid (Fig. 281). There is, however, this important difference. In the evaporation of a liquid the molecules are uncharged, but in the discharge of electricity from hot wires each escaping electron is the elementary charge of negative electricity. This discharge of electricity from hot wires has found many very important practical applications.

563. Thoriated-tungsten Filament.—If a filament is made of tungsten in which there is dissolved about 0.5 per cent of thorium oxide, the electron emission from the wire is greatly increased. In this case, the emission seems to be from a very thin layer of thorium on the surface of the wire. The tungsten wire serves to heat the thorium oxide. During this heating, the thorium oxide is reduced to metallic thorium, and this metallic thorium diffuses to the surface of the filament. The emission increases with the time as more thorium appears on the surface. After a monomolecular layer of thorium has been formed on the surface of the wire, the emission of electrons reaches a steady value beyond which it does not increase. For a temperature below a certain limit, the metallic thorium remains on the surface of the wire. If the temperature of the wire is raised above this value, the atoms of thorium evaporate, leaving an exposed surface of tungsten. Other atoms of thorium diffuse to the surface thus maintaining a monomolecular layer of thorium on the surface. If the layer of metallic thorium is in some way driven from the surface of the wire, the emission drops to the electron emission for tungsten. The metallic thorium may be removed from the surface of the wire by operating the filament at too high a temperature. The filament has thus been deactivated.

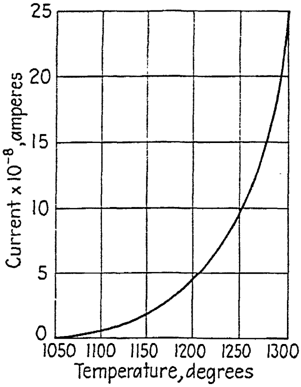


FIG. 532.—Increase of discharge of electrons with the temperature.

564. Distribution of Electrons at the Surface of a Hot Wire.—In Fig. 533 is given a rough diagrammatic representation of the way in which the electrons may be distributed near the surface of a hot wire. As the temperature of the wire is increased, the concentration of the electrons in the neighborhood of the wire also increases. At low temperatures, few electrons leave the surface and these have such low velocities that some of them are pulled back into the wire before they go far from it. If the temperature of the wire is increased, the mean velocity of the electrons is also increased so that more

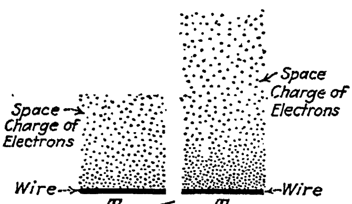


FIG. 533.—Space charge of electrons near a wire at different temperatures.

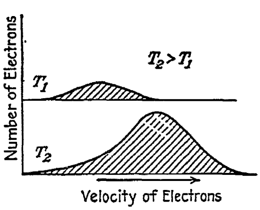


FIG. 534.—Relation between number of electrons and velocity at different temperatures.

of them move farther away from the wire before they are pulled back into it by the electrostatic forces in the neighborhood of the wire. The way in which the number of electrons and the distribution of velocities among the different electrons change with the temperature is represented diagrammatically in Fig. 534. The areas under the curves are proportional to the total number of electrons emitted by the wire at that temperature. The form of the curve indicates the way in which the number of electrons with a given velocity changes with the temperature. The higher the temperature, the greater the number of electrons with a given velocity.

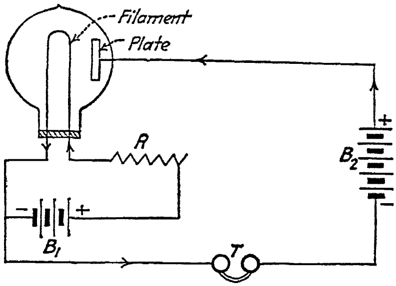


FIG. 535.—Thermionic valve.

565. The Thermionic Valve.—The thermionic valve (Figs. 535 to 537) consists of a bulb from which air or other gases have been completely exhausted. On the inside of the bulb is a tungsten filament much like the tungsten filament in a small electric light bulb. This filament is heated to incandescence by means of an external battery B_1 as an electric light filament is heated. As the filament is heated, electrons escape from it and fly out into the surrounding space. In the bulb there is mounted a plate which is connected to a wire passing through the side of the bulb. In some cases this plate takes the form of a cylinder surrounding the filament. There is no electrical connection between the plate and the filament inside of the bulb. If the filament is now connected to the negative terminal of a battery and the plate to the positive terminal, the electrons will be repelled from the filament and attracted to the plate. There will thus be a current of electrons from the filament to the plate through the circuit containing the battery B_2. If now the terminals of the battery B_2 are reversed so that the negative terminal is connected to the plate

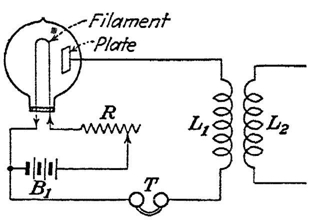


FIG. 536.—Rectification of signals by a thermionic valve.

and the positive terminal to the cell, the electrons will remain in the filament and no current will pass from the plate to the filament. The bulb has, therefore, the power to let a current pass in one direction through it but prevents the current from flowing in the opposite direction. The bulb when placed in an

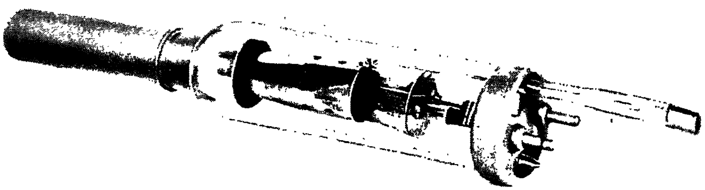


FIG. 537.—Thermionic tube with a large current-carrying capacity. (*Courtesy Bell Laboratories.*)

electric circuit behaves like an electric valve, permitting the current to flow in only one direction. Thus, when the coil L_2 induces alternating electromotive forces in the coil L_1 (Fig. 536), current in the coil L_1 flows in only one direction. The electronic current can pass only from the filament to the plate. This is equivalent to a positive current on the basis of the ordinary conventions with respect to the direction of the current flowing from the plate to the filament. The ability of such a bulb to rectify a current has important applications.

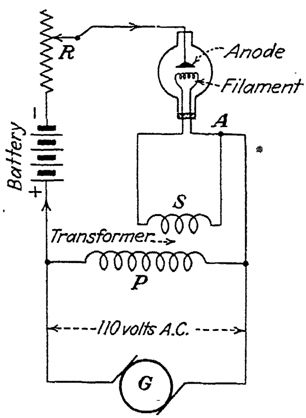


FIG. 538.—Diagram of tungar rectifier.

566. Tungar Rectifier.—In the tungar rectifier (Fig. 538), the generator G supplying the voltage to be rectified is connected to the terminals of the primary P of a transformer. The secondary terminals S of the transformer are connected to the filament of the vacuum tube. The voltage is transformed until it is sufficiently low to be suitable for heating the filament. If it is desired to charge a battery from the alternating-current generator, the positive terminal of the battery is connected to the primary of the transformer and the negative terminal through a regulating resistance R to the anode of the vacuum tube. The other terminal of the primary of the transformer is connected to the filament at A. The electrons from the hot filament are able to pass only from the filament to the anode. Hence, the electronic current can flow only from the filament to the anode, and the tube

acts as a valve which allows the current to pass in only one direction. Hence, the electronic current in the circuit containing the battery flows through the battery in the direction opposite to that indicated by the arrows. This direction is opposite to the direction in which the current would flow if the battery were delivering current. Because of the rectifying power of the tube, the current flows in only one direction through the battery although the generator G supplies an alternating voltage. Figure 540 gives an oscillogram of the current and voltage after rectification by a tungar rectifier. The upper curve represents the voltage, and the lower curve the current. By means of such a rectifier it is possible to charge a storage battery from an alternating generator. In other words, it is possible to obtain direct current from an alternating generator.

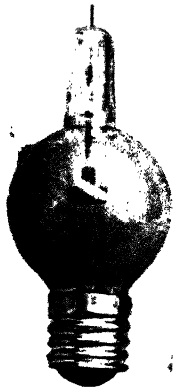


FIG. 539.—Tube of tungar rectifier.

567. The Audion.—The audion, now extensively used in receiving and sending wireless messages, differs from the thermionic valve (Fig. 541) in that a wire gauze or grid G is inserted between the plate and the filament. A wire passing through the side of the bulb is connected to this wire gauze or grid, but the grid is not in any way connected electrically to either the plate or the filament inside of the tube. The electrons emitted

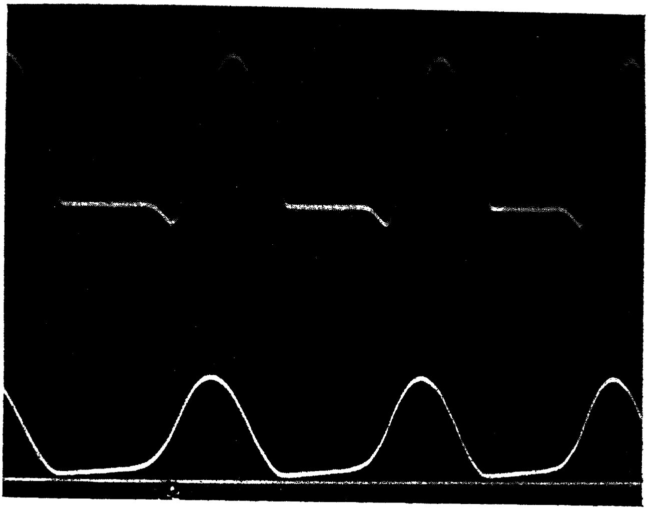


FIG. 540.—Voltage and current in a tungar rectifier. Upper curve, voltage. Lower curve, current.

by the filament will charge the gauze or grid negatively. The passage of electrons from the filament to the grid will then nearly cease because of the negative charge on the grid. This effect is increased if the grid is given a

negative potential with respect to the filament. The battery B_2 is then unable to produce a positive current from the plate to the filament.

If, on the other hand, the potential of the grid be made positive with respect to the filament, electrons pass freely toward and through the grid and reach the plate. Hence, the battery B_2 can now produce considerable current. A small negative potential of grid over filament nearly stops the current from the battery B_2, and a small positive potential of grid over filament allows a large current to be supplied by the battery B_2. Thus small changes in potential when applied to the grid are able to produce large changes in the current supplied by the battery. By this means an amplification of currents too feeble to be otherwise detected is produced.

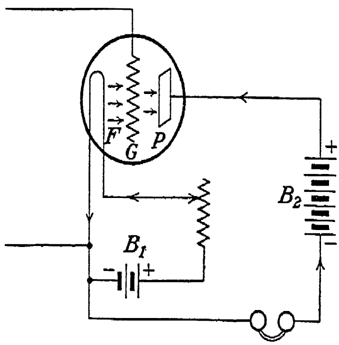


Fig. 541.—The action of the audion.

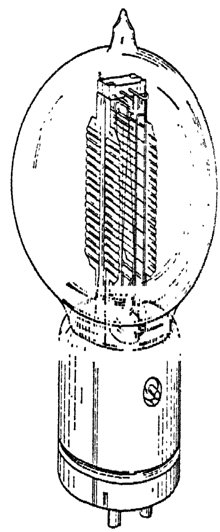


Fig. 542.—The audion. (*Courtesy Western Electric Company.*)

Such an amplifier is used in long-distance telephones. A telephone current usually varies in accordance with the speech of the user. It is an alternating current of varying frequency and wave form. When the current is made to impress a similarly varying voltage on the grid of an amplifying tube, the plate current varies in just the way the voltage impressed on the grid varies, except that the variation of the plate current is much greater than the variation of the grid current. A weak current may thus be amplified and the distance over which telephone messages can be heard much increased.

CHAPTER XLVIII

ELECTROMAGNETIC WAVES

568. Electrical Oscillations.—The oscillatory discharge of a condenser through a spark gap (Fig. 543) sets up electric oscillations which are transmitted through space as electric waves. These waves have a frequency and a wave length which are determined by the characteristics of the condenser and the inductance in series with it. In such a case, the frequency is given by the equation,

$$n = \frac{1}{2\pi\sqrt{LC}}$$

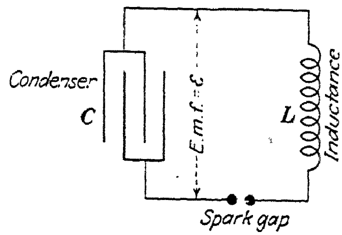


FIG. 543.—Oscillations in an electric circuit.

where n = the frequency.
L = the inductance in henrys.
C = the capacity in farads.

Example.—In a circuit containing inductance and capacity, the inductance is 0.02 millihenry, and the capacity is 0.01 mf. Find the frequency of oscillations in the circuit.

$$n = \frac{1}{2\pi\sqrt{LC}} \text{ cycles per second}$$

$$= \frac{1}{2\pi\sqrt{0.00002 \times 1 \times 10^{-8}}}$$

$$= \frac{1}{2\pi\sqrt{20 \times 10^{-14}}} \quad 3.6 \times 10^5 \text{ cycles per second.}$$

Example.—The radio antenna of a sending station has an inductance of 0.00002 henry and a capacity of 10^{-8} farad. Neglecting the effect of resistance find the natural frequency and the wave length of the electromagnetic waves.

$$\frac{1}{2\pi\sqrt{LC}}$$

$$\frac{1}{\frac{44}{7}\sqrt{2 \times 10^{-5} \times 1 \times 10^{-8}}}$$

$$= \frac{7}{44}\,\frac{1}{\sqrt{20}} \times 10^6$$

$$= 360{,}000 \text{ oscillations per second.}$$

$$\text{Wave length} = \frac{\text{velocity}}{\text{frequency}} = \frac{3 \times 10^{10}}{360{,}000} = 833 \text{ m.}$$

Figure 544 gives a more detailed picture of what happens at the spark gap. When the electric field is changing it produces a mag-

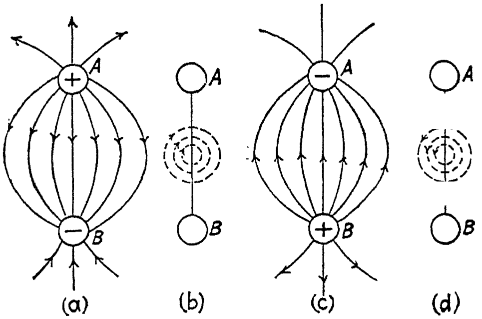


FIG. 544.—Electric oscillations.

netic field at right angles to it, and when the magnetic field is changing it also produces an electric field at right angles to itself. Thus, a varying electric force has associated with it a varying magnetic force, and a varying magnetic force has associated with it a varying electric force. These forces are always at right angles

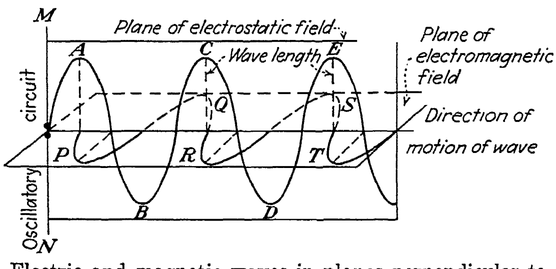


FIG. 545.—Electric and magnetic waves in planes perpendicular to each other near the oscillator.

to each other. For example, if the potential difference between the two neighboring conductors A and B (Fig. 544) becomes so large that a discharge occurs, there is set up in the space which surrounds these conductors a series of electric oscillations which are accompanied by magnetic oscillations, and these oscillations

are propagated through space with the velocity of light. At first, A is positive and B negative. Then, both are neutral, and, a little later, B is positive and A negative. The electric field thus reverses its direction. In like manner, the magnetic field which is associated with this electric field reverses its direction.

At the oscillator, the electric and magnetic fields are out of time phase by 90 deg. When one of them is a maximum, the other is zero. This relation between the electric and magnetic fields at the oscillator is shown in Fig. 545. At points far away from the antennae, the electric and magnetic fields are in time phase with each other so that at a given point in space both the electric and magnetic fields rise and fall simultaneously. This phase relation between the electric and magnetic fields at points distant from the oscillator is represented in Fig. 546. In this case, the electric and magnetic fields are in phase.

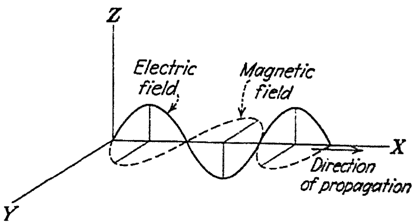


FIG. 546.—Relation between the electric and magnetic fields at some distance from the antenna.

If an antenna and a ground connection (Fig. 547) are electromagnetically coupled with a circuit in which there are electrical oscillations, induced electrical oscillations are set up in the antenna. As the current oscillates in the primary, it will alternately charge the upper end of the antenna positive and negative. These upward and downward surges in the antenna will produce magnetic lines of force which encircle the antenna and reverse their direction whenever the flow of current reverses its direction. In this way rapidly reversing electric and magnetic fields are produced. They are perpendicular to each other and travel through space with the velocity of light.

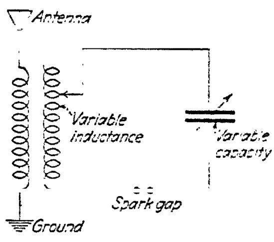


FIG. 547.—Electrical oscillations induced in a grounded antenna.

In order that the electrical oscillations in the secondary should be as large as possible, the two circuits should have the same frequency. The influence of the primary on the secondary will then be a maximum. The two circuits will then be in tune and energy will be transferred from the primary to the secondary with maximum efficiency. Since the frequency of each circuit is determined by the product of its capacity and its inductance, the two circuits will have the same frequency and be in tune when

$$n_1 = \frac{1}{2\pi\sqrt{L_1C_1}} = n_2 = \frac{1}{2\pi\sqrt{L_2C_2}}$$

In order to realize this condition, variable inductances and variable condensers may be inserted in one or both of the circuits. By varying the capacities and inductances the product of the inductance and capacity of one circuit can be made equal to the product of the inductance and capacity of the other circuit. The two circuits will then be tuned.

When such electromagnetic waves or disturbances encounter another electric circuit, they set up induced electromotive forces in it. These electromotive forces are also oscillatory but their frequencies are so high that they cannot be detected with the human ear and a telephone receiver because the human ear does not respond to sounds of frequencies greater than about 15,000 per second. Before these currents can be detected some means of rectifying them must be provided.

569. Sending and Receiving Electric Waves.—To communicate by means of electric waves it is necessary to provide: (1) a means of sending electromagnetic waves, and (2) a means of receiving such waves. In Fig. 548 one of the earlier methods of sending and receiving electric waves is represented diagrammatically. The sending system consists of a high-frequency generator G which is connected through a key K to the primary P of a transformer. The secondary S of the transformer is connected to the variable capacitance C_1. The terminals of this capacitance C_1 are also connected to the primary L_1 of the air-core transformer. One terminal of the secondary L_2 of this transformer is connected to the earth and the other to a number of parallel wires which form the **antenna.**

When the key K is closed, an alternating electromotive force is produced in the secondary S of the transformer. When by this means the condenser C_1 is charged up to a sufficiently high potential to cause a spark to jump across the gap g, there is set up a series of electric oscillations in the coil L_1. These electric oscillations set up in turn electric oscillations in the secondary L_2 of the air-core transformer and in the antenna to which it is attached.

The electromagnetic waves thus produced are radiated out into space from the antenna. Whatever their wave length, they travel with the velocity of light, *i.e.*, 186,000 miles per second.

The equipment at the receiving stations consists of another antenna connected through a variable condenser C_3 to one end of the primary L_3 of an air-core transformer. The other terminal of this primary is connected to the ground. The terminals of the secondary L_4 of this air-core transformer are connected to the terminals of the variable capacitance C_2. One of the terminals of this capacitance is then connected to the grid of the audion A and the other terminal to the filament. A battery B_1 supplies the necessary energy to heat the filament. A telephone T is inserted in series with the battery B_2 which is connected from the plate of the audion to the filament.

The electromagnetic waves radiating out from the antenna of the sending station reach the antenna of the receiving station and produce induced cur-

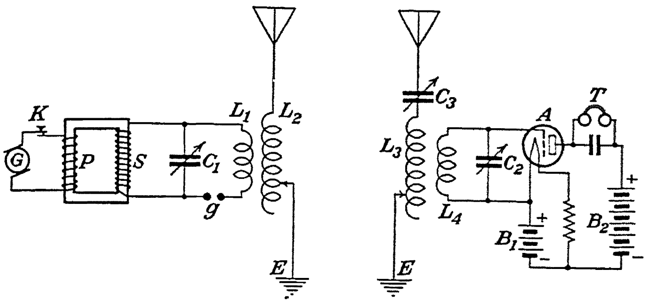


FIG. 548.—Diagram of system of wireless telegraphy.

rents in the secondary L_3 of the air-core transformer. By means of the condenser C_3, the frequency of the receiving circuit, consisting of the antenna and the coil L_3, can be made to have the same frequency as the corresponding part of the sending circuit. When the circuits are thus tuned, the receiving circuit responds much more forcibly to the waves sent out from the sending antenna. These induced currents in the coil L_3 produce electric oscillations of high frequency in the coil L_4 and thus set up variations of potential between the grid and the filament of the audion. Because of the amplifying and rectifying action of the audion, these variations of electric potential between the grid and the filament set up amplified currents in the circuit containing the telephone. In this way the electric waves sent out from the sending station are reproduced at the receiving station and after amplification produce audible sounds in the telephone.

High-frequency alternators, such as the one shown in Fig. 548, have now been replaced by *electron-tube oscillators*. In this form of oscillator an electron tube is used to produce the rapid oscillations of the electrons in a radio antenna. Ordinarily a three-element tube (Fig. 549) is used for this purpose, with the grid and the plate circuit so coupled that some of the energy of

the plate circuit is fed back to the grid circuit. A description of the operation of an electron tube connected in this way is rather complicated and beyond the scope of this book. The important point is, however, that electrical oscillations are set up in the antenna of the sending station and that these oscillations travel with the velocity of light to the receiving station where similar oscillations are set up in the antenna of the receiving station.

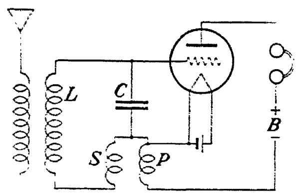


FIG. 549.—An electron-tube oscillator.

570. Radio Telephony.—When undamped waves are sent out continuously by the transmitting circuit, there is in the receiving circuit after rectification a pulsating direct current of constant amplitude. This current simply deflects the diaphragm of the telephone receiver without setting it into vibration, for the separate pulsations occur too rapidly to permit the diaphragm to return to its neutral position. If now the energy radiated by the transmitting antenna is varied, a corresponding variation takes place in the unidirectional current in the receiving circuit

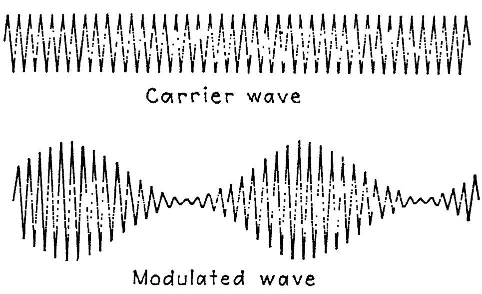


FIG. 550.—Carrier wave and modulated wave in wireless telephony.

and in the telephone. If these variations of current in the transmitting antenna are of audible frequency, a sound having the same frequency and characteristics will be produced in the telephone receivers of the receiving circuit. Thus, under suitable conditions, sounds or speech which have produced variations in the current of the transmitting circuit can be reproduced in the receiving circuit. Figure 550 represents the high-frequency carrier wave, unmodulated and modulated with audio-frequency.

Figure 551 represents a radiating set in which high-frequency oscillations are produced and maintained by a vacuum tube. Closing the plate circuit causes a current to start in the coil C. The currents in C induce currents in A which starts to vibrate with its natural frequency. In like manner the currents in A induce currents in B and these cause changes in the grid potential which again produce similar changes in the plate current. There is thus produced a set of actions and reactions in these coupled circuits which continue until high-frequency oscillations are built up to a point where the action of the tube becomes stable. When this condition is reached the energy of the high-frequency oscillations in the antenna is radiated into space.

If now sound waves are introduced into the transmitter, the resistance of the microphone will vary due to the varying pressure of the diaphragm. Low-frequency variations in the grid potential will be produced and cor-

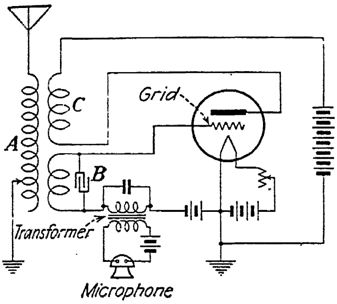


FIG. 551.—Vacuum-tube oscillator used to produce undamped electric oscillations.

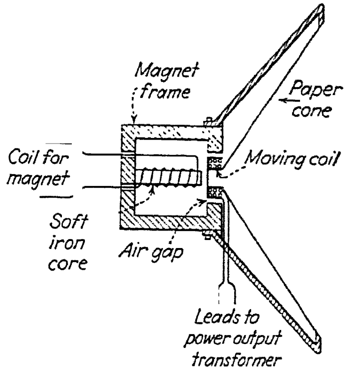


FIG. 552.—Dynamic loudspeaker.

responding variations in the plate current. Similar fluctuations are caused in the currents flowing in the antenna. The high-frequency waves radiated by the antenna undergo changes in amplitude. A modulated high-frequency wave is produced and radiated into space.

571. Reflection of Electric Waves by Kennelly-Heaviside Layer.—In order to explain why electric waves follow the curvature of the earth rather than go off into free space, it was suggested by Kennelly and also by Heaviside that there is a layer of ionized air in the upper atmosphere which acts as a reflector of these waves and sends them back toward the earth. Ions might be produced in this outer layer in a variety of ways, and the height of this ionized layer probably changes from time to time so that the efficiency of transmission of electric waves along the surface of the earth changes with the conditions of the air in the upper atmosphere. It has, for instance, been shown that there is a definite relation between sun-spot activity and the intensity of radio signals. This relation is explained by assuming that the presence of many sun spots on the sun causes the height of the Kennelly-

Heaviside layer to change and its reflecting power to be altered. The Kennelly-Heaviside layer is lower in the daytime than at night time.

The way in which the Kennelly-Heaviside layer is supposed to act is indicated in Fig. 553, where A represents a sending station. After reflection from this layer, the electric waves return to the earth and are really confined to an annular space included between the surface of the earth and the

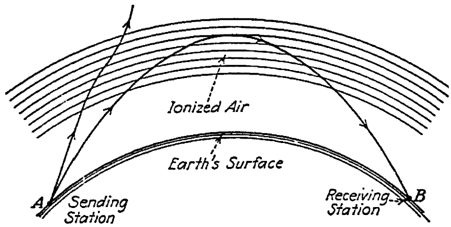


FIG. 553.—Reflection of electric waves at Heaviside layer.

Kennelly-Heaviside layer. This layer of ionized air probably does not reflect like a perfect mirror but turns the electric waves back by total internal reflection, as is the case where radiation is traveling from a medium of greater optical density to one of less optical density, with a medium of varying optical density in between these two media.

PART VI.—PHYSICAL AND GEOMETRICAL OPTICS

CHAPTER XLIX

NATURE AND PROPAGATION OF LIGHT

572. Nature of Light.—A hot body gives off radiations of different wave lengths. Some of these radiations are called heat waves. Others are known as light waves. These two kinds of waves are identical except with respect to their wave lengths. The heat waves have long wave lengths in comparison with the light waves. Both types of waves obey the same laws of reflection, refraction, interference, etc. Radiation with wave length longer than the wave length of red light is called infra-red radiation, and that with a wave length shorter than the wave length of the violet is called ultra-violet. Ultra-violet radiation produces chemical effects like those which are found when a photographic plate has been exposed to the light.

The theory that light consists of a wave motion in a hypothetical medium known as the ether is now ordinarily accepted as it is useful in explaining many of the observed phenomena of light. If an ordinary incandescent lamp bulb is evacuated of air and other gases, the incandescent filament illuminates its surroundings in spite of the fact that the light from the filament travels through a vacuum. The light from the sun could not reach the earth if a material medium were necessary for its transmission. That light is a transverse wave motion will be more evident from a discussion of the facts of polarization which is given in a subsequent chapter.

573. Velocity of Light.—The velocity of light, which is 186,000 miles per second, is so great that in 1 sec. it would travel more than seven times around the earth at the equator. Light travels from the sun to the earth in a little over 8 min., but it requires 4 years for light to travel from the nearest star to the earth. If the North Star were obliterated, the earth would continue to receive light from it for about 44 years. Several methods have

been devised for determining the velocity of light. It is sufficient for our purposes to describe two of them.

The method of Foucault, which has been modified and improved by Michelson, has given very accurate results for the velocity of light. An understanding of this method can be obtained by examining Fig. 554. A source of light, S, sends a beam of light to a mirror AB which can be rotated about an axis through its center O perpendicular to the plane of the diagram. The mirror AB reflects the light through the lens LN to the plane mirror M, which sends it back to the mirror AB and, thence, to S where an image is formed. When the mirror AB is not rotating, this image coincides with the source of light, S. When, however, the mirror AB rotates through an angle x while the light travels from the mirror AB to the mirror M and back again to the mirror AB, the beam after reflection at the mirror AB will not return to S but to some other point, S'. As the mirror AB turns through an angle x, its normal OP also turns through this same angle. The angle between the ray incident on the mirror AB after returning from the lens LN is thus decreased by x, and, since the angle of incidence is always equal to the angle of reflection, the angle of reflection will also be decreased by x. The ray after reflection at the mirror will, therefore, have been deflected through an angle of $2x$ from its position when the mirror AB was not in rotation.

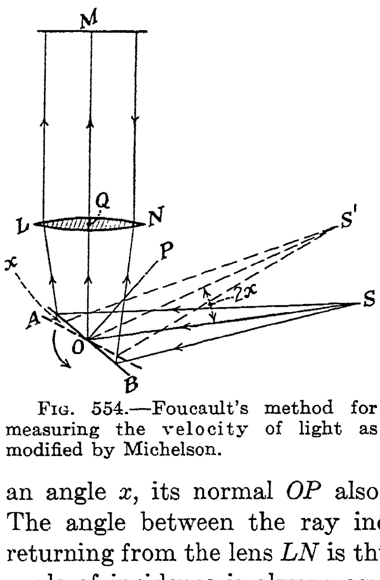


FIG. 554.—Foucault's method for measuring the velocity of light as modified by Michelson.

By measuring the distance SS' and the distance d from the mirror AB to S, the angle $2x$ can be found, for $2x = SS'/d$ radians. If the number of revolutions which is made per second by the mirror AB is observed and denoted by n, then $x = 2\pi nt$, where t is the time for the light to pass from the mirror AB to the mirror M and back again to the mirror AB. If V denotes the velocity of light and L the distance from the mirror AB to the mirror M,

$$2L = Vt.$$

$$t = \frac{x}{2\pi n}.$$

$$2L = V\frac{x}{2\pi n}.$$

$$V = \frac{4\pi nL}{}.$$

Example.—In determining the velocity of light by Foucault's method it was found that the rotating mirror made 100 revolutions per second. The distance from the fixed to the rotating mirror was 603 m. and the angle through which the mirror rotated was 0.0025 radian. Find the velocity of light.

$$V = \frac{4\pi nL}{x}$$
$$= \frac{4 \times 3\tfrac{1}{7} \times 100 \times 60{,}300}{0.0025}$$
$$= 3.02 \times 10^{10} \text{ cm. per second.}$$

574. Michelson's Latest Method.—The latest method used by Michelson for measuring the velocity of light is indicated in Fig. 555. A distance of

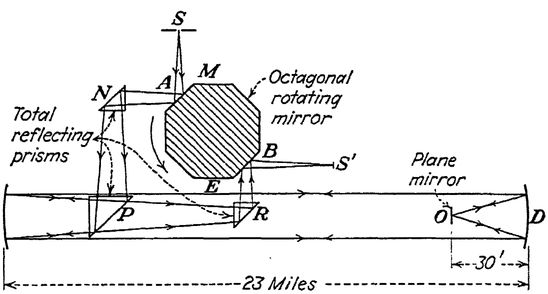


FIG. 555.—Michelson's latest method for measuring the velocity of light.

about 23 miles was surveyed with extraordinary care and skill from Mount Wilson to Mount San Antonio in California. In one case the distance used was 35,426.3 m., and this distance was accurate to within 0.1 m. The revolving mirror M of steel or quartz could be rotated at a very high speed and this speed could be kept extremely constant. This revolving mirror was in the form of a rectangular octagon, each side of which was polished, thus forming eight mirrors equally inclined to each other. Light from a brilliantly illuminated source S was reflected from the side A of the revolving mirror and, after reflection at the two totally reflecting prisms N and P, was received by the large concave mirror C and reflected from this mirror as a beam of parallel rays. To guarantee that these reflected rays be parallel, it was only necessary to place the source S at the principal focus of the mirror C. After the light had been reflected from the mirror C, it traveled

23 miles to a similar mirror D on Mount San Antonio, by which it was brought to a focus on a small mirror at the principal focus O of the large mirror D. From this small mirror, the light was reflected back to the mirror D and then back to the mirror C on Mount Wilson. From the mirror C, the light was again reflected to the totally reflecting prism R by which it was reflected to the face B of the revolving mirror M and then brought to a focus at S' where the image was observed by means of a telescope provided with cross wires suitable for measuring the position of the image in the eyepiece of the telescope.

Imagine that, at the outset, the mirror M is not in rotation about its axis. Light leaving the source S after reflection at the face A is reflected over and back between the mirrors C and D, and finally, after reflection at the face B, is received in the telescope and made to coincide with the cross wires of the telescope. Now imagine that the mirror M is set in rotation at a high speed: While the light reflected from the face A of the revolving mirror M travels from the face A to the mirror C and then to D, and back to C, and then to B, the mirror M rotates so that the returning light does not fall on the face B, as it did when the mirror M was stationary but on the face E. If now the speed of the rotating mirror M is such that the mirror turns exactly one-eighth of a revolution between the time of the reflection of the light at the surface A and its reflection by E, the light is reflected from the face E in exactly the same way it was formerly reflected from the face B. Hence, the image of the light again falls on the cross wires of the telescope. In other words, the speed of rotation is so adjusted that the face E just replaces the face B while the light is traveling twice between the mirror C and the mirror D. When this condition is realized, the time the light is traveling between its reflection at face A and at face E is known, for it is just equal to the time necessary for the mirror M to make one-eighth of a complete revolution. Now knowing both the distance the light has traveled and the time to travel that distance, it is possible to get the velocity of light accurately from the simple equation

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}.$$

575. Frequency and Wave Length.—The relation between frequency, velocity, and wave length is the same for light waves as it is for sound waves. Hence,

$$v = n\lambda$$

where v is the velocity, n the frequency, and λ the wave length. Waves of yellow light have been found to have a wave length equal to about 0.000059 cm. Taking the velocity of light to be 3×10^{10} cm. per second, the frequency of yellow light is

$$n = \frac{v}{\lambda} = \frac{3 \times 10^{10}}{0.59 \times 10^{-4}} = 5.01 \times 10^{14} \text{ per second.}$$

The wave length of light is often expressed in Angstrom units. **One Angstrom unit = 10^{-8} cm.**

576. Sources of Light.—The sun is the chief source of light and heat, but there are many artificial sources. Any body when heated to a sufficiently high temperature becomes a source of light.

As the temperature of a body is raised, the body emits invisible radiation. When it becomes red-hot, visible radiations begin to be emitted. The higher the temperature, the greater is the amount of both heat and light waves which are emitted, but the percentage of visible radiations becomes larger and larger as the temperature of the source of radiations is increased. For this reason, the modern tungsten lamp is much more efficient than the old carbon incandescent lamp. (Fig. 556.) Tungsten has a very high melting point, and when it is surrounded by nitrogen or when it is in a vacuum, it can be heated to a high temperature and its efficiency thus made large.

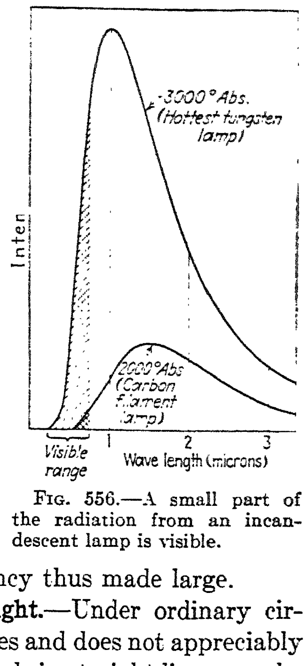


FIG. 556.—A small part of the radiation from an incandescent lamp is visible.

577. Rectilinear Propagation of Light.—Under ordinary circumstances light travels in straight lines and does not appreciably bend around objects. That light travels in straight lines may be shown by placing a candle or other source of light behind a screen having in it a small hole (Fig. 557). In front of this screen

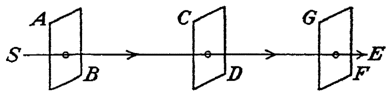


FIG. 557.—Rectilinear propagation of light.

AB are placed two screens *CD* and *GF*, each with a small hole at the center. When these screens are so adjusted that the eye *E* can see the source of light *S* distinctly, it will be found that the straight line joining *S* and *E* passes through the holes in the screens. This shows that light from *S* to *E* comes in a straight line.

The rectilinear propagation of light is also shown by the fact that a small source of light will cast a sharp shadow of an object. Thus a small source of light S (Fig. 558) will illuminate all points on a screen above and below A and B. No light arrives at the screen between A and B because it is stopped by the opaque body M. The fact that the boundary of the shadow is sharp shows

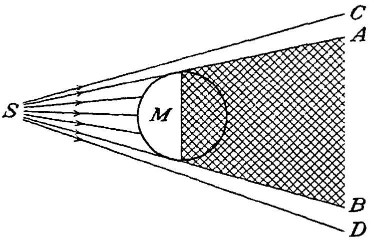


FIG. 558.—Shadows by opaque objects. Light from a point source gives sharp shadows.

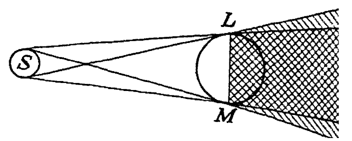


FIG. 559.—Shadows from finite sources. Light from finite sources gives shadows with no sharp edges.

that light from S does not bend appreciably into the shadow of the opaque body.

When the source of light is not small, the boundary of the shadow will not be sharp (Fig. 559). Points on the screen above A and below D will be fully illuminated by S. The part of the screen between B and C will receive no light and will, therefore, be a complete shadow. The screen between A and B and between C and D will be partly illuminated and will be a less dense shadow than the part between B and C. This outer shadow will gradually shade off from complete shadow at B to complete illumination at A. This gradual shading off gives the shadow that blurred appearance which characterizes most shadows.

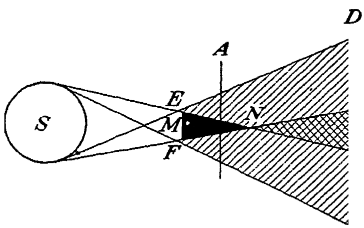


FIG. 560.—Shadows by objects smaller than the source of light.

When the source of illumination is larger than the object casting the shadow (Fig. 560), the only region of complete shadow is the cone FEN having the object as a base. If the screen is placed near the object, there will be cast on it a small complete central shadow surrounded by a large partial shadow. When the screen

is far from the object, there will be only a diffuse shadow with no complete shadow at the center.

578. Total, Annular, and Partial Eclipses.—The best illustration of shadows on a large scale is found in the eclipses of the sun or moon. When the moon happens to be interposed between the sun and the earth in such a way that its shadow falls wholly or partly on the earth, the sun is wholly or partly obscured by the moon and there is a total or partial eclipse of the sun. If the earth is sufficiently near to the moon and passes through the complete shadow represented by the cone *FEN* (Fig. 560), the eclipse is total. If, however, the earth is farther away from the moon and passes through only the partial shadow, the eclipse is said to be annular.

When the earth, moon, and sun are not in exactly the same straight line, there may be a partial eclipse. Sometimes the moon, when it is full, passes through the shadow of the earth. There is then an eclipse of the moon.

Problems

1. The nearest star is at a distance of 4 light years. Express this distance in miles. (A light year is the distance traveled by light in free space in 1 year.)

2. What frequency of light will result in a wave length of 0.00006 cm. in air?

3. Signals with a frequency of 600 kc. are sent out by a radio station. Find the wave length, assuming that the waves travel with the speed of light. (One thousand vibrations per second = 1 kilocycle.)

4. What minimum speed of rotation is necessary for an 8-sided mirror used by Michelson in measuring the speed of light, with a total path between reflections of 44 miles?

5. Light of a certain wave length has 12,000 light waves to the centimeter in air. Find the number of waves per centimeter when the light is traveling in water if speed of light in water is three-fourths as great as it is in air.

6. A beam of light of wave length 0.000059 cm. travels from one submarine to another under water a distance of 400 m. Find the time which elapses and the number of waves between the submarines? Velocity of light in water = 2.25×10^{10} cm. per sec.

7. In measuring the velocity of light a rotating mirror having 12 sides was used. The rotating mirror was 45 km. away from the stationary mirror. What speed of rotation must be imparted to the rotating mirror?

8. An astronomical unit of distance is the light year by which is meant the distance light travels in a year. Compute this distance in miles.

CHAPTER L

ILLUMINATION AND PHOTOMETRY

579. Standards of Illumination.—The intensity of illumination is measured by the amount of light which falls on unit area of a surface. The amount of energy received by unit surface can not be easily determined in absolute measure. The eye is the most sensitive means of detecting light, but it does not give a quantitative measure of it. By means of the eye it is, however, possible to make an accurate comparison of two intensities of illumination.

To compare sources of illumination, a standard source of illumination is necessary. The choice of these standards is more or less arbitrary. There are a number of such standards in use. The **British standard candle** is defined to be a candle made of spermaceti, weighing six to the pound and burning at the rate of 120 grains per hr. This standard does not have a sufficiently constant illuminating power to make it of scientific value. Its illuminating power changes with atmospheric conditions and with the conditions under which it is burned. For this reason it has been replaced by the **Harcourt pentane lamp** in which air is drawn over pentane and the mixture burned in a standard burner in which the flame is adjusted to a definite height. Corrections must be made for atmospheric conditions. The lamp has an illuminating power which is equal to that of ten standard candles. **From it an international candle is defined to be a light with an illuminating power equal to one-tenth of that of a Harcourt pentane lamp.**

An electric incandescent lamp when operated at a definite voltage is the most convenient standard of illumination. Such standards must be accompanied by a certificate giving their candlepower when they are operated under definite conditions.

580. Candlepower.—The candlepower of a lamp is a specification of its illuminating power in terms of some standard candle. For example, the illuminating power of an incandescent lamp

may be thirty times that of a standard candle and is then said to be a 30-cp. lamp. But the illuminating power of a light varies according to the direction from which it is observed. Such a variation of the intensity of the light in different directions is shown in Fig. 561 where the light from an incandescent gas mantle has been reflected by a glass shade It becomes, there-

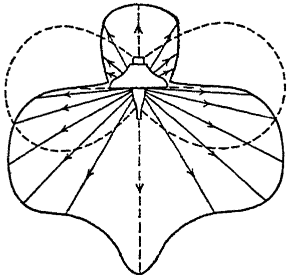


FIG. 561.—Distribution of light from a gas mantle.

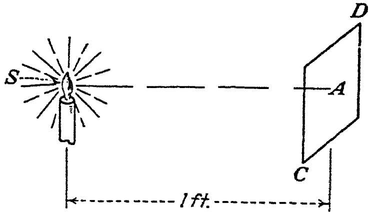


FIG. 562.—Illumination at A is 1 foot-candle.

fore, necessary to measure the average candlepower in a given plane. The average illuminating power in the horizontal plane is called the **mean horizontal candlepower.** The mean spherical candlepower denotes the average illumination by a source of light from all directions in space. Figure 561 shows how the illumination from the gas mantle varies in a vertical plane with and without the glass shade. The dotted curves show the distribution without the shade.

581. The Foot-candle.—A foot-candle is the intensity of illumination upon a surface at a point which is 1 ft. distant from a source of 1 candle, the surface being perpendicular to the light rays at that point. In Fig. 562, if the source S has 1 cp. and is 1 ft. from the screen DAC, the intensity of illumination on the screen at the point A is 1 foot-candle. A gray surface receiving light of an intensity equal to 1 foot-candle will not appear as bright as a white one under the same illumination, for more light is reflected back to the eye from the white surface.

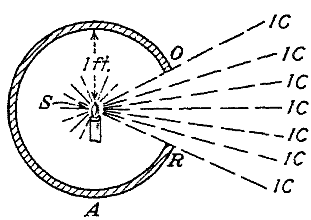


FIG. 563.—Opening OR has an area of 1 square foot and emits 1 lumen.

582. The Lumen.—The unit used to denote quantity or amount of light is the *lumen.* A lumen is the amount of light falling on a surface which has

an area of 1 sq. ft. when every point of the surface is 1 ft. from a point source of light of 1 candle. If in Fig. 563 the area OR is 1 sq. ft., the light escaping through it is 1 lumen, provided the distance of the surface from the source is 1 ft. and the illuminating power of the source is 1 candle.

583. Law of Inverse Squares.—Consider the light waves proceeding from a small source L (Fig. 564). As the spherical wave advances, the energy in the surface of the wave is distributed over a larger and larger area. Assuming that the medium through which the wave travels does not absorb any of the energy, the total energy distributed over any of these concentric spherical surfaces is the same.

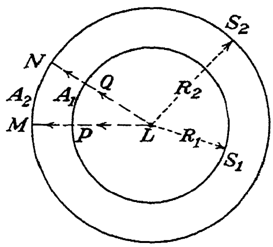


FIG. 564.—Law of inverse squares.

Let Q be the rate at which energy is sent out by the source.

Let R_1 and R_2 be the radii of two concentric spheres surrounding the source. The area of the smaller sphere is $4\pi R_1^2$ and that of the larger one is $4\pi R_2^2$. The amount of energy per unit area of the smaller sphere is

$$I_1 = \frac{Q}{4\pi R_1^2},$$

and the amount per unit area of the larger sphere is

$$I_2 = \frac{Q}{4\pi R_2^2}.$$

Hence,

$$\frac{I_1}{I_2} = \frac{R_2^2}{R_1^2}.$$

Therefore, **the intensity of illumination from a point source of light varies inversely as the square of the distance from the source.**

Example.—A small spherical lamp sends light in all directions. A screen 1 cm. square is placed 200 cm. from it, and a second screen of the same size is placed 400 cm. from it. Find the ratio of the energies received by the two screens.

$$\frac{I_1}{I_2} = \frac{R_2^2}{R_1^2} = \frac{(400)^2}{(200)^2} = \frac{16}{4} = \frac{4}{1} = 4.$$

Example.—The distance from the earth to the sun is 93,000,000 miles. On the average, each square centimeter of the earth's surface intercepts 1.93 cal. per minute. What is the total rate at which the sun radiates heat?

$$\text{Total energy radiated per minute} = 1.93 \times 4\pi R^2 = 45.6 \times 10^{26},$$

where

$$R = 1.37 \times 10^{13} \text{ cm.}$$

584. Photometer.—A photometer is a device for comparing the illuminating power of two sources of light. To make this comparison, the distances of the sources from a screen are adjusted until they produce the same intensity of illumination on the screen. Thus, if C_1 is the illuminating power of one source, C_2 that of the other source, and D_1 and D_2 their respective distances from the screen when they produce equal illumination on it, then

$$\frac{C_1}{D_1^2} = \frac{C_2}{D_2^2},$$

or

$$\frac{C_1}{C_2} = \frac{D_1^2}{D_2^2}.$$

By observing the distances D_1 and D_2 the candlepower of either source can be determined if that of the other is known.

There are many devices for aiding the eye in determining when the intensities of illumination from the two sources are the same. Usually, provision is made for varying the distance of one or the other of the sources until the eye can detect no difference in the intensities of the lights on the screen.

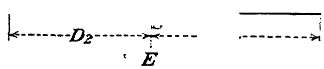


FIG. 565.—Bunsen photometer for comparing the intensities of lights.

585. Bunsen Grease-spot Photometer.—If a spot of grease or oil is made on a piece of plain paper, the spot is more translucent than the rest of the paper. When the paper is held between the observer and the light, the grease spot will appear brighter than the rest of the paper because light passes through it more readily than it does through the remainder of the paper. If this same paper is viewed by reflected light, the grease spot appears darker than the remainder of the paper because the grease spot transmits the greater part of the light falling on it, while the paper reflects most of the light falling on it. This difference in the behavior of the spot and the paper gives a means of comparing the intensities of two lights.

A screen S consisting of a piece of cardboard having a hole in it covered with oiled paper is placed between two lights L_1 and L_2 (Fig. 565). The screen is adjusted until the appearance of both sides is the same. It is then considered that both sources produce the same intensity of illumination on the screen. To make it possible to see both sides of the screen at the same

time two mirrors M and N are placed on either side of the screen, and the images of the spot in these mirrors are observed. By measuring the distances from the lights to the screen, the relative candlepower of the sources can be determined.

Let C_1 = the candlepower of L_1.
C_2 = the candlepower of L_2.
D_1 = the distance from the screen to L_1.
D_2 = the distance from the screen to L_2.
C_1/D_1^2 = the intensity of the light from L_1 on the screen.
C_2/D_2^2 = the intensity of the light from L_2 on the screen.

Since these two intensities are the same,

$$\frac{C_1}{D_1^2} \quad \frac{C_2}{D_2^2}$$

and

$$\frac{C_1}{C_2} = \frac{D_1^2}{D_2^2}.$$

Example.—In comparing the intensities of two sources by means of a Bunsen grease-spot photometer, it was found that when one light was 40 cm. from the screen, the other must be 60 cm. from it to produce the same intensity of illumination on the screen. If the weaker light has a candlepower of 16, what is the candlepower of the more intense light?

$$\frac{\text{Candlepower of } L_1}{\text{Candlepower of } L_2} = \frac{\text{square of distance of } L_1 \text{ from the screen}}{\text{square of distance of } L_2 \text{ from the screen}}.$$

$$\frac{C_1}{C_2} = \frac{D_1^2}{D_2^2} = \frac{(60)^2}{(40)^2} = \frac{36}{16}.$$

$$\frac{C_1}{16} = \frac{36}{16}.$$

Hence,

$$C_1 = 36 \text{ cp.}$$

586. Lummer-Brodhun Photometer.—Because the adjustments with a Bunsen photometer cannot be made with great accuracy, a Lummer-Brodhun photometer is frequently used when greater accuracy is required. In this photometer (Fig. 566), light from two sources L_1 and L_2 is received on the two sides M and N of a milk-white screen. Light from L_1, after irregular reflection from the surface M of the screen, is received by the prism B where it is totally reflected and passes through the prisms D and C where they are in intimate contact at the middle. On the other hand, the light from L_2, after irregular reflection at the surface N, is totally reflected by the prism A and then enters the prism C. That part of the light from L_2 which falls on that

part of the rear surface of the prism C which is in contact with the prism D passes on through the prism D. But that light from L_2 which strikes that part of the rear surface of the prism C which is not in contact with the prism D is totally reflected and enters the telescope T. On looking into the telescope, it is found that the central part of the field of view is illuminated by light from L_1 after reflection by the prism B. The outer portion of the field of view is illuminated by light which came from L_2 after reflection at A and the rear surface of the prism C.

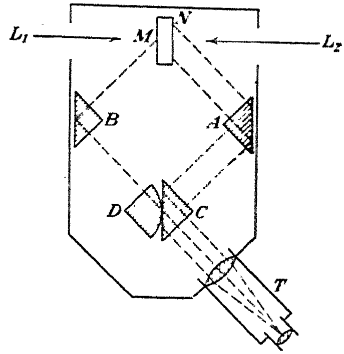


FIG. 566.—Lummer-Brodhun photometer. Light from both sources can be seen simultaneously.

By moving the photometer box with respect to the sources and observing the two parts of the field of view in the telescope, the intensities of the two parts of the field of view can be made the same. When the two parts of the field of view are equally illuminated, an accurate measure of the candlepower of one source in terms of the other can be obtained by measuring the distances of the sources from the screen MN. The relative candlepowers are calculated as in the Bunsen photometer, from the equation

$$\frac{C_1}{C_2} = \frac{D_1^2}{D_2^2}.$$

Problems

1. An arc lamp, at a distance of 60 ft. from a screen, produces the same illumination as a 25-cp. lamp at a distance of 5 ft. from the screen. What is the candlepower of the arc lamp?

2. A screen is illuminated by a 25-cp. lamp at a distance of 4 m. At what distance will the same illumination be produced by a 40-cp. lamp?

3. Two lamps of 50 and 40 cp. are placed 150 cm. apart. At what position between them will the illumination on both sides of an interposed screen be equal?

4. A lamp of unknown candlepower is placed at a distance of 300 cm. from a 32-cp. standard. The setting of a Lummer-Brodhun photometer head is found by experiment to be 180 cm. from the standard lamp. What is the candlepower of the unknown?

5. A small screen at a distance of 40 ft. from a 100-cp. source has its surface making an angle of 60 deg. with the line drawn from the source. What is the intensity of illumination?

6. A screen is placed 90 cm. from a lamp. When a sheet of smoked glass is placed between the screen and the lamp, the lamp must be moved so that it is at a distance of 60 cm. from the screen in order to produce the same intensity of illumination on the screen. Calculate the percentage of light transmitted by the sheet of glass.

7. What is the equivalent candlepower of the full moon if it produces the same intensity of illumination on a screen as is produced by a standard candle at a distance of 4 ft. from the screen. Assume that the distance from the moon to the earth is 240,000 miles.

8. Find the ratio of the intensity of illumination produced by the sun when it is directly overhead and when it is 45 deg. above the horizon.

CHAPTER LI

REFLECTION OF LIGHT

587. Laws of Reflection.—When a beam of light, traveling in a homogeneous medium, comes to a second medium, some of the light is reflected. At a polished or silvered surface, nearly all of the light is reflected. At the surface of clear glass, only a small part of it is reflected. The greater part of it enters the glass and passes through. In Fig. 567, let AB represent the reflecting surface, MP the perpendicular or normal to this surface, OP the incident ray, and PN the reflected ray. The angle OPM between the incident ray and the normal to the surface is called **the angle of incidence.** The angle MPN between the reflected ray and the normal to the surface is called the **angle of reflection.** Reflection at such a surface occurs according to the following two laws:

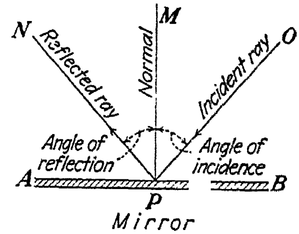


FIG. 567.—Reflection of light from a plane mirror. The angle of incidence is equal to the angle of reflection.

First Law of Reflection.—The incident ray, the reflected ray, and the normal to the surface lie in the same plane.

Second Law of Reflection.—The angle of incidence is equal to the angle of reflection.

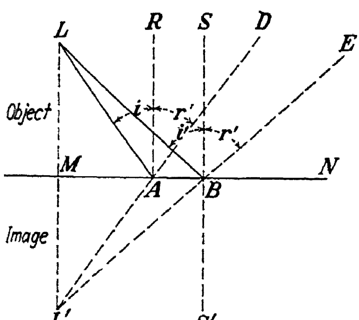


FIG. 568.—Images in a plane mirror. The image is behind the mirror and the same size as the object.

588. Image in a Plane Mirror.—When a luminous object like a small candle flame is placed in front of a plane mirror MN (Fig. 568), any point L of the source sends light in all directions. Two of these rays of light LA and LB, after reflection by the mirror, travel in the directions AD and BE respectively. To an observer in front of the mirror these reflected rays seem to

diverge from a point behind the mirror. By drawing other rays, it can be shown that they all seem, after reflection, to diverge from the same point behind the mirror. The observer will, therefore, see a bright spot behind the mirror, and for every other spot in the luminous source there will be a corresponding bright spot behind the mirror. There will thus be seen behind the mirror an image of the flame or any other luminous body which is in front of the mirror.

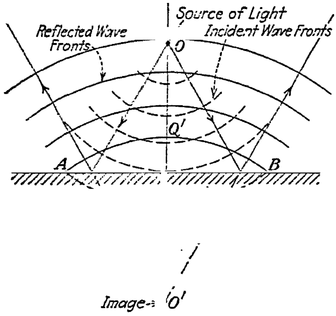


FIG. 569.—Reflection of spherical waves at a plane surface. The curvature of the wave front is reversed by reflection.

From L, drop a perpendicular to the mirror MN and produce this perpendicular until it intersects AD. The angles LAM, DAN, and MAL' are equal. The angle at M is a right angle. Hence, the triangles LAM and MAL' are similar. Since the side MA is common to both triangles, $LM = ML'$. Next draw the line BL'. The two triangles LBM and MBL' have $LM = L'M$, the side MB in common, and the angles at M right angles. Hence, these triangles are equal in all their parts. Hence, $LBM = MBL'$, $LBS = L'BS' = SBE$, and the line EBL' is a straight line. Consequently, it is possible to say that the line joining the object and the image is perpendicular to the mirror and that the image is as far behind the mirror as the object is in front of it.

Reflection from a plane surface may also be considered in terms of the wave front of the disturbance (Fig. 569). At the surface AB, the direction of propagation of the disturbance is reversed. The dotted lines indicate the incident wave fronts and the continuous lines the reflected wave fronts.

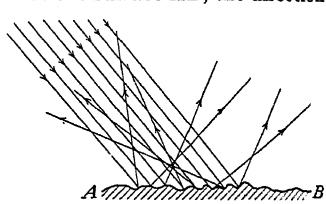


FIG. 570.—Irregular reflection. The light is scattered or diffused.

589. Reflection of Light at an Opaque Surface.—When light falls on a rough opaque surface (Fig. 570), the incident light is scattered in all directions. When the surface is so smooth that the distances between the successive elevations on the surface are less than about one-quarter wave length of light, there will be very little scattering of light and the surface is said to be polished. Thus, a surface which may be considered to be polished for light of long wave length is not polished for light of short wave length. Such a polished surface which reflects light with little

scattering is called **a mirror.** Radiation which is invisible (Fig. 571) is also reflected.

590. Selective Reflection.—The fraction of the light which is reflected depends on the wave length of the light, the angle of incidence, and the medium surrounding the body. No substance has been found which reflects light of only one wave length. Hence a substance ordinarily reflects light composed of several wavelengths. A substance which reflects completely light of all wave lengths is white when it is illuminated by white light. Substances which reflect wave lengths unequally appear colored in white light.

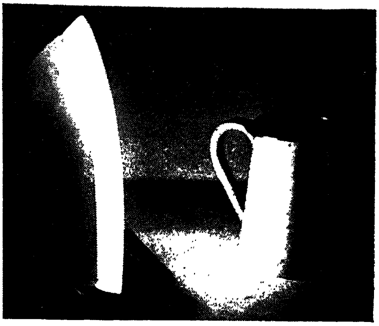


FIG. 571.—Reflection of infra-red radiations. Neither the flat iron nor the cup was visible in a dark room. (*Photographed on infra-red plate by F. W. Davis, Ohio State University.*)

591. Concave Spherical Mirrors.—A concave spherical mirror is part of a spherical shell with its inner surface polished. The center of the sphere from which the mirror was taken is called the **center of curvature** of the mirror, and the radius of the sphere is called the **radius of curvature** of the mirror. The middle point M of the mirror (Fig. 572) is known as the **vertex of the mirror,** and the straight line MN through the vertex of the mirror and its center of curvature is **the principal axis of the mirror.** From the property of the mirror it is evident that any line from C, the center of curvature, to the mirror is perpendicular to the surface of the mirror at the point to which it is drawn. Such a line is the radius of the sphere of which the mirror is a part, and the radius is perpendicular to the sphere.

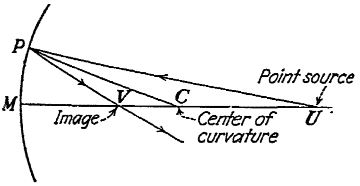


FIG. 572.—Spherical mirror. The angle of incidence is equal to the angle of reflection.

Suppose that a luminous point U is placed on the principal axis and that this point sends a number of rays to the mirror. Consider two of these rays, UM and UP. The first of these rays, after passing through the center of curvature, strikes the mirror

normally and is reflected back over its former path. The other ray UP will be reflected by the mirror in such a way that the angle of incidence is equal to the angle of reflection. Hence, the incident ray UP and the reflected ray PV will make equal angles with the radius of curvature CP. The reflected ray crosses the principal axis at some point V. The ray UM is also reflected through V. The point V is the image of U, and all the rays from U will pass through it if the aperture of the mirror is small. This relation holds for all positions of the point U.

592. Principal Focus.—If the luminous point such as U (Fig. 572) is removed very far from the mirror so that the rays reaching the mirror are nearly parallel, the light after reflection, comes to a single point F which is easily located by observing the image which the mirror makes of the sun. This point at which all the rays parallel to the principal axis meet after reflection is known as **the principal focus** (Fig. 573). It is half way between the mirror and its center of curvature.

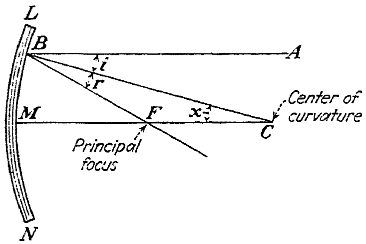


FIG. 573.—Principal focus of a concave mirror.

Let the ray AB (Fig. 573) parallel to the principal axis strike the mirror at B and be reflected in the direction BF, such that the angle of reflection r is equal to the angle of incidence i. Since AB is parallel to the axis, the angles i and x are equal and the triangle FBC is isosceles so that BF is equal to FC. If B is not too far from M, BF and MF are nearly equal. Hence, MF is nearly equal to FC, and, therefore, F is halfway between M and C.

Where a spherical mirror of large aperture is used, the rays parallel to the axis do not all meet at the principal focus. This gives rise to an imperfection known as **spherical aberration.** This imperfection is small when the concave mirror is only a small part of the sphere. This imperfection is overcome, where large mirrors are necessary, by using parabolic mirrors (Sec. 597).

593. Construction of Images in a Concave Mirror.—If in a concave mirror the center of curvature and the principal focus are given, it is not difficult to construct the image of an object AB placed in front of the mirror. There are three cases which arise: (1) The object is beyond the center of curvature; (2) the

object is between the center of curvature and the principal focus; (3) the object is between the principal focus and the mirror.

Suppose that AB (Fig. 574) is outside the center of curvature of the mirror. From A draw a ray parallel to the principal axis. After reflection, this ray will pass through the principal focus F. Now from A draw a second ray through the center of curvature. This ray will be reflected back on itself and pass again through the center of curvature. Where these two rays intersect,

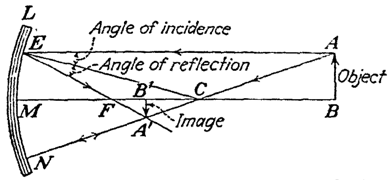


Fig. 574.—Location of images in a concave mirror. The image is real, inverted, and diminished.

there will be formed an image of the point A. The image of any other point in the arrow AB may be located in the same way so that the image of AB is found to be $A'B'$.

If the object lies between the center of curvature and the principal focus (Fig. 575), the rays are drawn as in the preceding case, but the image now lies back of the center of curvature and is magnified. Let AB be the object lying between the principal focus and the center of curvature. From A draw the

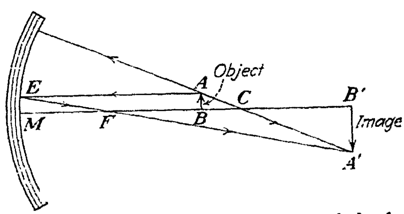


Fig. 575.—Object between the center of curvature and the focus. The image is real, inverted, and enlarged.

ray AE parallel to the principal axis. After reflection, it passes through the principal focus F. Through the center of curvature draw the ray AC which is reflected back on itself and intersects the ray reflected from E at A', forming an image of A at that point. In a similar manner the images of the other points on the arrow AB are found, giving $A'B'$ as the magnified and inverted image of AB.

When the object is inside the principal focus (Fig. 576), the ray from A parallel to the principal axis is drawn as before and as before it passes through the principal focus after reflection. The ray AEC through the center of curvature strikes the mirror normally and is reflected back through C. These reflected rays EC and DF diverge and seem to have come from a point A' behind the mirror. There is thus formed behind the mirror an

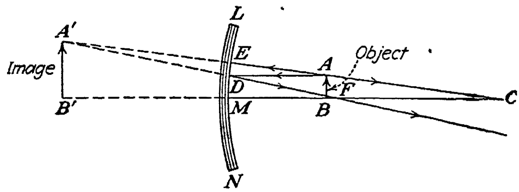


FIG. 576.—Object inside of the focus. The image is virtual, upright, and enlarged.

image of the point A at the point A' where these two rays, extended backward, intersect. Locating the images of the other points of AB in the same way, we have behind the mirror the **magnified** and **erect image** $A'B'$ of the arrow AB. In this case, the image appears to be behind the mirror. It is a **virtual image** because light only *appears* to come from it. To an eye in front of the mirror the effect is the same as if light actually came from the image $A'B'$.

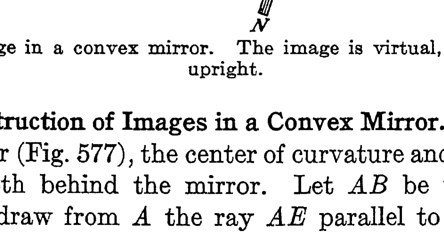


FIG. 577.—Image in a convex mirror. The image is virtual, diminished, and upright.

594. Construction of Images in a Convex Mirror.—In case of a convex mirror (Fig. 577), the center of curvature and the principal focus are both behind the mirror. Let AB be the object as before, and draw from A the ray AE parallel to the principal axis BC. After reflection at E, this ray leaves the mirror as if it came from F, the principal focus behind the mirror. Now draw AD perpendicular to the surface of the mirror. It is reflected

from the mirror at D and appears to have come from C, the center of curvature behind the mirror. At A', where the backward extensions of these two rays intersect, there is formed a **virtual image** of the point A. Locating the images of the other points in AB in the same way, it is found that the object AB forms the image $A'B'$ in the mirror LN. This image is behind the mirror. **It is virtual, erect, and diminished in size.**

595. Formula for a Spherical Mirror.—A simple formula which expresses the relation between the radius of curvature of the

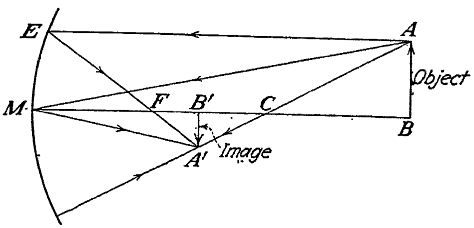


FIG. 578.—Derivation of mirror formula.

mirror, the object distance, and the image distance can be obtained from Fig. 578 as follows.

By similar triangles,

$$AB:A'B'::BC:CB'.$$
$$AB:A'B'::BM:B'M.$$

Hence,

$$BC:B'C::BM:B'M.$$

Let u and v denote the object distance and the image distance, respectively, that is, the distances of B and B' from the mirror, and r denote the radius of curvature of the mirror.

Then, $BC = u - r.$
$B'C = r - v.$
$BM = u.$
$B'M = v.$

$$u - r:r - v::u:v.$$

Multiplying means and extremes together,

$$uv - vr = ur - uv.$$
$$2uv = vr + ur.$$

Dividing by uvr,

$$\frac{2}{r} = \frac{1}{u} + \frac{1}{v}.$$

If AB is at a great distance from the mirror, that is, if u is infinitely large, $1/u = 0$ and, therefore,

$$v = \frac{r}{2} = f = \text{the principal focal length of the mirror.}$$

When $u = r$,

$$\frac{1}{r} + \frac{1}{v} = \frac{2}{r}$$

and

$$v = r.$$

This means that when the object is at the center of curvature, the image is also at the center of curvature. When $u = f$,

$$\frac{1}{f} + \frac{1}{v} = \frac{1}{f}$$

and

$$v = \infty.$$

Hence, the image lies at infinity when the object is at the principal focus.

When u is less than the focal length f, the image is virtual and lies behind the mirror. The image distance is therefore negative, and this negative value must be substituted in the formula.

This formula can be also applied in the case of a convex mirror if account is taken of the fact that in this case the radius of curvature is negative. In convex mirrors the image is always virtual and behind the mirror, so that the image distance is also always negative. Hence, in the use of this formula for a convex mirror, a negative sign must always be prefixed to both the radius of curvature and the image distance. As in the case of the concave mirror, when $u = \infty$, $v = f$, and the image is at the principal focus, but in this case the image is virtual and behind the mirror, and both f and v are negative.

Example.—An object is situated at a distance of 80 cm. from a concave mirror of radius of curvature of 60 cm. Find the position of the image.

$$u = 80,\ f = \frac{60}{2} = 30.$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$$\frac{1}{80} + \frac{1}{v} = \frac{1}{30}.$$

$$v = \frac{2{,}400}{50} = 48 \text{ cm.}$$

Example.—A bright spot situated at a distance of 40 cm. in front of a convex mirror forms an image 20 cm. behind the mirror. Find the focal length of the mirror.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$$u = 40,\ v = -20.$$

$$\frac{1}{40} - \frac{1}{20} = \frac{1}{f}.$$

$$f = -40.$$

596. Size of Object and Image.—In the case of either the convex or the concave mirror, there is a simple relation between the size of the image and the size of the object. In Fig. 578, the triangles ABM and $A'B'M$ are equi-angular and, therefore, similar, because the acute angles are the angle of incidence and the angle of reflection, respectively. The corresponding sides of the triangles are therefore proportional.

Hence,

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{distance of image from mirror}}{\text{distance of object from mirror}} = \frac{v}{u}.$$

Example.—Find the size of the image of a body 2.5 cm. high when placed 50 cm. in front of a concave mirror whose focal length is 20 cm.

$$u = 50,\ f = 20.$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{50}.$$

$$v = 33.3 \text{ cm.}$$

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{33.3}{50} = 0.66.$$

$$\therefore \text{Size of image} = 0.66 \times 2.5 \quad 1.66 \text{ cm.}$$

597. Parabolic Mirror.—If the width of a mirror (Fig. 579) is comparable to its radius of curvature, parallel rays after reflection do not all meet at the single point F, the principal focus. Rays

which are reflected from a limited region of the mirror in the neighborhood of its vertex M are brought to a focus at F, but the rays which strike the mirror at points distant from the vertex of the mirror cross the axis at points nearer to the mirror than F and are not brought to a focus. The effect of this is to destroy the sharpness of the image which would otherwise be formed by the mirror.

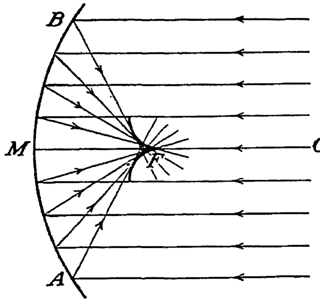


FIG. 579.—Spherical aberration in a spherical mirror.

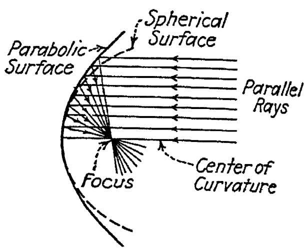


FIG. 580.—Parabolic mirror corrects for spherical aberration.

If the section of the mirror is a parabola instead of a circle, parallel rays after reflection are all focused at a single point (Fig. 580). On the other hand, a point source of light at the focus will send out parallel rays after reflection from the surface of the mirror. The chief property of a parabola is the fact that a line FB (Fig. 581) through the focus and a line BS parallel

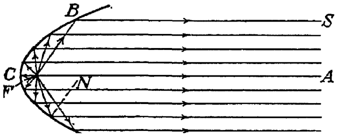


FIG. 581.—Parabolic mirror, source of light at the principal focus.

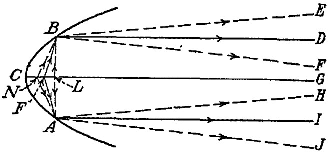


FIG. 582.—Parabolic mirror, light not at the principal focus.

to the axis make equal angles with the normal. From this, it follows that all rays parallel to the axis CA will pass through the point F after reflection, and the whole beam, however large, will be brought to a common focus. The distortion found in the spherical mirror is thus avoided. Since mirrors of large aperture are needed for searchlights, parabolic mirrors are used for such purposes. If the source of light is placed inside the focus of a

parabolic mirror (Fig. 582), the rays of light diverge after reflection. If it is outside the focus, the rays converge after reflection. This fact is of importance in the adjustment of the headlights of an automobile.

Problems

1. A plane mirror lies face up making an angle of 20 deg. with the horizontal. A ray of light shines down vertically on the mirror. What is the angle of incidence? What will be the angle between the reflected ray and the horizontal?

2. A short plane mirror used for looking at shoes is placed with its lower edge against the floor. If the mirror is 4 ft. from the shoes, and the level of the eyes is 5 ft. above the shoes, what is the greatest angle the mirror can make with the vertical?

3. An object is placed 12.5 cm. from a concave mirror which forms an image 25 cm. from the object on the side away from the mirror. What is the radius of curvature of the mirror?

4. An object is placed 80 cm. from a concave mirror, and an erect virtual image twice the size of the object is formed. What is the radius of curvature of the mirror?

5. An electric light bulb is placed at a distance of 6 in. from a concave mirror with a focal length of 3 in.; where is the image located, and how large is it?

6. Where must an arc light be placed with reference to a concave mirror with a radius of curvature of 8 ft., in order to have its image focused on a screen 20 ft. from the mirror?

7. What will be the magnification obtained by using a concave mirror with a focal length of 1.5 ft., if the mirror is held 8 in. from the face?

8. An incandescent lamp is located 6 ft. from a wall. It is desired to throw on the wall an image magnified three diameters, using a concave mirror. What must be the radius of curvature of the mirror, and where must it be placed?

9. An object is placed 9 ft. away from a convex mirror which has a focal length of 2 ft. Find the position and relative size of the image.

10. The distance of distinct vision for a normal eye is 10 in. A person wishes to look at his own eye with a plane mirror. How far must the mirror be placed from the eye?

11. An object is placed in front of a concave mirror, having a radius of curvature of 30 cm. It is desired to produce both a real and a virtual image which is three times as large as the object. Find the two distances at which the object must be placed in front of the mirror.

CHAPTER LII

REFRACTION OF LIGHT

598. Refraction.—Experiments have shown that light travels with the greatest speed in a vacuum, and that it travels with different speeds in different media. When it passes obliquely from one medium to another in which it has a different velocity

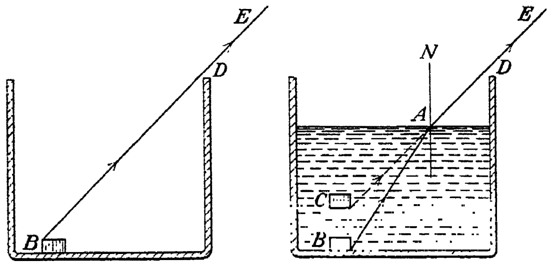


FIG. 583.—Refraction of light. The rays bend away from the normal on leaving the water.

there occurs a change in the direction of propagation of the light. This bending of a ray of light when passing from one medium to another is known as **refraction.**

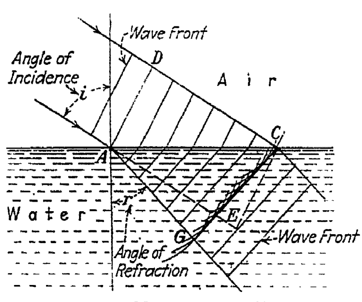


FIG. 584.—Change in direction of wave front at surface of separation of media.

Refraction can be illustrated by taking a cup which is opaque (Fig. 583) and placing a coin on the bottom of it at the point B, so that the far edge of the coin can just be seen when the eye is at E. If now, without moving the eye, water is poured into the cup, the coin will come completely into view. The ray BA as it leaves the water is bent away from the normal NA. Other rays are bent in a similar manner, and there is formed an image of the coin at C, so that the depth of the coin below the surface of the water seems to have been lessened. Here it is seen that rays coming from the water to the air are bent away from the normal. The rays are always bent away from the normal when

they enter a medium in which their velocity is greater than in that from which they came. In Fig. 584, refraction at a plane surface is described in terms of the change in direction of the wave front. The direction of the wave front changes as the wave front enters the refracting medium.

599. Laws of Refraction.—Let *RS* (Fig. 585) be a boundary surface separating two media, as air and glass. Take a point *O* on the surface and draw *MN* perpendicular to the surface at *O*. If now *AO* represents the incident ray, *OB* represents the refracted ray. If, on the other hand, *BO* represents the incident ray, *OA* represents the refracted ray. In the first case, the ray goes from a medium where the velocity is greater to a medium in which the velocity is less and is bent toward the normal. In the second case, the ray goes from a medium where the velocity is less to one in which the velocity is greater and is bent away from the normal. The angle *AON* is called the **angle of incidence,** and the angle *MOB* the **angle of refraction.**

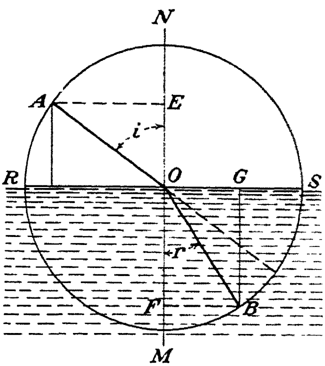


FIG. 585.—Laws of refraction.

Index of Refraction.—The ratio of the velocity in the medium from which the wave came to the velocity in the medium into which the wave enters is called the **index of refraction of the second medium with respect to the first.**

The index of refraction of any medium with respect to a vacuum is called the absolute index of refraction of that medium. This index is so nearly the same as the index with respect to air that the latter is ordinarily thought of when the term "index of refraction" is used without qualification.

Let V = the velocity in air.
V' = the velocity in glass.

$\frac{V}{V'} = n$ = **the index of refraction of glass.**

First Law of Refraction.—The refracted ray, the incident ray, and the normal to the surface lie in the same plane.

Second Law of Refraction. Snell's Law.—Let AB (Fig. 586) represent a surface separating one medium such as glass, in which the velocity is less, from a second medium like air, in which the velocity is greater. As soon as a ray enters the glass at D, it is retarded, while the ray CL continues to advance with the same velocity as before. Consequently, the direction of all the rays between D and L will be changed on entering the glass. If CL is the distance the wave travels in the upper medium in the time t and DH the distance it travels in the lower medium in the same time, and if V is the velocity in the upper medium and V' that in the lower medium, then

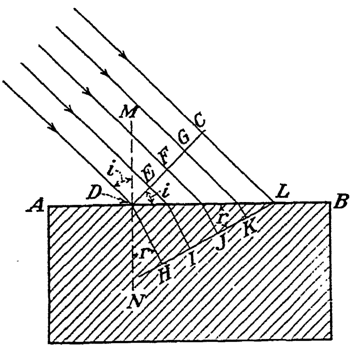


FIG. 586.—Proof of law of refraction.

$$CL = Vt = DL \sin i.$$
$$DH = V't = DL \sin r.$$
$$\frac{CL}{DH} = \frac{V}{V'} = \frac{\sin i}{\sin r} = \text{index of refraction.}$$

It thus appears that the ratio of the **sine of the angle of incidence to the sine of the angle of refraction is equal to the ratio of the velocities of the light in the two media and is, therefore, a constant for all angles of incidence.** This constant is the index of refraction of the medium. This law is known as **Snell's law.**

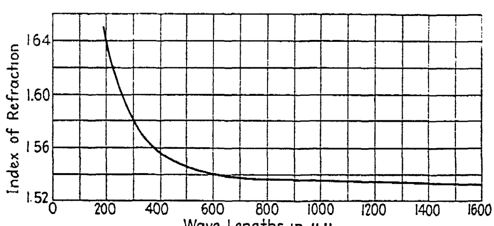


FIG. 587.—Change of index of refraction of quartz with the wave length.

The index of refraction depends on the wave length of light. The relation between index and wave length for quartz is represented in Fig. 587.

600. Refraction through a Slab with Parallel Faces.—Suppose that a ray of light AB (Fig. 588) falls on a plate of glass with parallel faces. As the ray enters the glass, it is bent toward the normal, according to Snell's law, so that

$$\frac{\sin i}{\sin r} = n.$$

The ray now travels to the opposite face, and, as it emerges at C, it is refracted away from the normal in such a way that

$$\frac{\sin i'}{\sin r'} \quad n$$

where n is the index of refraction of glass with respect to air. The ray of light is, therefore, refracted once toward the normal to

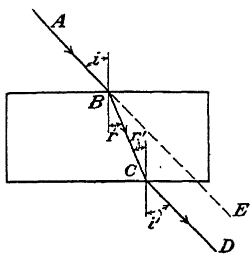


Fig. 588.—Refraction through a thin slab. Direction of ray AB and ray CD is the same.

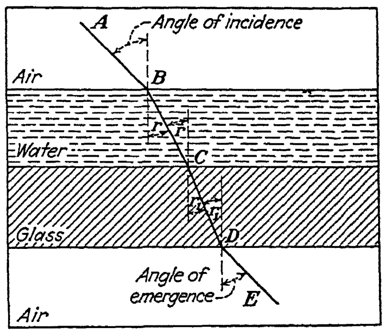


Fig. 589.—For several parallel slabs, the angle of incidence equals the angle of emergence.

the surface and once away from this normal. Since the faces of the plate are parallel to each other, the normals to the front and back faces are parallel to each other and therefore $r = r'$. Since the index of refraction is the same whether the light goes from air to glass or from glass to air,

$$\frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r'}$$

and since $r = r'$,

$$\sin i = \sin i' \text{ and } i = i'.$$

Consequently, the direction of the ray on emerging from the glass is the same as its direction on entering the glass. It has, however, been displaced laterally.

In case a ray of light passes through several parallel slabs of transparent substances the paths are as indicated in Fig. 589.

There is also reflection from the lower surface of the glass plate (Fig. 590).

601. Apparent Thickness of a Transparent Body.—One of the consequences of the refraction of light is found in the fact that on looking through a transparent material the body under observation appears nearer to the observer than it actually is. Let CA (Fig. 591) be a surface separating air from some other denser medium like glass or water. A ray of light, coming from a luminous point O in the denser medium and striking the surface of separation at A where OA is normal to the surface AC, will emerge without

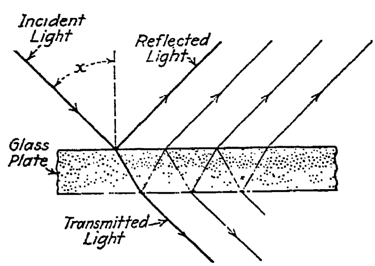


FIG. 590.—Multiple reflection and refraction at parallel plane surfaces.

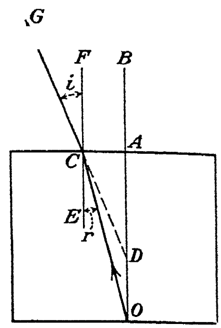


FIG. 591.—Apparent depth of substance such as water or glass is less than the real depth.

change of direction. Any other ray striking the surface at some point C at any angle less than 90 deg. will be bent away from the normal to the surface on entering the air. By prolonging this ray backward, it will intersect the normal ray at D. To an eye situated in the air above the surface AC and viewing the object normally to the surface, the light will appear to come from D instead of from O. By a similar construction for the other rays coming from O and striking the surface in the neighborhood of A, it will be found that these rays also appear to come from D. Hence, the object O from which the rays actually come appears to be at D which is above O. The apparent depth of O below the surface is, therefore, less than its true depth. From Fig. 591 it is seen that

$$\angle GCF = \angle DCE = \angle ADC$$

and

$$\angle OCE = \angle AOC.$$

$$n = \frac{\sin GCF}{\sin OCE} = \frac{\sin ADC}{\sin AOC} = \frac{AC}{DC} \div \frac{AC}{OC} = \frac{OC}{DC}.$$

The eye receives only rays very near the normal to the surface, so that without sensible error we may write AO for CO, and AD for CD.

$$\frac{AO}{AD} = \frac{\text{real depth}}{\text{apparent depth}}.$$

The apparent depth of the object below the surface depends on the angle (Fig. 592) at which the object is viewed.

To spear or shoot a fish under water it is necessary to aim below where the fish appears to be. If a fish were at O, it would appear to be higher up at D in case it is viewed normal to the surface.

By looking down into clear water it is easily observed that the water appears to be less deep than it really is. The cause of this shallowing effect is the refraction of the light at the surface of separation of the water and air. The greater the inclination at which the object is viewed, the less is its apparent depth.

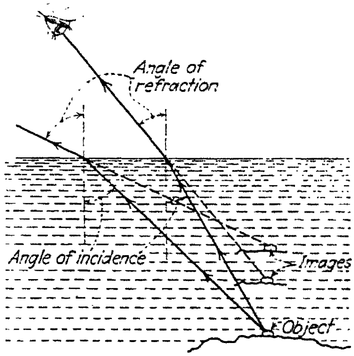


FIG. 592.—The apparent depth depends on the angle at which light comes to the eye.

602. Atmospheric Refraction.—Light is refracted in going from a vacuum to air. Let a ray of light from the sun (Fig. 593) enter the earth's atmosphere at B. This ray will be bent toward the radius of the earth and will reach an observer at O as if it had come in the direction AO instead of in its true direction. For this reason the observed altitude of the sun appears too great, so that it must be corrected to get the true altitude. On account of the irregular change in the density of the atmosphere, this correction is not easy to calculate. When the sun is at the zenith, no correction is necessary. A body which is already below the horizon may appear to be above the horizon. Consequently, the sun really sets before its last rays disappear. On the other hand, the rays of the sun become visible in the morning a little while before the sun is above the horizon. Because the bending of a ray is greater the more nearly horizontal the ray becomes, the rays from the lower edge of the sun are bent more than those from the upper edge. This produces a shortening of the vertical diameter, while the horizontal diameter is unchanged. The result is that the sun or moon appears elliptical when near the horizon.

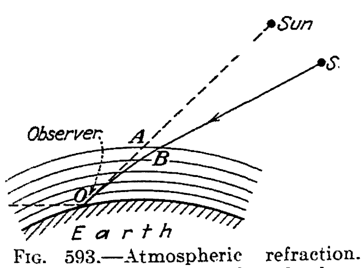


FIG. 593.—Atmospheric refraction. The sun appears too high in the sky.

603. Refraction through a Prism.—A wedge-shaped portion of a refracting medium bounded by two plane surfaces is called a

prism. If the medium of which the prism is composed is optically denser than the surrounding medium, a ray of light incident on one of the faces will be bent toward the normal to that face on entering the prism. On emerging from the opposite face, the ray will be going from a denser to a rarer medium and will be bent away from the normal at that face. The angle D (Fig. 594) through which the ray has been deflected in passing through the prism is called the **angle of deviation.** When the angle at which the ray enters one face is equal to the angle at which it leaves the opposite face, the angle of deviation has its least value and is known as the **angle of minimum deviation.**

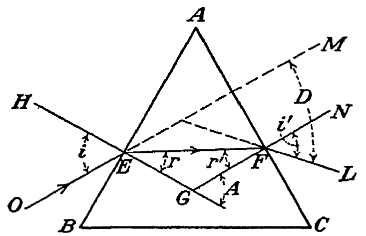


FIG. 594.—Refraction by a prism. The angle of deviation is least when the angle of incidence equals the angle of emergence.

604. Determination of Index of Refraction of a Prism.—By observing the angle of minimum deviation and the angle between the faces of the prism, it is possible to find the index of refraction of the material out of which the prism is made.

Let A = the angle of the prism (Fig. 594).
D = the angle of minimum deviation.
n = the index of refraction of the material out of which the prism is made.

Let i and r denote the angles of incidence and refraction at the first face of the prism, and r' and i' the angles of incidence and emergence at the second face.

Then, since the angle between the faces of the prism is the same as that between the normals HG and NG,

$$A = r + r'.$$

The deviation at the first face is $i - r$ and at the second face $i' - r'$. Hence, the total deviation is

$$\begin{aligned} D &= i - r + i' - r' \\ &= (i + i') - (r + r') = i + i' - A. \end{aligned}$$

It can be shown by experiment and also from theoretical considerations that the angle of deviation has a minimum value when

the angle of incidence is equal to the angle of emergence. In this case, $i = i'$ and $r = r'$.

$$2r = A.$$

Hence

$$r = \frac{A}{2}.$$

$$D = 2i - A.$$

$$i = \frac{A + D}{2}.$$

From the law of refraction,

$$\text{Index of refraction } = n = \frac{\sin i}{\sin r}.$$

Substituting for i and r,

$$\text{Index of refraction } = n = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}.$$

Example.—In a glass prism, the angle of minimum deviation was found to be 58 deg. for the *D*-lines of sodium and the angle of the prism was 60 deg. What is the index of refraction of the prism?

$$\text{Index of refraction } \frac{A + D}{\sin \frac{A}{2}}$$

$$\text{Angle of prism } = A = 60 \text{ deg.}$$

$$\sin \frac{1}{2} A = \sin \frac{1}{2}(60) = \sin 30 \text{ deg.} = 0.5.$$

$$\sin \frac{1}{2}(A + D) = \sin \frac{1}{2}(60 + 58) = \sin 59.$$

$$\frac{\sin 59}{\sin 30} \quad \frac{0.8572}{0.5} = 1.714 = \text{index of refraction of glass.}$$

605. Critical Angle.—When a ray of light passes from a dense medium such as water to a rarer medium such as air, it is bent away from the normal so that the angle of refraction is greater than the angle of incidence [Fig. 595(1)]. If the angle of incidence is made larger and larger, the angle of refraction will also become larger and larger and will always be greater than the corresponding angle of incidence. When the angle of incidence is increased sufficiently, the angle of refraction becomes 90 deg.,

and the refracted ray travels along the surface of separation between the two media [Fig. 595(2)]. **That angle of incidence**

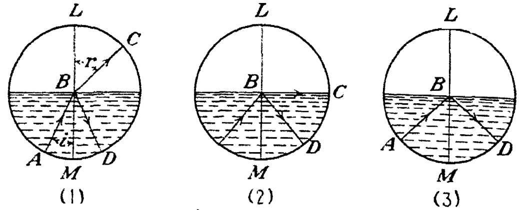


FIG. 595.—Total reflection. Angle of incidence must exceed the critical angle.

for which the angle of refraction is 90 deg. is called the critical angle.

606. Total Reflection.—If the angle of incidence is made larger than the critical angle, as in Fig. 595(3), light no longer enters the rarer medium since the angle of refraction cannot exceed 90 deg. The ray in this case is reflected back into the medium from which it came. In this reflection the ordinary laws of reflection are obeyed so that the angle of incidence is equal to the angle of reflection. Since none of the light in this case enters the second medium, this type of reflection is known as **total reflection.** It always :es place at a surface separating a rarer from a denser lium, when the light comes from the denser to the rarer

FIG. 596.—Total reflection by a right-angled prism.

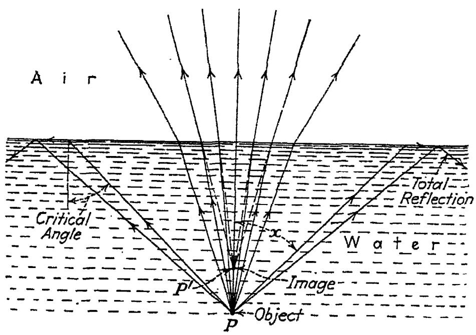


FIG. 597.—Total reflection at a plane surface.

medium and the angle of incidence exceeds the critical angle. The prism ABC (Fig. 596) produces total reflection of the ray OL.

In Fig. 597, light emerges from a point source P below the surface of water. Rays nearly normal to the surface emerge as if

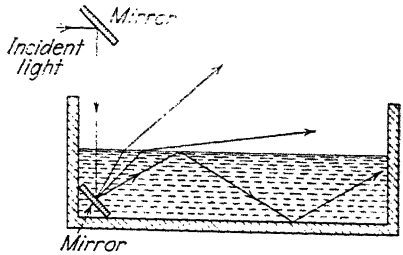


FIG. 598.—Apparatus for demonstrating total internal reflection.

they came from the point P'; other rays, for which the angle of incidence is greater than the critical angle, are totally reflected.

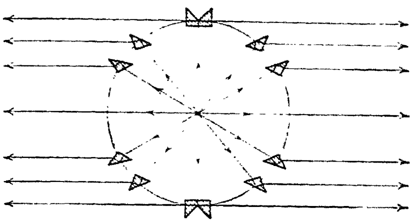


FIG. 599.—Lighthouse reflector. The light is totally reflected by the prisms.

Figure 598 gives an easy method of demonstrating the conditions under which total reflection takes place.

607. The Lighthouse Reflector.—The lighthouse reflector is an application of total reflection. Right-angled prisms are placed around the light (Fig. 599), so that they form an enclosed sphere. These prisms are so close together that light does not get out between them. The rays of light coming from the lamp at the center of the sphere strike one leg of the right-angled prism, enter the glass, strike the hypotenuse at an angle greater than the critical angle, and are then totally reflected in such a way that the rays of light are parallel to each other. The light is reflected without loss because it is a case of total internal reflection.

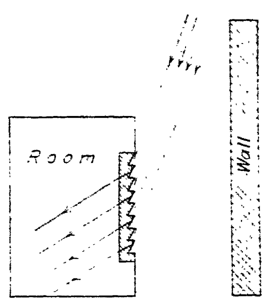


FIG. 600.—Prismatic window glass is another case of total reflection.

608. Prismatic Window Glass.—Where it is impossible to get sunlight into a room by means of ordinary windows, prismatic window glass may be used. The light coming almost straight down (Fig. 600) strikes the prismatic window glass in such a

way that it is totally reflected into the room. This type of window glass is useful in the illumination of basements below the level of the street.

Problems

1. Calculate the index of refraction of a substance for which light incident at an angle of 55 deg. is refracted at an angle of 40 deg.

2. Light passes from water into carbon disulphide. If the direction in the water makes an angle of 40 deg. with the normal to the surface between the two media, what will be the direction in the carbon disulphide?

3. A prism of glass with an index of refraction of 1.52 is made to have an angle of minimum deviation for yellow light of 30 deg. What is the angle of the prism?

4. Given a 45-deg. prism of crown glass, find the angle of minimum deviation. At what angle of incidence must the light strike the prism? Find the deviation if the angle of incidence is 5 deg. less; 5 deg. greater.

5. Determine the index of refraction of a substance for which the critical angle is 43 deg.

6. Find the critical angle between water and carbon disulphide if the index of refraction, air to water, is 1.33, and that of air to carbon disulphide is 1.67.

7. A prism of crown glass with angles 90, 45, and 45 deg., respectively, is used as a totally reflecting prism. At what angle of incidence on one of the short faces will the light strike the hypotenuse at the critical angle? Illustrate with a diagram.

8. The index of refraction of glass with respect to air is 1.53, and the index of refraction of water with respect to air is 1.33. What is the index of refraction of glass with respect to water?

9. The critical angle for a certain kind of light is 48 deg. from water to air. Find the velocity of light in water.

10. A ray of light passes from oil to water. The index of refraction of the water is 1.33 and that of oil is 1.45. What is the critical angle for light passing from oil to water?

11. A hollow prism is made of plates of glass whose surfaces are parallel. The angle between the faces of the prism is 60 deg. What is the angle of minimum deviation when yellow light having a wave length of 0.00005896 cm. is passed through the prism which is filled with water?

12. What is the index of refraction of a glass prism which produces an angle of minimum deviation of 53 deg. for yellow light, assuming that the angle of the prism is 60 deg.?

13. A crown-glass prism which has an angle of 20 deg. is used with a flint-glass prism in such a way that the *D*-lines of sodium are undeviated when light passes through both prisms. What angle must the flint-glass prism have?

CHAPTER LIII

LENSES

609. Lenses.—Lenses are bodies made of transparent material and bounded by faces having a cylindrical or spherical form. Although lenses differ much in form, they may be divided into

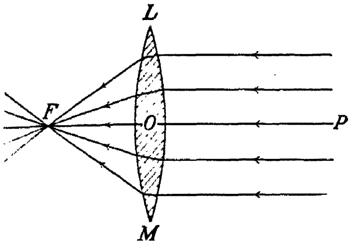


FIG. 601.—Principal focus of a converging lens. The incident rays are parallel to the principal axis *POF*.

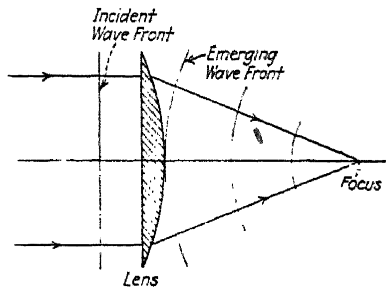


FIG. 602.—Change in curvature of a plane wave front by a converging lens.

two classes according to the way in which they act on a parallel beam of light. Consider the lens in Fig. 601 on which parallel rays are incident. Each ray is bent toward the normal to the surface on entering the lens and away from the normal on emerging from the lens. In this way, the rays above the axis *PO* are bent downward and those below it are bent upward. After leaving the lens, the rays converge to a point *F*, called the principal focus. Such a lens is **a converging lens.** If the incident rays are parallel to each other, the incident wave front (Fig. 602) is a plane perpendicular to the incident rays. When this wave front emerges from the lens, it has been changed to a concave wave front which converges to the focus. When the

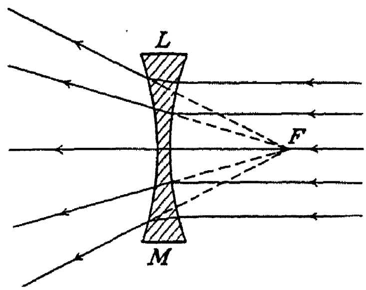


FIG. 603.—Principal focus of a diverging lens.

bounding surfaces of the lens are very convex, the lens converges the rays rapidly. This gives the lens a short focal length. When the bounding surfaces are only slightly convex, the lens has a long

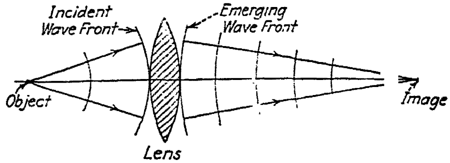


FIG. 604.—Reversal of curvature of a spherical wave front by a converging lens.

focal length. In order that all the rays may come to a point after leaving the lens, the beam of light must be restricted to a narrow bundle near the principle axis of the lens. For an

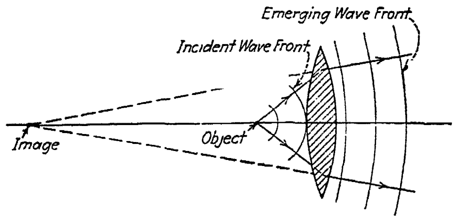


FIG. 605.—Change of curvature of a spherical wave front by a converging lens.

extended beam the outer rays will not pass through the same point as the rays near the axis.

When the surfaces of the lens are concave instead of convex, the lens makes the rays which pass through it more divergent, and for

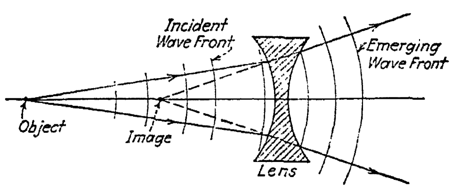


FIG. 606.—Change of curvature of a spherical wave front by a diverging lens.

this reason it is known as a **diverging lens.** In Fig. 603, parallel rays are incident on a concave lens. On entering the lens, the rays are bent toward the normal as before, and on leaving they are bent away from the normal. In this case, however, the

emerging rays are bent away from the principal axis. They appear on leaving the lens to come from a point F behind the lens. When the incident rays are parallel to each other and to the principal axis, that point from which the rays appear to come on leaving the lens is **the principal focus.** This is only an apparent focus because the light does not really come from it, but the effect on the left-hand side of the lens is the same as if the light actually came from this point behind the lens. This kind of focus from which the light appears to come is a **virtual focus.** It is to be carefully distinguished from a real focus through which the light actually goes.

If the lens is convex, and the incident rays come from a point source, the incident wave front is spherical, and the emerging wave front is the surface of a sphere with its center at the image (Figs. 604 and 605). If the lens is concave, the curvature of the wave front is increased in passing through the lens (Fig. 606), and the image is behind the lens.

610. Spherical Aberration of a Lens.—When rays of light parallel to the principal axis of a spherical lens pass through zones near its edge, they cross the axis nearer the lens than those which

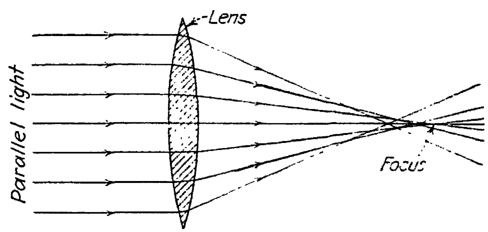


FIG. 607.—Spherical aberration in a convex lens. Rays do not converge to same point.

pass through its center. For this reason the refracted rays do not form a perfect cone of light. Instead of converging to a sharp point (Fig. 607), they cross the axis at different points and form a blurred image of the source. This imperfection of a lens is known as **spherical aberration.** The scattering arising from it increases with the square of the aperture of the lens and is inversely proportional to the cube of the focal length. For lenses of small aperture and long focal length it becomes negligible. It can be corrected by a proper choice of the curvature of the surfaces of

the lens. The curvature at different points must be so chosen that rays which have come from a single point on the object will converge to a point after passing through the lens.

611. Diopter.—The reciprocal of the focal length of a lens is called its "power" by opticians. The unit of power in common use is called a **diopter.** A lens which has a focal length of 1 m. has a power of 1 diopter. The power of a lens in diopters is the reciprocal of the focal length in meters. If the focal length of a lens is 0.25 m., its power is $1/0.25 = 4$ diopters.

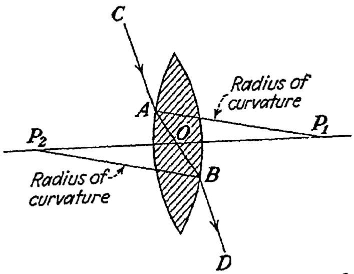


FIG. 608.—The optical center is at O where AB crosses the axis.

612. Optical Center.—For each lens there is some point such that the rays passing through it are not deviated by the lens. For example, a ray of light $CABD$ (Fig. 608) emerges from the lens in the same direction as that in which it entered. This is seen by drawing a tangent plane at A and another at B parallel to the one at A. The direction of the ray in the lens is the line joining these two points of tangency. The lens behaves for rays which enter at A and leave at B as a plate of glass with parallel faces. Such a plate of glass (Fig. 588) causes only a displacement of the ray without a change in its direction. **That point where an undeviated ray in passing through the lens crosses the axis is known as the optical center of the lens.**

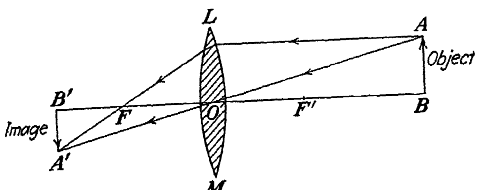


FIG. 609.—Image by a convex lens when object is some distance from the lens. The image is real, inverted, and diminished.

613. Graphical Construction of Images.—In order to obtain the position and size of the image formed by a lens, select a point A of the object AB (Fig. 609). Of the rays going out from this point select two, one of which is parallel to the principal axis while the other passes through the optical center of the lens.

The ray parallel to the principal axis will pass through the principal focus of the lens, and the other ray will pass through the optical center O without deviation. Where these two rays intersect, there will be formed an image A' of the point A from which these rays came. In like manner, other points on the object AB can be located, giving the image $A'B'$.

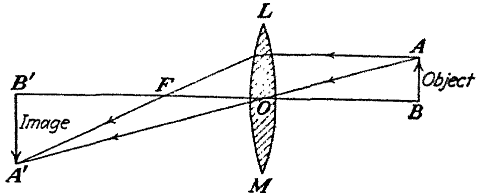


FIG. 610.—Image by a convex lens when the object is near the lens but not inside the principal focus. The image is real, inverted, and magnified.

In a convex lens three different cases will arise: Suppose the object is at a considerable distance from the lens; then the location of the image by this method will give a **real and diminished image** as in Fig. 609. In case the object is near the lens but still farther from it than its focus, the lens will form a **real and magnified image** (Fig. 610). When the object lies between the principal focus and the lens (Fig. 611), the rays of light on leaving the lens seem to diverge from a point A' on the same side of the lens

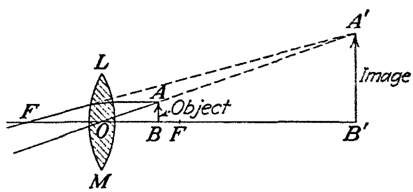


FIG. 611.—Image by a convex lens when the object is inside the principal focus. The image is virtual, upright, and magnified.

as the object A. There is thus formed at $A'B'$ **a virtual and magnified image** of the object AB.

For the concave lens there is only one kind of image possible (Fig. 612). Wherever the object is placed, the image lies between the principal focus and the lens. The rays from A on leaving the lens diverge as if they had come from a point A' behind the lens. There is thus formed an image $A'B'$ inside of the principal focus of the lens. This image is **virtual, erect,** and **diminished.** If

the object is taken farther away from the lens, the image $A'B'$ approaches the focus of the lens. Whatever the position of AB, the image always lies between the focus and the lens.

614. Magnification of Image by a Lens.—The magnifying power of a lens is the ratio of the linear dimensions of the image to the linear dimensions of the object.

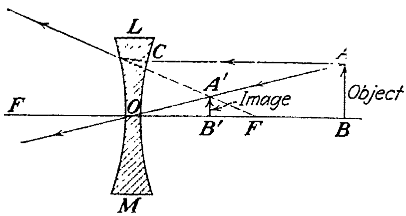


FIG. 612.—Image by a concave lens. The image is virtual diminished, and upright.

In Fig. 611 by similar triangles,

$$\frac{A'B'}{AB} = \frac{OB'}{OB} = \frac{\text{image distance}}{\text{object distance}}.$$

In general,

$$\frac{\text{Length of image}}{\text{Length of object}} = \frac{\text{image distance}}{\text{object distance}}.$$

615. Formulae for Lenses.—The formula connecting the object distance, the image distance, and the focal length of a lens is similar in form to the corresponding formula for mirrors.

Let U = the distance of the object from the lens.
V = the distance of the image from the lens.
F = the focal length of the lens.

For a converging lens in which the object is farther away from the lens than the principal focus (Figs. 609 and 610),

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{F}.$$

If the object is very far from the lens, $U = \infty$ and

$$\frac{1}{\infty} + \frac{1}{V} = \frac{1}{F}.$$

$$V = F.$$

Hence, the image is at the principal focus for very distant objects. If the object lies at the principal focus,

$$U = F \text{ and } V = \infty$$

so that no image is formed, or the rays which leave the lens are parallel to each other.

In the case of a converging lens when the object lies between the principal focus and the lens (Fig. 611), the image lies on the same side of the lens as the object, and the image distance is **negative.** This fact must be taken account of by prefixing a **negative** sign to the image distance when the object is inside the principal focus of the lens.

For a diverging lens (Fig. 612) both the image distance and the focal length are **negative,** and a **negative** sign must be prefixed

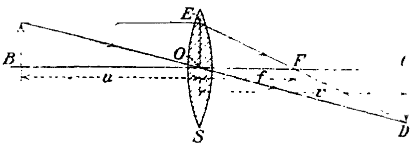


Fig. 613.—Relation between object distance and image distance for a converging lens.

to each of them whenever the lens formula is used for a diverging lens.

When the object is far from the lens, $U = \infty$ and

$$\frac{1}{V} = \frac{1}{F}.$$

$$V = F.$$

Hence, the image is at the principal focus. Here both V and F are inherently negative.

616. The Lens Formula.—The relation between the object distance, the image distance, and the focal length of a lens has been derived in Appendix E-15, but a simplified and more approximate derivation will serve the present purposes.

In Fig. 613 let $OB = u$ = the object distance.
$OC = v$ = the image distance.
$OF = f$ = the focal length.

The triangles AOB and COD are similar. Hence,

$$\frac{AB}{CD} = \frac{BO}{OC}.$$

$$\frac{\text{Length of object}}{\text{Length of image}} = \frac{\text{object distance}}{\text{image distance}}.$$

618. Telephoto Lens.—In order to produce as large an image as possible of a distant object, the focal length of the lens must be as large as possible. This fact makes it necessary that a camera of inconvenient length be used, since the length of the camera must be approximately equal to the focal length of the lens. This difficulty can be avoided by a telephoto lens (Fig. 615) which consists of a combination of a convergent lens A and a divergent lens LM placed at a distance d from each other. The convergent lens A would form an image of a distant object just outside its focus F_1, but the divergent lens LM causes the image to be formed at F_1'. The image formed at F_1' is larger than the one that would have been formed at F_1. In this way, the magnification of the lens system is increased without increasing too much the length of the camera.

Problems

1. A concave lens has a focal length of 8 cm. An object 4 cm. high is placed 20 cm. in front of the lens. Find the position and size of the image.

2. A candle is placed at a distance of 1 m. from a diverging lens with a focal length of 1 m. Where is the image located, and how large is it? Draw a diagram to illustrate this case.

3. A metric scale is placed at a distance of 25 cm. from the eyes and observed with one eye unaided, the other looking at a similar scale through a converging lens placed close to the eye. It is seen that a magnified millimeter division appears the same size as 6-mm. divisions seen with the naked eye. Find the magnification and the focal length of the lens.

4. A converging lens of focal length 4 cm. is used as a magnifying glass. Find the magnification if the image is at infinity; at 25 cm. from the lens.

5. A lamp and screen are 12 ft. apart. Where must a converging lens with a focal length of 2 ft. be placed so as to form an image of the lamp on the screen? Show that there are two solutions, and find the relative size of the image in each case.

6. A beam of sunlight falls on a diverging lens of focal length 10 cm.; 15 cm. beyond this is placed a converging lens of 15-cm. focal length. Find where a screen should be placed to receive the final image of the sun.

7. A double-convex lens is used to form the image of an object which is at a distance of 40 cm. from the lens. The front surface of the lens has a radius of 15 cm. and the back surface a radius of 18 cm. The glass has an index of refraction of 1.6. Find the position of the image.

8. The radii of curvature of a double-concave lens are 40 and 25 cm., respectively. It is made of glass which has an index of refraction of 1.55. Find its focal length.

9. A plano-convex lens is used to form the image of an object. The image is formed 12 cm. from the lens. The glass of which the lens is made has an index of refraction of 1.6, and the convex surface has a radius of curvature of 15 cm. Find the position of the object.

10. A double-convex lens has radii of curvature which are equal in magnitude. The index of refraction of the glass is 1.56 and the strength of the lens is 5 diopters. What is the radius of curvature of each of the convex surfaces?

11. A convex lens produces an image which is the same size as the object. The distance from the object to the screen on which the image is cast is 36 in. When another lens is put in contact with the first lens, the image is one-quarter of its original size and the screen must be moved accordingly. Find the focal length of each lens.

12. A glass lens having an index of refraction of 1.65 and radii of curvature of $+10$ and $+20$ cm., respectively, is cemented to another glass lens having an index of refraction of 1.56 and radii of curvature of $+5$ and -10 cm., respectively. Find the focal length of the combination.

13. A double-convex lens is made of glass having an index of refraction of 1.56 for the *D*-lines of sodium. If its surfaces have radii of curvature of $+5$ and $+10$ cm., respectively, what is the focal length of the lens?

14. What is the index of refraction of a plano-convex glass lens which has a focal length of 30 cm. and a radius of curvature of 18 cm.?

15. A plano-convex lens is made of flint glass which has an index of refraction of 1.69 for violet light and 1.64 for red light. If the radius of curvature of the curved surface of the lens is 20 cm., what is the difference in focal length of the lens for these two colors?

16. A convex and a concave lens are placed 16 cm. from each other. Where will this combination of lenses produce an image of a distant object, if the convex lens has a focal length of 20 cm. and the concave lens a focal length of 5 cm.?

17. The index of refraction of a glass lens is 1.65. Its radii of curvature are 12 and 25 cm., respectively. Where will it produce an image of an object which is 28 cm. in front of it?

Also the triangles EOF and CDF are similar.

$$\frac{EO}{CD} = \frac{OF}{FC} = \frac{f}{v - f}.$$

$$EO = AB.$$

$$\frac{EO}{CD} = \frac{AB}{CD} = \frac{OF}{FC} = \frac{f}{v - f}$$

and

$$\frac{AB}{CD} = \frac{OB}{OC} = \frac{u}{v}.$$

$$\frac{f}{v - f} = \frac{u}{v}.$$

$$\frac{v - f}{f} = \frac{v}{u}.$$

Dividing by v,

$$\frac{1 - \frac{f}{v}}{f} = \frac{1}{u}.$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}.$$

In the case of a divergent lens (Fig. 614).

$$\frac{AB}{CD} = \frac{u}{v}.$$

$$\frac{EO}{CD} = \frac{f}{f - v}.$$

$$EO = AB.$$

$$\frac{u}{v} = \frac{f}{f - v}.$$

$$\frac{f - v}{f} = \frac{v}{u}.$$

$$1 - \frac{v}{f} = \frac{v}{u}.$$

$$\frac{1}{v} - \frac{1}{f} = \frac{1}{u}.$$

$$\frac{1}{u} - \frac{1}{v} = -\frac{1}{f}.$$

Example.—A double-convex lens is made of glass having an index of refraction of $3/2$. The front surface has a radius of curvature 10 cm., and the back surface has a radius of curvature of 20 cm. Find its focal length. See Appendix for the formula.

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{1}{20}\right)$$

$$= \frac{1}{2}\left(\frac{3}{20}\right) = \frac{3}{40}.$$

$$f = \frac{40}{3} \quad 13.3 \text{ cm.}$$

617. Combination of Two Thin Lenses.—Let two lenses A and B be at a distance d apart; let f_1 and f_2 denote the focal lengths, u_1 and v_1 the object

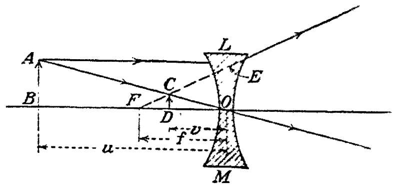


FIG. 614.—Relation between object distance and image distance for a diverging lens.

distance and the image distance for one lens, and u_2 and v_2 the object distance and the image distance for the other lens. Then.

$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f_1}$$

and

$$\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f_2}$$

where $u_2 = d - v_1$.

To find the image distance of the combination of these two lenses, suppose that parallel light is incident on the lens A. Then $u_1 = \infty$ and $v_1 = f_1$, and $u_2 = d - f_1$.

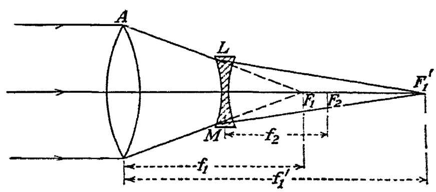


FIG. 615.—A telephoto lens. The effective focal length is increased by the addition of a concave lens.

Hence,

$$\frac{1}{d - f_1} + \frac{1}{v_2} = \frac{1}{f_2}.$$

$$\frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{d - f_1} = \frac{d - f_1 - f_2}{f_2(d - f_1)}$$

$$v_2 = \frac{f_2(d - f_1)}{d - f_1 - f_2}$$

In this case v_2 is the image distance for parallel light falling on the system of lenses. This distance is measured from the optical center of the lens B. In case the lenses are in contact, $d = 0$ and $v_2 =$ the focal length of the combination of lenses and

$$= \frac{f_1 f_2}{f_1 + f_2}.$$

CHAPTER LIV

OPTICAL INSTRUMENTS

619. The Photographic Camera.—The simplest application of lenses for optical purposes is in an ordinary photographic camera (Fig. 616). A lens or combination of lenses in one end of the camera produces a real image of an external object on the photographic plate or film at the other end of the camera. The distance between the lens and the photographic plate or film can be altered so as to focus the image on the plate. Sometimes the photographic plate is replaced by a ground-glass screen on which the image is cast and then the lens is moved into such a position that the image is sharply focused on the ground glass. The plate or film is then substituted for the ground-glass plate. The

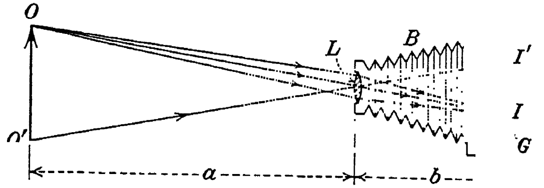


FIG. 616.—Photographic camera.

bellows B serves to exclude all the light except that which comes from the object. It also makes possible the to-and-fro motion of the lens necessary in the focusing.

To avoid spherical aberration, that is, the distortion of the image because all the rays do not come to the same focus, and to limit the quantity of light, a diaphragm is placed in front of the lens. This diaphragm restricts the rays to those which pass through the central portion of the lens. The smaller the diameter of the hole in the diaphragm, the better is the definition of the image. On the other hand, the smaller this opening, the less the intensity of the light forming the image on the plate. Where it is desired to have the image as bright as possible, the choice of the size of the opening in the diaphragm becomes a compromise between diminished brightness on the one hand and definition on the other hand.

A photographic plate consists of a silver compound spread on a glass plate or celluloid film. The image formed by the lens acts on this sensitive film, but the image is not visible until the plate or film is placed in a mixture of chemicals, known as the "developer."

Example.—The focal length of the lens in a camera is 20 cm. How far back must the plate be placed to take an object which is 10 m. away? If the object is 1 m. high, how high is the image?

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$$f = 20 \text{ cm.}$$

$$u = 10 \text{ m.}$$

$$\frac{1}{1{,}000} + \frac{1}{v} = \frac{1}{20}.$$

$$v = \frac{20{,}000}{980} = 20.4 \text{ cm.}$$

$$\text{Magnification} = \frac{\text{length of image}}{\text{length of object}} = \frac{v}{u} = \frac{20.4}{1{,}000} = 0.0204.$$

$$\frac{\text{Length of image}}{100} = 0.0204.$$

$$\text{Length of image} = 2.04 \text{ cm.}$$

620. The Relative Aperture of a Lens.—The diameter of a lens divided by its focal length is known as the **relative aperture** of the lens. For example, if a lens has a diameter of 2 cm. and a focal length of 10 cm. its relative aperture is $2/10 = 0.2$. If another lens has a diameter of 2 cm. and a focal length of 20 cm., it has a relative aperture of $2/20 = 0.1$. If these lenses are used to form images of the same object, the lens which has a relative aperture of 0.1 produces an image which has four times the area of the image produced by the lens which has a relative aperture of 0.2. The intensity of the image in the latter case is four times that in the former case. If the lens with the larger relative aperture is used in a camera, the time of exposure for the same object is only one-fourth as great as with the lens with the smaller relative aperture.

If the ratio of the diameter of the lens to its focal length is $1/5$, the relative aperture of the lens is $f/5$, where f is the focal length of the lens, *i.e.*, the diameter of the lens is equal to one-fifth of its focal length. A camera lens with a relative aperture of $f/3$ collects four times as much light as a camera lens with a relative aperture of $f/6$ and it is therefore four times as fast for taking pictures.

621. Projection Lantern.—The projection lantern, which is used to throw an image of a brilliantly illuminated object on a screen, consists of a powerful source of light S (Fig. 617), a large condensing lens NP, and a front or objective lens LM. The condensing lens NP collects the light from the source S and sends it through the slide OI so that this slide is brilliantly illuminated. The objective LM then produces a real image of the slide OI

on the screen XY. Since the slide OI is just outside the principal focus of the lens LM, the image on the screen is enlarged. This image is also real and inverted. The magnification produced by the lantern is obtained from an application of the law for the magnification of a lens.

Let u = the distance of the slide from the lens LM.
v = the distance between the screen XY and the lens LM.

$$\text{Magnification} = \frac{v}{u}.$$

If the distance from the slide to the screen and the magnification to be produced by the lantern are given, it is possible to find the focal length of the lens LM which must be used to produce

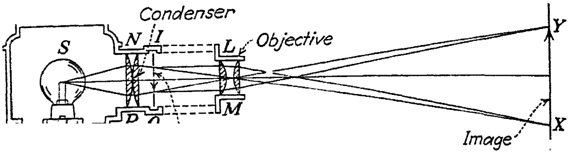


FIG. 617.—Projection lantern. It produces real, inverted, and magnified images.

an image on the screen. It is only necessary to apply the lens equation, .

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f},$$

together with the fact that the distance from the slide to the screen is $u + v$.

Example.—Find the focal length of a lens which must be used in a lantern to produce the image of a slide 8 cm. square upon a screen 3 m. square at a distance of 10 m. from the lantern. Assume that the image occupies the entire screen.

$$\text{Magnification} = \frac{v}{u} = \frac{300}{8}.$$

Since v = 1,000 cm. nearly,

$$\frac{v}{u} = \frac{1{,}000}{u} = \frac{300}{8} \text{ and } u = 26.6.$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$$\frac{1}{26.6} + \frac{1}{1{,}000} = \frac{1}{f}.$$

$$f = 25.9 \text{ cm.}$$

622. Simple Microscope.—When an object is placed slightly nearer to a converging lens than its focus, an eye brought up to the lens sees a virtual, erect, and magnified image $A'B'$ (Fig. 618). A lens used in this way constitutes a simple microscope. In order to obtain the greatest advantage, the eye should be as near as possible to the lens. In this way, the field of view is made as large as possible and the distance of the virtual image $A'B'$ from the eye for distinct vision is made as large as possible.

In order to find the magnification of a lens used in this way, it is necessary to consider that the apparent size of an object is determined by the angle which it subtends at the eye. Now for most

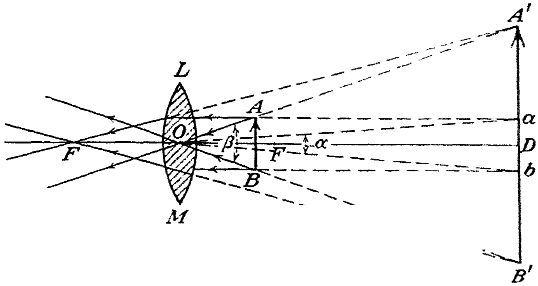


Fig. 618.—Simple microscope. It produces virtual, magnified, and upright images.

distinct vision, an object must be about 10 in. from a normal eye. If an object is placed at a greater distance than this, the image on the retina is smaller and its details are not seen so distinctly. When the object is placed nearer than 10 in., the image on the retina is blurred. When an object is examined with the aid of a magnifying glass, the object is brought nearer to the eye than would be possible for distinct vision without the magnifying glass. In Fig. 618 the angles subtended at the center of the lens by the object AB and the image $A'B'$ are the same. But OD is the distance of distinct vision, and, if the lens were absent, the eye could not see AB distinctly until it was removed to ab. Hence, by using the lens, the visual angle has been increased from α to β, and consequently the magnification is

$$\frac{A'B'}{AB} = \frac{v}{u}.$$

For most distinct vision, $v = 25$ cm.

$$\frac{A'B'}{AB} = \frac{25}{u}.$$

Since $v = -25$ and $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$,

$$\frac{1}{u} = \frac{1}{f} + \frac{1}{25}.$$

$$\frac{25}{u} = \frac{25}{f} + 1.$$

Hence,

$$\frac{A'B'}{AB} = 1 + \frac{25}{f}.$$

Example.—Find the greatest magnification that can be produced by a converging lens of focal length 2.5 cm.

$$\text{Magnification} = \frac{A'B'}{AB} = \frac{v}{u} = 1 + \frac{25}{f} = 1 + \frac{25}{2.5} \quad 11.$$

623. Compound Microscope.—In order to obtain very great magnification, a compound microscope is used. It consists

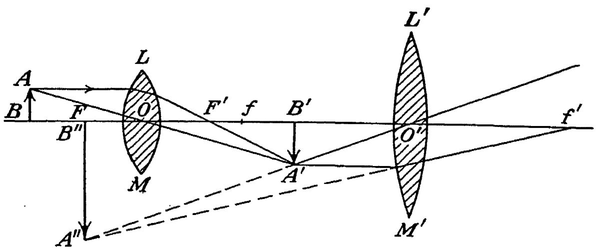


FIG. 619.—Diagram of compound microscope.

of a converging lens LM of short focal length (Fig. 619). This lens produces a real and magnified image of a small object placed just outside of its focus. This lens is called **the objective.** The image $A'B'$ produced by it is viewed through the eyepiece $L'M'$, which acts like a simple magnifying glass. The eyepiece produces an enlarged and virtual image $A''B''$ of the real image $A'B'$. The real image $A'B'$ of the object AB falls just inside the focus of the eyepiece. The lens $L'M'$ then acts on this real and inverted image, to magnify it still further.

The magnification of a compound microscope depends on the focal length of both the objective and eyepiece. If the objective

has a focal length of 5 mm., the real image may be at a distance of 20 cm. from the objective. Since the object is near the focus of the objective, the object distance may be taken as nearly equal to 5 mm. The magnification produced by this lens is then

$$\frac{v}{u} = \frac{20}{0.5} = 40.$$

If now the eyepiece has a focal length of 2.5 cm., it will magnify the real image $A'B'$ into the image $A''B''$ in accordance with the relation

$$\frac{A''B''}{A'B'} = 1 + \frac{25}{f} = 1 + \frac{25}{2.5} = 11.$$

$$A'B' = 40 \times AB.$$

$$A''B'' = 40 \times AB \times 11 = 440AB.$$

$$\frac{A''B''}{AB} = 440.$$

The approximate magnification may be written down as follows:

Let L = the length of the microscope tube.
F = the focal length of the objective.
f = the focal length of the eyepiece.

$$\frac{A'B'}{AB} = \frac{OB'}{OB}.$$

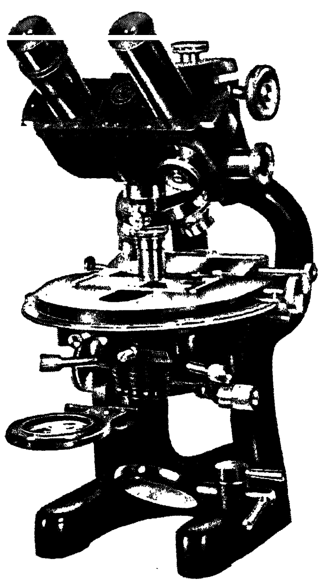


FIG. 620.—Compound microscope. (*Courtesy Bausch and Lomb Optical Company.*)

Since the object is nearly at the principal focus of the objective LM and the focal length of the eyepiece is small in comparison with the length of the microscope tube,

$$\frac{OB'}{OA} = \frac{L}{F} \text{ approx.}$$

The magnifying power of the eyepiece is

$$\frac{A''B''}{A'B'} \quad 1 + \frac{25}{f}.$$

Multiplying these equations together, we have as the magnifying power of both lenses.

$$\frac{A''B''}{AB} = \frac{L}{F}\left(1 + \frac{25}{f}\right) = \frac{25 \times L}{F \times f} \text{ approx.}$$

Example.—The focal length of the objective of a microscope is 0.5 cm., the distance between the objective and the eyepiece is 20 cm., and the focal length of the eyepiece is 1 cm. Find the magnification of the microscope.

$$\text{Magnification} = \frac{L}{F}\left(1 + \frac{25}{f}\right) = \frac{20}{0.5}\left(1 + \frac{25}{1}\right) = \frac{20}{0.5} \times 26 = 1{,}040.$$

624. Astronomical Telescope.—The principle of the astronomical telescope is the same as that of the compound microscope. The objective lens LM is modified to meet the fact that the instrument is used to view distant objects instead of nearby objects. This objective, which is a converging lens of long focal length, forms a real, inverted image $A'B'$ of a distant object AB. This real image is formed just inside the focus of the eyepiece NP by which it is viewed. By means of this eyepiece an enlarged virtual image of the real image $A'B'$ is produced at $A''B''$.

On account of the great distance of the object from the telescope, the rays from any point of it are essentially parallel when they reach the objective. Hence the image formed by the objective lies just outside the principal focus of the objective.

The magnification of the telescope is determined by the angle which the object subtends or appears to subtend at the eye (Fig. 621). The angle which the object AB subtends at the eye when no lenses are present is $\angle AOB = \angle B'OA'$. The angle which the image $A''B''$ subtends at the eye is $\angle B''RA'' = \angle A'RB'$.

$$\begin{aligned}\text{Magnification} &= \frac{\angle A'RB'}{\angle A'OB'} = \frac{A'B'/RD}{A'B'/OD} = \frac{OD}{RD} \\ &= \frac{\text{focal length of objective}}{\text{focal length of eyepiece}} \\ &= \frac{F}{f}.\end{aligned}$$

Hence, the magnification of an astronomical telescope is found approximately by dividing the focal length of the objective by the focal length of the eyepiece. The image formed by such a

telescope is always inverted. For terrestrial uses of the telescope, it is desirable that the image be erect. This condition is realized by inserting a third convex lens between the objective and the

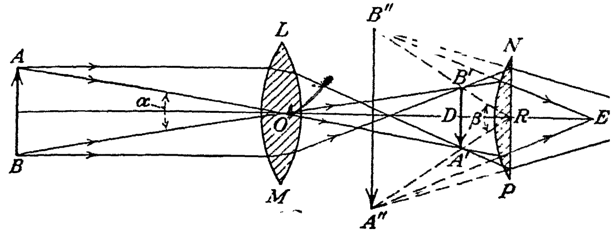


FIG. 621.—Astronomical telescope.

eyepiece in such a way that the image $A'B'$ formed by the objective is again inverted before it is viewed by the eyepiece.

Example.—The focal length of the objective of a telescope is 150 cm. and the focal length of the eyepiece is 2 cm. Find the magnifying power of the telescope for distant objects.

$$\text{Magnifying power} = \frac{\text{focal length of objective}}{\text{focal length of eyepiece}}$$
$$= \frac{150}{2} = 75.$$

In large telescopes used in the great astronomical observatories the object lens is replaced by a large concave mirror. The arrangement of this mirror and the eyepiece is evident from Fig. 622. The mirror in use at the Mount Wilson Observatory has a diameter of 100 in.

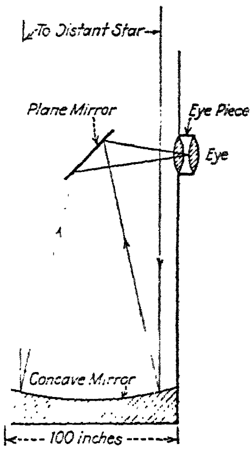


FIG. 622.—Reflecting telescope used in astronomical observatories.

625. Opera or Field Glasses.—The opera glass like the telescope consists of an objective LM which converges the rays toward a point S (Fig. 623), but before they reach this point they pass through the diverging lens NP which replaces the eyepiece of the telescope. In passing through this diverging lens, the rays which were converging on entering it are made to diverge on leaving it. To an eye on the right-hand side of the lens NP (Fig. 623) the rays seem to have come from a point A' behind the concave lens NP which thus forms the virtual and erect image $A'B'$ of the

object from which light was received. Unlike the astronomical telescope, the opera glass gives an erect image.

To have the opera glass in focus, the lens NP must be so placed with respect to the objective that the rays emerging from the lens NP are nearly parallel. The magnification is

$$\text{Magnification} = \frac{\text{focal length of objective}}{\text{focal length of eyepiece}}.$$

626. Prism Binoculars.—Field glasses of higher magnifying power and larger field of view are made of the optical parts

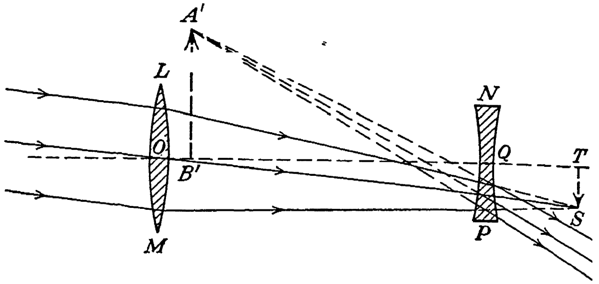


FIG. 623.—Opera or field glasses. They produce virtual and upright images.

represented in Fig. 624. The beam of light OB from the objective is reflected internally at B and C by a right-angled prism. In this way, its direction is reversed and it travels back to a second right-angled prism which is placed at right angles to the

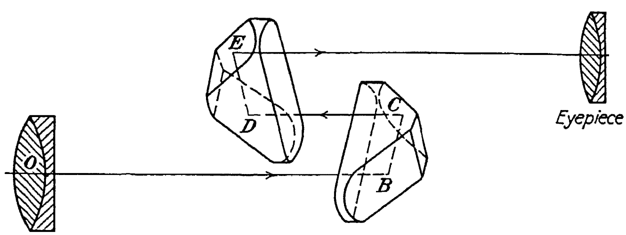


FIG. 624.—Prism binoculars.

first one. Here it is again twice reflected internally, at D and E, and then passes through the eyepiece. The reflections at B and C interchange the two sides of the image so that it is no longer perverted like the image in an ordinary plane mirror. Reflection by the second prism at D and E makes the image upright. Hence, the image formed by the object after the reflec-

tions by the two right-angled prisms is restored completely to its natural position. The eyepiece then magnifies this image without inversion. Objects viewed with such an opera glass thus appear in their natural position.

Because the ray passes back and forth three times between the focus, the focal length of the objective can be increased, and the magnifying power of the instrument is correspondingly increased.

627. Sextant.—The sextant is an instrument for measuring the altitude of heavenly bodies. It consists of a fixed mirror D (Fig. 625) which is silvered only on one-half of its surface so that an observer looking through the telescope T can see the horizon by means of light traveling in the direction CDT and passing through the unsilvered part of the mirror at D. The second mirror E is mounted on an arm which rotates about E as an axis. It carries a vernier at F by means of which its position can be read on the graduated circular scale. Light from a distant object, after reflection at E, is again reflected from the silvered part of D and passes into the telescope T. The eye observing through the telescope thus receives light simultaneously from two sources, that which has come in the direction CDT and that which has come in the direction $AEDT$. When the mirrors E and D are parallel to each other, the vernier reads zero and the two sources of light lie in the same straight line.

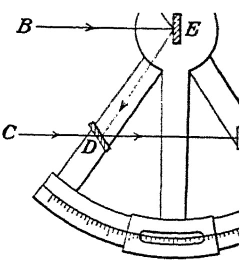


Fig. 625.—Sextant for measuring angular elevation of bodies above the horizon.

To find the position of a star, the mirror E is rotated by means of the arm on which it is mounted until the light from the star after reflection at E and D passes down the axis of the telescope T forming an image of the star which seems to rest on the horizon. At the same time, light is received in the telescope along the line CDT. This light comes from a reflecting surface which serves as an artificial horizon. By observing the angle through which it was necessary to rotate the mirror E from the position in which it was parallel to the mirror D to the position at which it reflects light from the star down the axis of the telescope while the axis of the telescope points in the direction of the horizon, the angle of elevation of the star above the horizon is at once obtained. The angle through which the mirror E is rotated is one-half the angle of elevation of the star above the horizon. The scale of the instrument can be so numbered that it reads the elevation of the star directly. To accomplish this, each degree of rotation of the arm is marked on the fixed scale as 2 deg. of rotation.

628. The Ultra-microscope.—The smallest distance between two points which can just be seen to be separated under an ordinary microscope depends upon the diffraction of light by these two points. The value of this smallest distance ϵ is

$$\epsilon = \frac{\lambda}{2a}$$

where λ = the wave length of the light used.
a = the numerical aperture of the microscope.
= index of refraction $\times$ sine of angle of aperture.

It is not easy to increase the numerical aperture of the microscope over that in use in microscopes at the present time. By decreasing the wave length of the light, the distance between the two points which can just be seen as separated can be decreased. By using ultra-violet radiations and making observations by photographic methods, the minuteness of detail that can be realized by the microscope can be still further increased. There

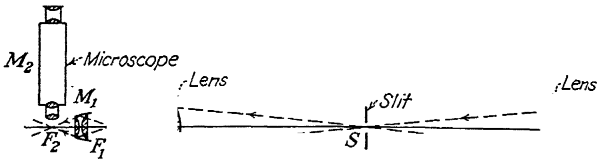


FIG. 626.—Diagrammatic representation of an ultra-microscope.

is even here a limit to the smallness of the objects, the presence of which can be revealed by a microscope.

Tyndall showed that small particles can be seen by passing a beam of light through a solution in which these particles are in suspension. These particles scatter the light in all directions, and by viewing them at right angles to the direction of the light the individual particles become visible. An optical apparatus which makes use of this principle is called an ultra-microscope. Figure 626 shows the essential parts of such an instrument.

A beam of light from the arc lamp is focused by means of the lens O_1 on a narrow horizontal slit S. A reduced image of the slit is formed by means of a second convex lens O_2 at the point F_1. Then a narrow conical beam of light is formed at F_2 by means of a convex lens M_1 of short focal length. If now colloidal gold particles are placed at F_2, the light scattered by the particles can be seen with the aid of the microscope M_2 whose optical axis is at right angles to the axis of the illuminating beam. What one observes is not the individual particles but diffraction disks formed by the light scattered from them. The general background is dark so that these bright spots of light can be seen on it.

629. Optical Pyrometer.—The intensity of the radiation emitted by an incandescent body varies rapidly with the temperature. For example, a small change in the voltage across the terminals of an incandescent lamp produces a relatively large change in the brightness of the filament. This

fact is often made use of in measuring high temperatures. The advantage of this method lies in the fact that it is not necessary to heat any part of the measuring apparatus to the temperature of the body being studied. Such an instrument is called an *optical pyrometer.*

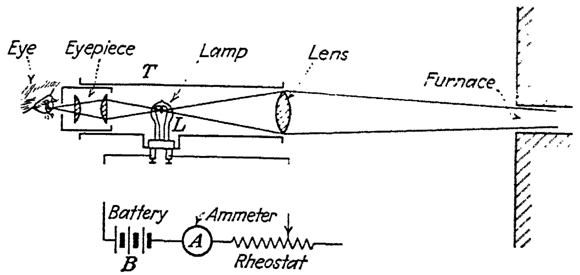


Fig. 627.—Diagrammatic representation of an optical pyrometer.

One of the most convenient forms of optical pyrometer is represented in Fig. 627. It consists of a telescope T with a small 6-volt lamp L mounted in the focal plane of its eyepiece. This lamp is heated to incandescence by means of an electric current from the battery B. When the telescope is focused on the furnace and the filament is lighted, the observer sees a uniform field of illumination with the bright image of the lamp extending across it. If the filament is hotter than the furnace, it appears as a bright line on a darker background. If, on the other hand, the filament is colder than the furnace, it appears as a darker line. When the filament and furnace are at the same temperature, it is not possible to see the image of the filament. By adjusting the current through the filament by means of the regulating rheostat, the image of the filament in the field of the eyepiece can be made to disappear. The current in the filament when the filament is no longer visible is measured by means of the ammeter A. This current then gives a measure of the temperature of the furnace. In order to know the temperature of the furnace in degrees centigrade, the pyrometer must be calibrated by measuring a number of known temperatures and the current in the ammeter corresponding to each of these temperatures. A curve is then plotted connecting these temperatures with the corresponding currents. From this curve an unknown temperature can be found as soon as the current in the filament at which the filament is no longer visible is determined.

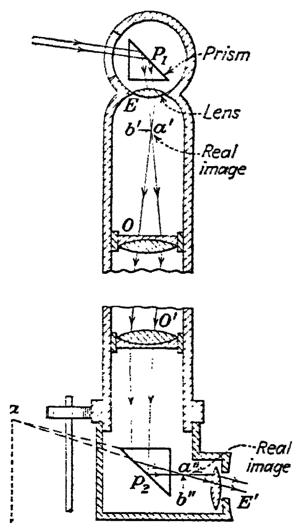


Fig. 628.—A submarine periscope.

630. Periscope.—In the periscope (Fig. 628), the rays from the objects which are being observed fall on a 45-deg. prism P_1 and are reflected downward and then brought to a focus by the lens E. The image thus produced is situated at the principal focus of the lens O so that the rays of light which emerge from O are parallel to each other and nearly parallel to the axis of the tube. After passing through the lens O' and being reflected by the prism P_2, these rays are again brought to a focus at $a''b''$. This image is then magnified by means of the lens E' which forms a virtual image of the object at ab.

631. Anatomy of the Eye.—The eyeball is a hollow sphere of dense fibrous tissue (Fig. 629). In front there is a window, also formed of fibrous tissue, but so modified as to be transparent. Its radius of curvature is much less than that of the remainder of the eyeball. Inside the eyeball is a second layer, consisting of the vascular portion of the eye. It is loaded with pig-

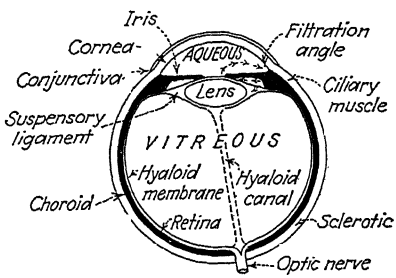


FIG. 629.—Anatomy of the human eye.

ment which performs the function of the black lining of optical instruments and prevents the reflection of light in the interior of the eye. The iris in the human eye contains a circular aperture called the pupil. It is provided with muscular fibers which diminish or dilate the pupil, thus regulating the amount of light which enters the eye. At the back of the eyeball the fibers of the optic nerve spread out to form the retina, which is a sensitive screen on which the images of external objects are brought to a focus. It corresponds to the photographic plate in a camera. Just behind the iris is the crystalline lens. It is a double-convex lens and with its attachments separates the eyeball into an anterior chamber filled with a clear fluid called the *aqueous humor* and a posterior chamber containing a rather more jelly-like substance known as the *vitreous humor*. The anterior chamber is divided into two compartments by the iris. Between these compartments, however, there is free communication through the pupil.

632. The Eye.—Like the camera, the eye can be looked upon as a light-tight enclosure having a lens at one end and a sensitive film made of nerve fibers at the other end. The lens produces on this sensitive screen a real and inverted image of external objects. The eye is provided with means of adjustment by which it is possible for it to produce distinct images in the proper posi-

tion for distinct vision. With a fixed-lens system, only objects at a definite distance from the eye could be sharply focused on the retina. For any other distance the image would be blurred. In the camera and projection lantern, the necessary focusing is done by moving the lens forward or backward. In the eye, instead of altering the position of the lens, the lens has been provided with muscles by which its convexity can be changed in such a way that the desired focus is obtained. In this way, the

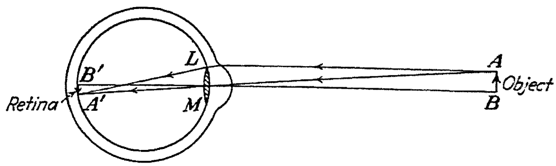


FIG. 630.—Normal eye. The image is focused on the retina.

eye adapts itself to different conditions not by varying the position of the lens with respect to the retina but by changing the focal length of the lens. When the eye receives light from near-by objects, the muscles contract in such a way as to make the lens more convex. When distant objects are viewed, the muscles relax, making the crystalline lens less convex and thus allowing the image to fall on the retina as before. This adjustment of the effective focal length of the lens to form a focus is called the **accommodation** of the eye.

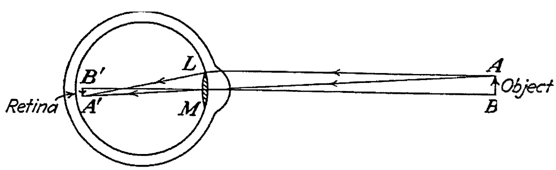


FIG. 631.—Near-sighted eye. The image is focused in front of the retina.

633. Defects of Vision.—The lens of an eye may have too short a focal length, or the eyeball may be too long so that the image formed by the lens falls in front of the retina. Such a condition is represented in Fig. 631. The eye in this case converges the light too rapidly so that the image is formed too soon. Bringing the object closer to the eye causes the image to move back toward the retina and thus it may be made to fall in its proper place. In this way, near-sighted persons find it possible to see distinctly. To correct for this defect, a concave lens

(Fig. 632) is placed in front of the eye. By choosing a concave lens of suitable focal length, the image is made to fall on the retina when the object is at the normal distance from the eye.

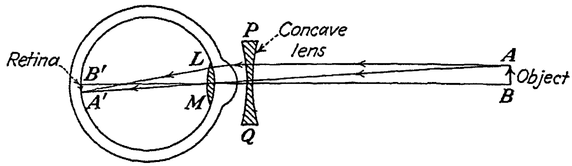


FIG. 632.—Correction for near-sighted eye. Use a concave lens which makes the image fall on the retina.

In the far-sighted eye the image from an object at a normal distance from the eye (Fig. 633) is formed behind the retina, because the crystalline lens is too flat or the eyeball is too short. As the object is moved farther away, the image moves nearer to

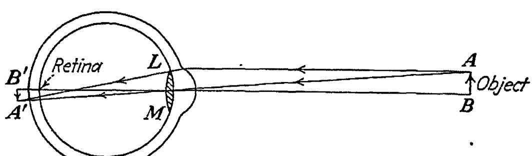


FIG. 633.—Far-sighted eye. The image is focused behind the retina.

the retina and may be made to fall on it. Hence, distant objects can be seen when near-by objects are indistinct. In order to correct for this defect, a convex lens (Fig. 634) is placed in front of the eye. It makes the equivalent focal length of the eye

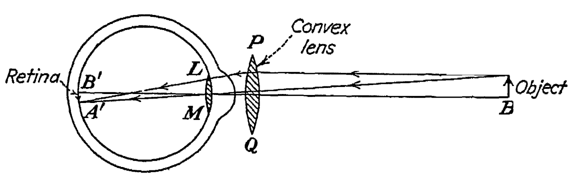


FIG. 634.—Correction for far-sighted eye. Use a convex lens which makes the image fall on the retina.

shorter and thus hastens the formation of the image. By choosing a lens of suitable convexity, the image can be brought on the retina of the eye for objects at a normal distance from the eye.

The distance at which the normal eye sees most distinctly is called the **distance of distinct vision.** This distance is about

25 cm. For greater or smaller distances an attempt to see fine detail causes an excessive strain on the eye.

Example.—A far-sighted person can see objects distinctly at a distance of 30 in. Find the focal length of glasses to be used in order that the same person may read print at a distance of 10 in.

Let f = the focal length of the lens in the eye.
u = the object distance.
v = the image distance.
F = the focal length of the spectacle lens.
F' = the equivalent focal length of the lens of the eye and the spectacle lens.

Then

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{v}.$$

With the spectacle lens before the eye

$$\frac{1}{F'} = \frac{1}{10} + \frac{1}{v}.$$

The equivalent focal length of the lens in the eye and the spectacle lens is

$$\frac{1}{F'} = \frac{1}{F} + \frac{1}{f}$$

and

$$\frac{1}{F'} \qquad \frac{1}{F}.$$

Eliminating :

$$\frac{1}{F'} - \frac{1}{f} \quad \frac{2}{30} \quad \frac{1}{15} = \frac{1}{F}.$$

$$F \quad 15 \text{ in.}$$

634. Astigmatism.—Another common defect of the eye is known as **astigmatism.** This defect occurs when the lens of the eye does not have a truly spherical surface. In such a case the curvature of the crystalline lens is not the same in different planes through the axis of the eye. The focal length of the lens will not be the same in different planes. Rays of light from a vertical luminous wire will not have the same focus in the eye as rays from a horizontal luminous wire. If light from a luminous area is received by the eye, there will be an attempt to adjust the eye so that the light from all parts of the object will form a sharp image. With such an eye it is not possible to make such an adjustment because the lens has different focal lengths for

light in different planes. Consequently, the image formed will be indistinct, and the eye will be strained in forming this image. This defect is overcome by using lenses which have cylindrical surfaces. If these lenses are properly placed with respect to

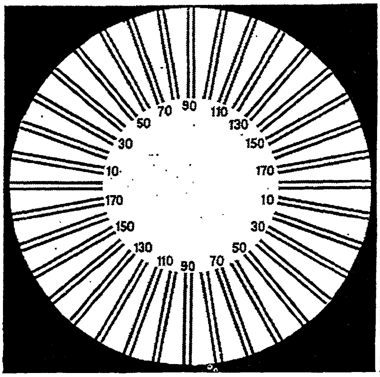


FIG. 635.—Chart viewed by normal eye.

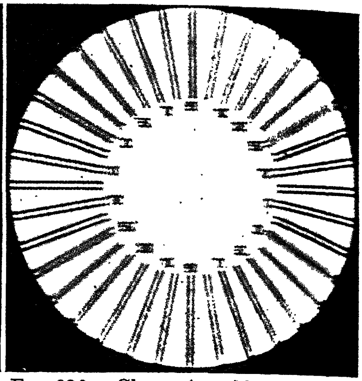


FIG. 636.—Chart viewed by astigmatic eye.

the eye, they combine with the lens in the eye to make a nearly normal lens.

635. Convergence and Divergence.—When the eyes are focused on near objects, there is a movement of the eyeballs causing the visual axes to converge from the normal parallel position toward the middle line (Fig. 637). Such a movement is necessary in order that the images in both eyes may fall on the correct position in the retina, **the spot of distinct vision.** The amount of convergence varies inversely as the distance of the object. In normal circumstances the visual axes never diverge, for they are in parallel adjustment for objects at infinity. Divergence can be brought about by interposing prisms in front of the eyes to render the rays of light divergent. For distinct vision there then results a corresponding divergence of the visual axes.

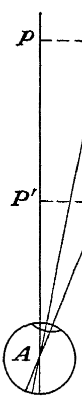


FIG. 637.—Convergence of the optical axes of the eyes.

636. Visual Judgments.—Because of the distance between the eyes, objects are viewed from slightly different angles from each eye. This fact is easily demonstrated by looking at an object first with one eye and then with the other. The result is easily evident when two objects nearly in alignment are observed. Each retina receives a slightly different picture; and, because of this fact, it becomes possible to judge distances and solidity. Objects seen with only one eye have a very flat appearance.

The combination of two slightly different pictures to form an apparently solid object is illustrated in the stereoscope. This instrument consists of two prisms or half lenses, placed with the thin edges inward about as far apart as the eyes. Two slightly dissimilar pictures P_1 and P_2 (Fig. 638) are fixed so that one is in front of each lens or prism. A screen S separates these pictures. When the two pictures are viewed through the instrument there seems to be a single picture at P. It stands out in pronounced relief in marked contrast to the flatness of the separate pictures P_1 and P_2.

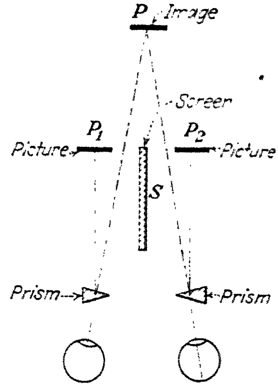


FIG. 638.—The stereoscope. Almost identical pictures viewed at slightly different angles make the picture stand out in relief.

637. Binocular Vision.—When an observer looks at an object, his two eyes receive light from the same point on the object. Experience in judging the angle which the two rays form with each other makes it possible to infer the distance of the object from the observer. A person with only one eye has difficulty in estimating distances. Simultaneous observations with both eyes make it possible to see objects in relief so that they appear in three dimensions instead of in two dimensions. In such cases, one eye receives a view of an object which is slightly different from that received by the other eye. The impressions due to these two views are combined in such a way as to make the object appear in relief.

638. Sensitivity of the Human Eye.—For equal amounts of energy a yellow-green light of wave length $\lambda = 0.56\mu$ appears the brightest to the

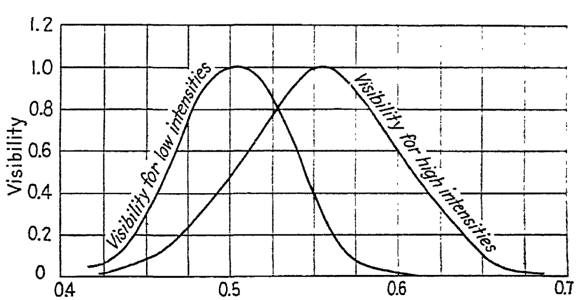


FIG. 639.—Visibility of radiations of different intensities. The maximum visibility depends on the intensity of the light.

normal light-adapted eye. When the light is made fainter, the region of maximum sensitivity shifts toward the green and then to the blue. The dark-adapted eye can see green or blue better than yellow. For this reason, weak lights appear more green or blue than stronger lights of the same energy distribution. This shift in the sensitivity of the eye occurs for

intensities of light which are not less than 0.5 m.-candle or more than 50 m.-candles. Below 0.5 m.-candle and above 50 m.-candles the sensitivity of the eye changes but little with the change in the intensity of the light. This variation of the sensitivity of the eye for lights of different intensities is represented in Fig. 639.

639. Absorption of Ultra-violet Radiation by the Eye.—Since the eye is sensitive to only a small portion of the radiation from a source of light, the question arises as to whether the limits of vision are determined by the wave lengths that are able to affect the retina, or whether the fluids and tissues of the eye actually do not transmit these certain wave lengths. Good vision requires radiation between 0.41 and 0.75μ. If the source of light is sufficiently intense, radiation as far out as wave length about 0.32μ in the ultra-violet, or 1μ in the infra-red, may be seen. It has been found that the combined tissues of the eye absorb the radiations in the ultra-violet when the wave length is shorter than about 0.31μ. Absorption is greatest in the crystalline lens and least in the vitreous humor. Any injury which tends to increase the content of salt in the eye also increases the absorption in the ultra-violet. The absorption of ultra-violet radiations by the eyes of animals does not materially differ from their absorption in the human eye. The limit of vision in the ultra-violet thus seems to be determined by the absorption of the ultra-violet radiations by the tissues of the eye.

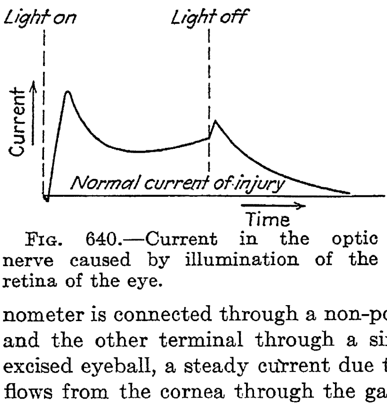


FIG. 640.—Current in the optic nerve caused by illumination of the retina of the eye.

640. Electrical Response of the Retina.—If one terminal of a galvanometer is connected through a non-polarizable electrode to the optic nerve and the other terminal through a similar electrode to the cornea of an excised eyeball, a steady current due to the injury of the nerve and tissues flows from the cornea through the galvanometer back to the optic nerve. This current is called the **current of injury.** If light is now allowed to fall on the retina of this eyeball, this current shows an increase both when the stimulation begins and when it ends. The character of these changes is represented diagrammatically in Fig. 640 where the current through the galvanometer is plotted on the vertical axis and the time on the horizontal axis. It is seen from this curve that when the light first falls on the retina there is a quick decrease in the current of injury followed by a rapid increase to a maximum with a subsequent decrease. When the light is shut off, there is a second sudden rise to a maximum, followed by a decrease to the normal current of injury.

It is seen from these observations that nerve impulses in the rods and cones of the retina and in the optic nerve are accompanied by electrical changes which can be measured by suitable electrical apparatus. These electrical changes are undoubtedly inseparable from the functional response of the conducting tissues, and a knowledge of these electrical variations gives some information with respect to the nerve impulses which go to the brain. It may be that after all vision is an electrical phenomenon.

Problems

1. The lens of an aerial camera has a focus of 9 in. What will be the dimensions on the plate of 1 sq. mile of the earth's surface, photographed from a height of 3 miles?

2. A camera with a lens of 6-in. focal lens is used to take a picture of an object 12 ft. away. How far from the eye must the picture be held so that the image will subtend the same angle at the eye as the object subtended at the lens of the camera?

3. A camera is provided with interchangeable lenses, one of 8-in. focus and the other of 20-in. focus. Find the size of the image of a man 6 ft. tall 20 yd. from the camera, with each of the lenses.

4. A certain subject can be photographed with an exposure of $1/25$ sec. at *f*-11. What would be the proper time of exposure with a lens working at *f*-7.7? (NOTE: The *f*-number is the ratio between the focal length and the diameter of the lens.)

5. A projection lantern is to be placed at a distance of 15 ft. from a screen on which an image with a height of 4 ft. is desired, of a slide 3 in. high. What should be the focal length of the projecting lens?

6. A projection lantern is desired which will throw an image 6 ft. wide on a wall 30 ft. distant, of a slide 3 in. wide. What must be the focal length of the projecting lens selected?

7. A convergent lens used as a reading glass has a focal length of 5 cm. What is the magnifying power of the lens, if it is used so as to have the image at a distance of 25 cm. from the eye, and the lens right at the eye?

8. A simple microscope, with a focal length of 6 cm. and a diameter of 0.8 cm. is placed 1 cm. from the eye, and a scale with millimeter divisions is placed so that its image is 50 cm. from the lens. How many of the millimeter divisions can be seen without moving the eye?

9. A compound microscope has an objective lens of focal length 4 mm., which forms an image 20 cm. from the lens. The eyepiece produces a magnification of 10. What is the total magnification?

10. The objective lens of an astronomical telescope has a focal length of 8 ft., and an eyepiece with a focal length of 1.5 in. Calculate the magnification obtained when the telescope is used for viewing distant objects.

11. The closest distance of distinct vision for a certain eye is 2 m. What kind of lens will permit this eye to see clearly an object which is 40 cm. from the eye? State the power of the lens in diopters.

12. The lens of a camera has focal length of 8 in. It is used to photograph an object which is 20 ft. away from it, and then to photograph an object which is 120 ft. from it. How much must the plate of the camera be changed to secure a focus in the two cases.

13. If a camera with a focal length of 6 in. produces a sharp image of a distant object, how far must the lens be moved to have the camera in focus for an object which is 8 ft. away?

14. When a camera was used to make a picture of an object 9 ft. away, the lens was $4\frac{1}{2}$ in. from the film. What should be the focal length of a lens to be placed immediately in front of the objective lens of the camera so that an object $2\frac{1}{2}$ ft. away from the camera can be photographed without changing the position of the film with respect to the lens?

CHAPTER LV

SPECTRA AND COLOR

641. Separation of Light by a Prism.—When a beam of light passes through a prism, it is bent through a certain angle from the direction in which it was originally traveling. This bending arises out of the fact that the velocity of the light in air is not the same as it is in the material out of which the prism is made. If white light, which is composed of a number of colors, passes through a prism, each color will be bent by the prism. Now it happens that lights of different colors do not travel with the same

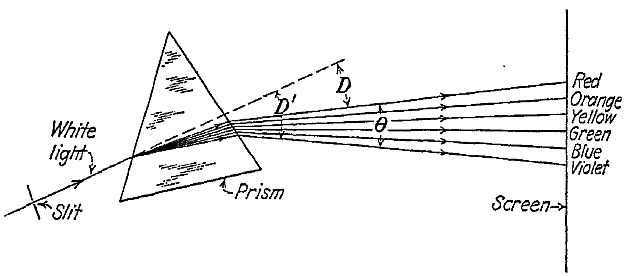


FIG. 641.—Dispersion of light by a prism. Red is deviated less than violet.

velocity in the glass. Hence, these different colors will be bent different amounts on entering and leaving the prism; and, although all of the colors come to the prism in the same direction, they will leave it in different directions because of the unequal bending which they suffer. White light in passing through a prism (Fig. 641) is thus spread out into a band of colors known as a **spectrum.** The violet light which has the shortest wave length is bent most and the red which has the longest wave length is bent least. Light with wave lengths between the red and the violet occupies intermediate positions in the spectrum.

642. The Spectrometer.—For the study of spectra a spectroscope or a spectrometer is used. One of the essential parts of a spectrometer (Fig. 642) is a collimator which consists of a tube with a convex lens *B* at one end and a slit at the other. This

slit is located at the principal focus of the lens so that the light which emerges from the lens when the slit is illuminated is parallel light. There is also a telescope with an eyepiece L and an objective A. This telescope is mounted so that it can rotate about the vertical axis of the instrument. The angle through which the telescope is rotated is read on a divided circle. At the center

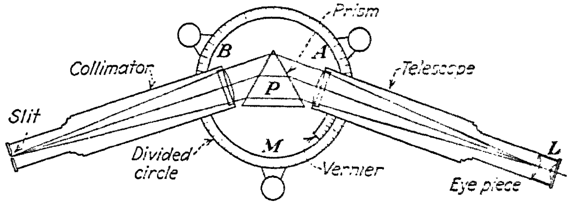


FIG. 642.—Diagram of a spectrometer. The collimator produces parallel rays. The telescope focuses them.

of this divided circle is placed a prism P which disperses the incident light into a spectrum that is observed through the telescope.

643. Achromatic Prism.—When light of more than one color falls on a prism, the emerging beam is not only deviated but

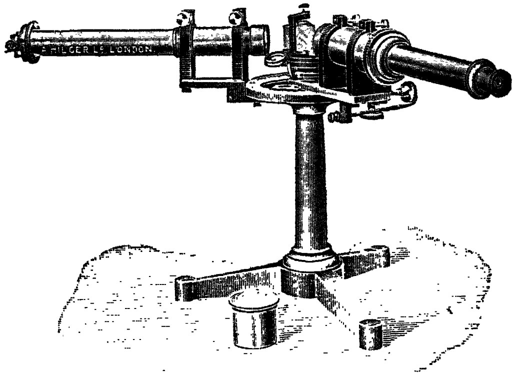


FIG. 643.—A spectrometer. (*Courtesy A. Hilger & Co.*)

spread out into a variety of colors called a spectrum. If a second prism is placed in front of the first one with its vertex to the base of the first one, the second prism tends to gather the different colors together again and combine them into white light. The net dispersion produced by the two prisms is the difference

between the dispersions produced by the individual prisms. The deviation produced by the second prism is in the opposite direction to that produced by the first prism, and the net deviation is the difference between the deviation produced by the first prism and the deviation produced by the second prism. In this way, it is possible to construct a prism which deviates the light without spreading it out into a spectrum. Such a prism is called an **achromatic prism.**

Such prisms are made by combining a prism of crown glass with a prism of flint glass (Fig. 644). The angles of the two prisms are so chosen that the dispersion produced by the crown glass is just equal and opposite to that produced by the flint glass. This is possible because in prisms of equal angles flint glass produces greater dispersion than crown glass does. When the two prisms are placed as in Fig. 644, the net dispersion is zero. On the other hand, the deviation produced by the crown glass is greater than that produced by the flint glass, so that there is a net **deviation without any dispersion.** All the rays emerge parallel to each other and combine to form white light. A consideration of Fig. 645 shows how parallel rays after passing through the prisms and after being split up into a spectrum superpose to form white light again. The rays, however, have all been bent from their original direction.

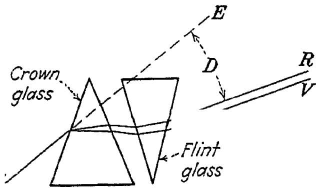


FIG. 644.—An achromatic prism. Dispersion of the prisms the same, but deviations different.

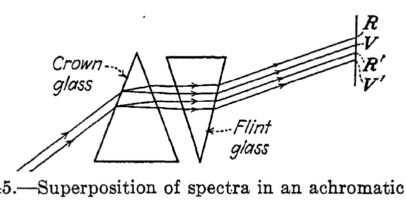


FIG. 645.—Superposition of spectra in an achromatic prism.

644. Dispersion without Deviation.—If the angle of the flint-glass prism is made larger, the deviation which it produces is increased and its dispersion is also increased. When its angle is made large enough to make its deviation equal to that produced by the crown-glass prism, the dispersion of the flint-glass prism

exceeds that produced by the crown-glass prism. When two such prisms are put together the combination produces **dispersion without deviation.** When white light travels through such a prism, it will be split up into a spectrum but its general direction will be unchanged.

This principle is made use of in the construction of a **direct-vision spectroscope.** Such a spectroscope (Fig. 646) is made up

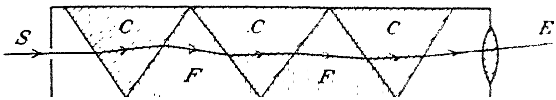


FIG. 646.—Direct-vision spectroscope. Deviations of the prisms the same, but dispersions different.

of a number of prisms of crown and flint glass, arranged alternately. The angles of the prisms are so chosen that the deviation of the combination is zero, allowing the light to emerge in the direction in which it entered. Since the dispersion of the flint glass exceeds the dispersion of the crown glass, the light is spread out into a spectrum. This type of spectroscope is very convenient in spectrum analysis. Figure 647 shows one type of direct-vision spectroscope.

645. Achromatic Lenses.—When white light passes through an ordinary lens, it is not only refracted and brought to a focus

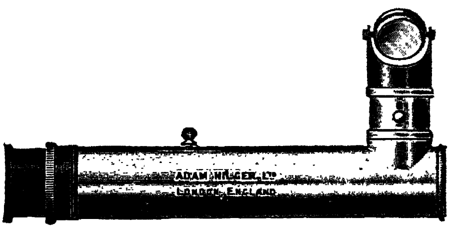


FIG. 647.—Direct-vision spectroscope. *(Courtesy A. Hilger & Co.)*

as in Fig. 648 but is separated into colors just as by a prism. Thus the red rays and the violet rays are bent unequal amounts by the lens and will not, therefore, come to the same focus. There will be one focus for the red rays, another for the violet rays, and a focus for each of the other colors present in the light. Since the violet rays are bent more than the red rays, the focus for the violet light will be nearer the lens than the focus for the red light. Such a lens cannot give a sharp image of an object

illuminated by white light. Because of the unequal bending of the different colors, the image will not be white but will be more or less colored. This defect in a lens is called **chromatic aberration.**

It may be corrected by combining a convex lens of crown glass with a concave lens of flint glass. These lenses are so chosen that the power of one to separate the light is just equal to the power of the other to recombine it. In this way, no separation of the light into colors occurs. On the other hand, the bending or refractive power of one of the lenses exceeds the

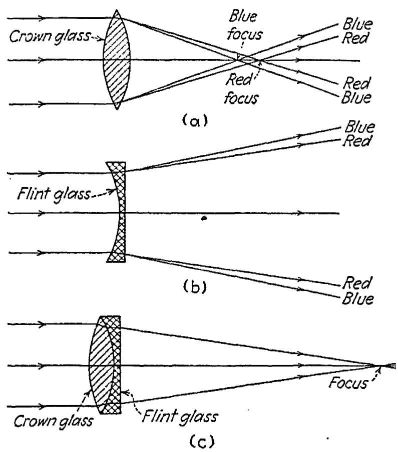


FIG. 648.—(a) Chromatic aberration. The blue rays come to a focus before the red rays do. (b) Blue rays are refracted more than the red rays. (c) An achromatic lens. All the rays focus nearly at the same point.

refractive power of the other; thus, there is a resulting bending of the rays, in spite of the fact that one of the lenses bends the rays in one direction and the other bends them in the opposite direction. Of course, the bending power is not so much as it would be for a single lens by itself. Such lenses which give bending without dispersion are called **achromatic lenses.**

646. Continuous Spectra.—When the spectrum of the light from an incandescent solid or liquid is examined, it is found to contain all colors from red to violet. It shows no regions of darkness such as are found in the line spectrum. Spectra from the filament of an incandescent lamp or from the electrodes of an arc light are spectra of this kind. Such spectra are known as

continuous spectra to distinguish them from line spectra. They contain light of every wave length and, therefore, light of every color. Figure 649 gives a complete chart of the electromagnetic wave spectra. It extends from the very short electromagnetic radiations given off during the transformations of radioactive substances to the electromagnetic waves of importance in wireless

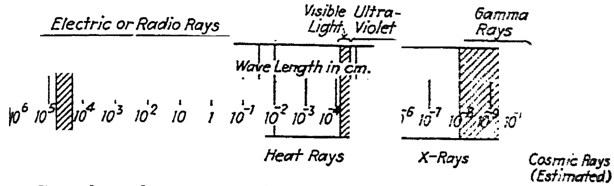


FIG. 649.—Complete electromagnetic spectrum. It extends from very short to very long wave lengths.

telegraphy and telephony. These waves differ only in frequency and wave length. They are essentially electromagnetic pulses.

Kinds of Waves	Limit of Wave Lengths
Waves from oscillatory circuits...	10^4 km. to 10^{-2} cm.
Infra-red radiation..............	10^{-1} to 7×10^{-5} cm.
Visible spectrum................	7×10^{-5} to 4×10^{-5} cm.
Ultra-violet radiation...........	4×10^{-5} to 10^{-6} cm.
X-rays........................	10^{-6} to 10^{-9} cm.
Gamma rays..................	10^{-8} to 10^{-10} cm.
Cosmic rays..................	10^{-11} to 10^{-12} cm.

647. Ultra-violet and Infra-red Spectra.—Besides the visible part of the spectrum which affects the retina of the eye, there are wave lengths which are too short and others which are too long to affect the eye. The former are known as ultra-violet light and the latter as infra-red radiation. The ultra-violet is most easily detected by its action on a photographic plate. It also produces a number of chemical effects. The infra-red or heat rays are studied by allowing them to be absorbed and noting the heat effects which they produce.

Since glass readily absorbs infra-red radiations, it is not possible to use glass prisms or glass lenses on a spectrometer which is used to study the longer wave lengths in the spectrum. In an infra-red spectrometer a rock-salt prism is used instead of a glass prism, and the lenses are replaced by concave mirrors which are silvered on the front surface so that the radiation does not penetrate into the mirror but is reflected at its surface. Figures

650 and 651 show a constant-deviation instrument of this type. The light entering the slit S_1 is rendered parallel by the mirror M_1 and then passes through the rock-salt prism to the plane

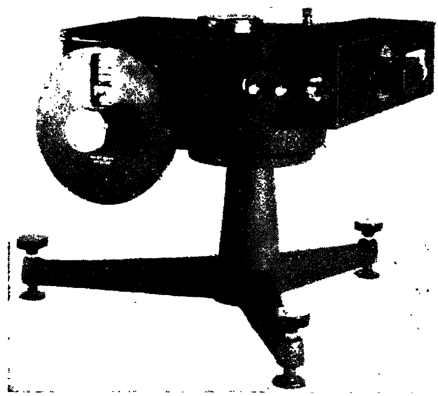


FIG. 650.—Infra-red spectrometer. (*Courtesy Gaertner Scientific Corporation.*)

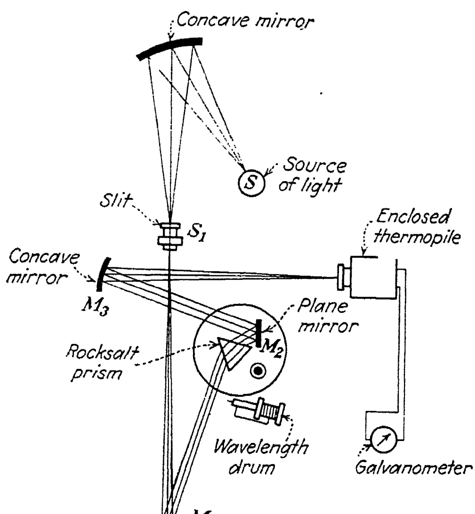


FIG. 651.—Diagrammatic representation of infra-red spectrometer.

mirror M_2 where it is reflected to the concave mirror M_3. This second concave mirror reflects the radiation so that it is brought to a focus on a linear thermopile which is immediately behind a

narrow slit. This slit and the thermopile are enclosed to prevent stray radiation from reaching the thermopile. The prism and plane mirror are mounted on a table which is rotated by a fine screw carrying a drum on which the wave lengths can be read directly. This instrument can be used for wave lengths up to about 10μ.

When the ultra-violet region of the spectrum is studied, the lenses and prisms of the spectrometer must be made of quartz because glass absorbs the ultra-violet radiations. A quartz

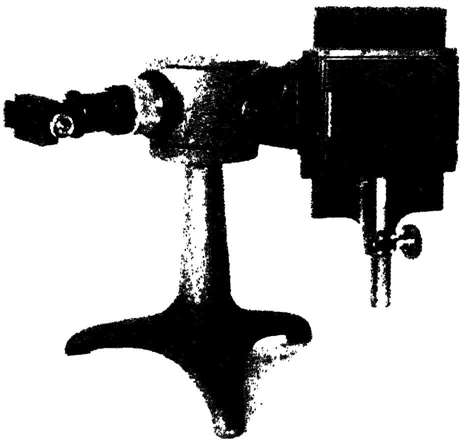


FIG. 652.—Small quartz spectrograph. (*Courtesy Gaertner Scientific Corporation.*)

spectrograph which can be used between 6000 and 2100 Å. is shown in Fig. 652.

648. Bright-line Spectra.—If a solution of some salt, like sodium chloride, potassium chloride, or lithium chloride, is introduced into the non-luminous flame of a Bunsen burner the flame emits light which is characteristic of the salt that has been introduced into it. If the light thus emitted is examined with a spectroscope, it is found to consist of a number of narrow bright lines with wave lengths which are characteristic of the element which emits them. In the case of lithium, it is red light.

When a substance like copper, zinc, or iron is introduced into an electric arc, the metal is vaporized and the spectrum of the vapor is emitted. The lines which are characteristic of the spectrum of carbon are also present but can be distinguished from the spectrum of the element introduced into the arc. Figure

653 shows the arc and spark spectrum of potassium and Fig. 654 the spark, arc, and furnace spectra of gadolinium.

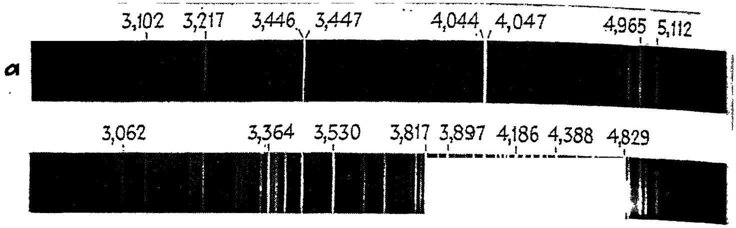


FIG. 653.—Arc and spark spectrum of potassium. (*Foote and Mohler.*)

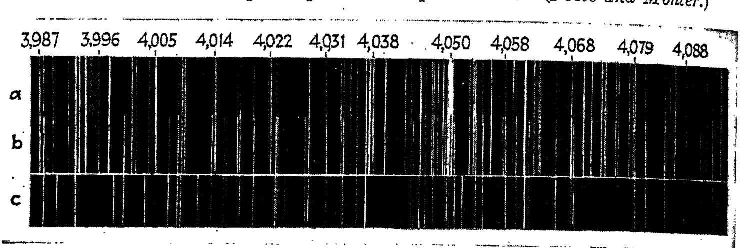


FIG. 654.—Spectra of gadolinium. *a.* Spark spectrum. *b.* Arc spectrum. *c.* Furnace spectrum. (*A. S. King.*)

Another way of exciting the spectrum of an element which exists in the form of a gas or a vapor is by means of an electric discharge between terminals enclosed in a glass tube (Fig. 655). The gas or vapor to be excited is enclosed in the tube at a reduced pressure. When a sufficiently intense discharge from the terminals of a small transformer or an induction coil passes through the gas or vapor, the line spectrum of the element enclosed in the tube is emitted. To increase the intensity of the light which is emitted, the central portion is a capillary tube in which the discharge is more concentrated and the intensity of the illumination is therefore increased. The spectrum of helium (Fig. 656) is a good illustration of this type of spectral excitation. (See Plate I.)

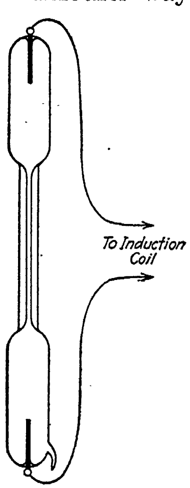


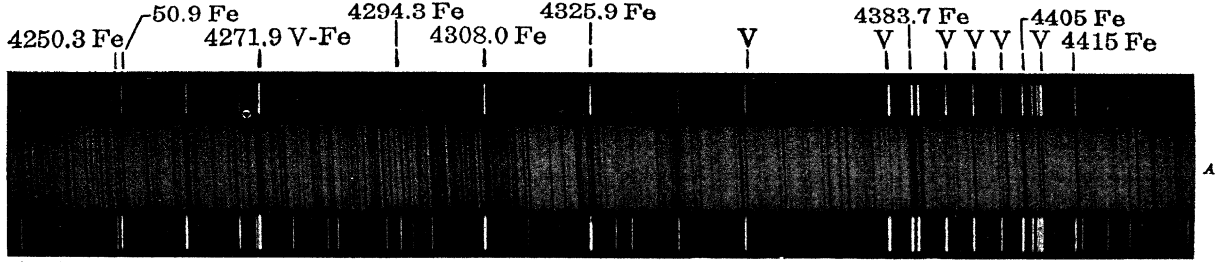
FIG. 655.—Simple spectrum tube.

Such spectra are known as **bright-line spectra.** Each substance has its own characteristic spectrum which is not

Spectrum of Jupiter

The slant of lines is due to Doppler effect

Comparison spectra are from iron-vanadium spark.



The slit of the spectrograph was over Jupiter's equator, and as his axial rotation brings the east half of his disk toward us and the west half away, his dark lines in the upper part of spectrum are shifted toward the violet and in the lower (west) part toward the red, longer wave-lengths. Such a marked slant of the dark lines means a very rapid rotation, *i.e.*, a short day for Jupiter, which is known to be $9^h\ 50^m$. Thus the spectrograph can tell us the rotation periods of the planets. (*By V. M. Slipher, Lowell Observatory, Flagstaff, Ariz.*)

Spectrum of Ball and Rings of Saturn

The slant of lines is due to Doppler effect.

Comparison spectra are from iron-vanadium spark.

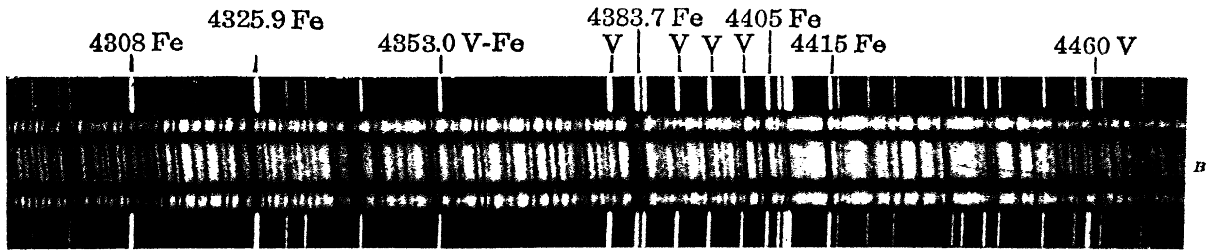


Plate III.—The slit of the spectrograph was over the planet's equator. The upper part of the planetary spectrum is from the eastern part of Saturn and the lower from the western part. Hence the dark lines of the planet and rings are shifted relative to the bright comparison lines, toward the shorter wave lengths above and, oppositely, redward below, in consequence of the eastern portion of the system approaching the earth and the western portion going away owing to rotation of the ball on its axis and the revolution of the rings about the planet. It will be noted that the rings are moving more rapidly on their inner edges, proving that they are multitudes of small moons streaming around the planet, those nearer to the planet of necessity having the higher speeds. (*By V. M. Slipher, Lowell Observatory, Flagstaff, Ariz.*)

SPECTRUM OF PROCYON

Showing the Doppler velocity shift.
Comparison spectra are from iron-vanadium spark.

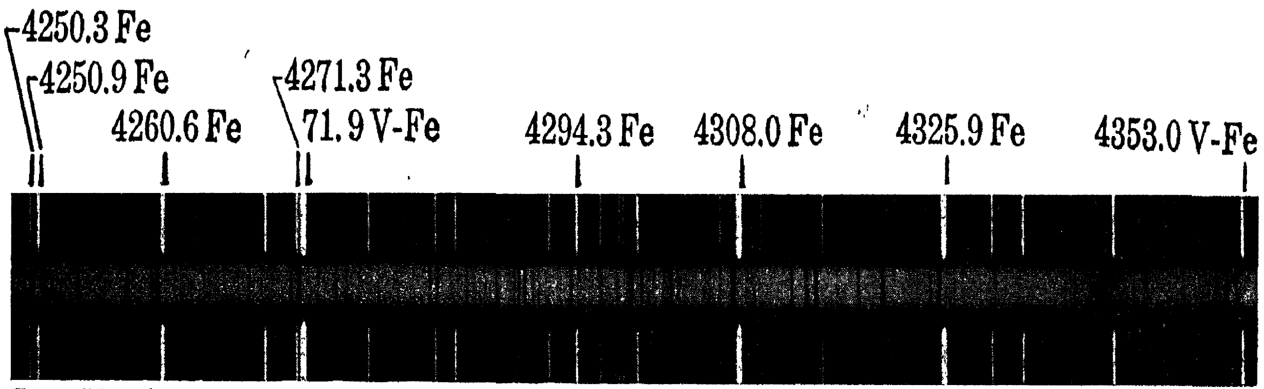


PLATE IV.—The central (light) band with its dark vertical lines is a record of the spectrum (in the indicated region of wave lengths) of the star Procyon. The bright lines above and below the star spectrum are due to the spark between terminals of iron and vanadium. To many of the dark lines in the star are corresponding (bright) comparison lines—chiefly of iron—but between the two there is a slight shift. This is due to the fact that the star and the earth were approaching one another with a relative velocity of about 30 km. second. (*By V. M. Slipher, Lowell Observatory, Flagstaff, Ariz.*)

exactly like that of any other substance. With great resolution, each individual line may split up into a number of lines so that what appears to be a single line in a spectroscope of low resolving

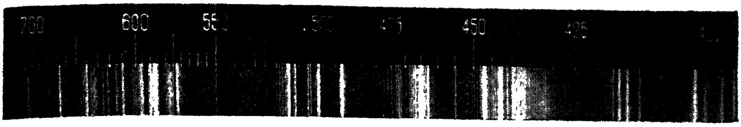


FIG. 656.—Spectrum of helium. (Goldstein.)

power may actually be a number of lines lying very close together. Figure 657 shows how a single line of praseodymium, having a wave length 4,578, splits up into six components which are very close together, so close that all six of them lie in a space of about 0.3 Å.

The analysis and interpretation of these spectra will be considered in Sec. 734.

649. Doppler Effect for Spectral Lines.—In Sec. 244 it was seen that in the case of sound, when the observer or the source of sound is moving, the apparent frequencies of the sound waves are changed. A similar effect is observed on the frequencies and wave lengths of light waves. This effect, known as the Doppler effect for light waves, gives a valuable means of determining the motions of distant luminous bodies like planets or stars. The motions of these bodies produce shifts in the wave lengths and the frequencies of the spectral lines which are being emitted. By measuring the displacements of these lines, the speed and direction of motion of the moving planet or star can be determined. Plates III and IV give some of the interesting results which can be obtained by this method of analysis.

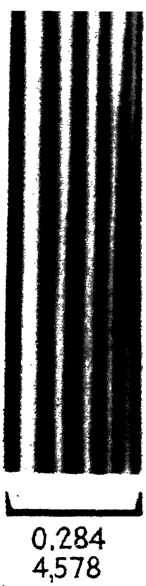


FIG. 657.—Resolution of a single spectral line into its components.

650. Spectrum Analysis.—Since each substance has a spectrum which depends entirely on the nature of the substance, an examination of the light emitted by a substance gives direct evidence of the composition of the substance. When the bright-line spectrum of a substance is definitely known, it is possible to

conclude that the substance is present whenever its spectrum is found. Thus suppose that the spectrum of calcium is known, and that there is now introduced into the colorless flame of a Bunsen burner an unknown salt. If the spectrum of calcium is now found in the spectroscope, it follows at once that calcium was present in the salt. This method is very useful in detecting small quantities of a substance. It is a very rapid and a very sensitive method. The least trace of sodium in a Bunsen flame is sufficient to give the sodium lines. Because of its great sensitiveness this method of analysis has made possible the discovery of a number of elements which were present in such small quantities that they could not be detected by other methods. Caesium, rubidium, thallium, indium, and gallium were discovered in this way.

651. Origin of Light Waves.—In later sections there will be found a discussion of the atomic and molecular processes which are responsible for the emission of spectral lines. Energy changes which produce a bright-line spectrum are due to the rearrangement of the electrons within the atom so that the positions and intensities of these lines are determined in each case by the characteristics of the atom. The lines found in band spectra are produced by molecular agitation.

652. Color of Bodies.—When a piece of red glass is held in a beam of white light so that light passes through the glass, all wave lengths are absorbed except those that together produce what we call red light. In the same way, a piece of blue glass, placed in the path of a beam of white light, transmits only those wave lengths in the spectrum which are characteristic of blue or violet light, the remainder being absorbed. If both pieces of glass are inserted in the beam of white light at the same time, the entire spectrum may be almost completely absorbed, the red glass absorbing all the wave lengths except those in the red end of the spectrum and the blue glass all the wave lengths except those in the blue end of the spectrum. The color of a body is determined by the wave lengths of the light which the body does not have the power to absorb. If a body when it is illuminated with white light does not absorb any of this light but transmits or reflects all wave lengths in equal measure it appears to be white. If, however, the same body were illuminated with red light, it would appear red because it would reflect or transmit only red

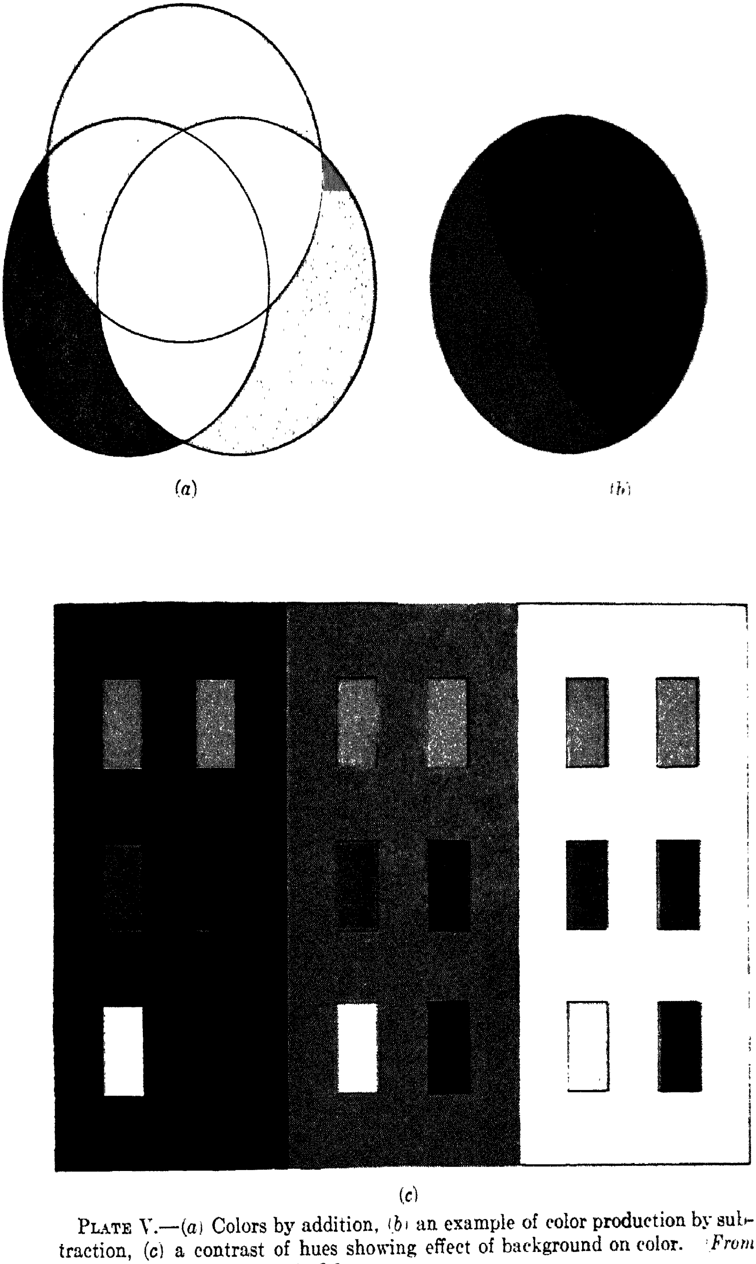
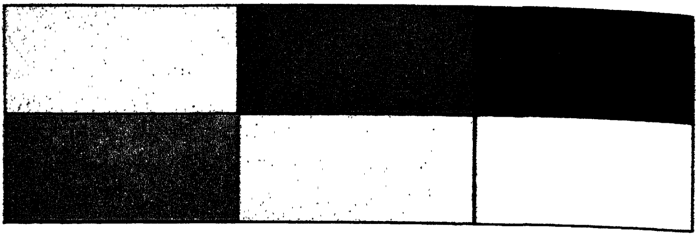


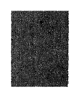
PLATE V.—(*a*) Colors by addition, (*b*) an example of color production by subtraction, (*c*) a contrast of hues showing effect of background on color. (*From Physics of the Home, by F. A. Osborne.*)

(a)



(b)

(c)



(d)

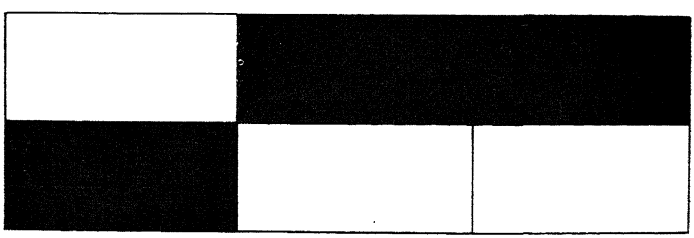


PLATE VI.—(*a*) Complementary colors, (*b*) print from a blue-filter negative, (*c*) print from a green-filter negative, (*d*) print from a red-filter negative. (*Courtesy Eastman-Kodak Company.*)

light to the eye. Since no other colors would be present to be reflected, the color of the body would be completely determined by this reflected or transmitted red light. If a body appears red in daylight it is due to the fact that it reflects or transmits the red rays more abundantly than it does the other light; in other words, it absorbs the lights with wave lengths other than the wave length of the red light and reflects red light. Similarly, a green object is one which reflects or transmits green light and absorbs light of other wave lengths. An object which is red in white light will also be red in red light, but it will be black in green light or in any other light which it does not reflect or transmit. A body which appears white in white light will be red in red light, green in green light, etc., since a white body has the power of reflecting or transmitting all colors of the spectrum to an equal degree.

Bodies which seem to have the same color when viewed in daylight may not have the same color when viewed in lamp light. Light from an ordinary incandescent lamp is richer in red rays than in blue rays. An attempt is made to correct this defect in some of the so-called "daylight lamps" in which a blue glass surrounding the lamp reduces the red and yellow radiation until the distribution of energy is nearly the same as it is in sunlight.

653. Compounding Colors.—When two or more distinct colors are superposed on each other, the resultant color differs from either of the colors used to produce it. For example, if red and blue are superposed, the resultant color is purple. The resultant color also depends on the relative intensities of the component colors. If all the colors which are present in the spectrum of white light are recombined in the same proportions as they were originally present in the light, there results white light. If one of the parts of the spectrum is removed by absorption all the other colors are combined, but the result is not pure white light (Plate V).

The effect of compounding two or more colors may be shown by means of a color disk. The disk consists of sectors of different colors. The disk is rotated about its axis so rapidly that the effect on the eye is precisely the same as if all the colors were seen at the same time. If one half of the disk is red and the other half green, the disk appears to be yellow. If one half of the disk is green and the other half violet, the disk appears to be blue. If one third of the disk is red, one third blue, and one third green, the disk appears to be grayish white. To the eye these colors appear to be the same as those observed in the spectrum. As a matter of fact they are quite different. The colors observed in the spectrum consist of a single wave length. The colors produced by mixing colors may consist of several wave lengths mixed in such proportions that they produce the same effect on the eye as the light of a single wave length in the spectrum.

654. Mixing Pigments.—A clear distinction must be made between mixing colored lights and mixing colored pigments. When yellow light and a proper shade of blue light are combined in a suitable ratio of intensities, the result will be a white sensation. The mixing of lights of different colors is an additive process. If a pigment which is "yellow" because it absorbs the blue light, transmitting other colors to some degree, is mixed with a pigment which is "blue" because it absorbs red light, transmitting other colors, the mixture is a "green" pigment, that is, a pigment which absorbs red and blue, transmitting a green mixture. The colors of pigments are thus determined by the light which they absorb. Each pigment subtracts certain colors from white light. The color of the mixture is, therefore, the color which has not been absorbed by either pigment.

655. Absorption of Light by Liquids, Solids, and Gases.—Numerous liquids and solutions absorb different parts of the spectrum. One liquid or solution may absorb the red and the violet, while another may absorb all colors but the green. Figure 658 shows the absorption spectrum of an aqueous solution of cobalt chloride at six different temperatures: 5, 18, 38, 52, 64, and 83°C. The absorption spectrum of cobalt chloride is characterized by two wide bands in the visible spectrum and a band in the ultra-violet. As the temperature rises, these bands widen slightly.

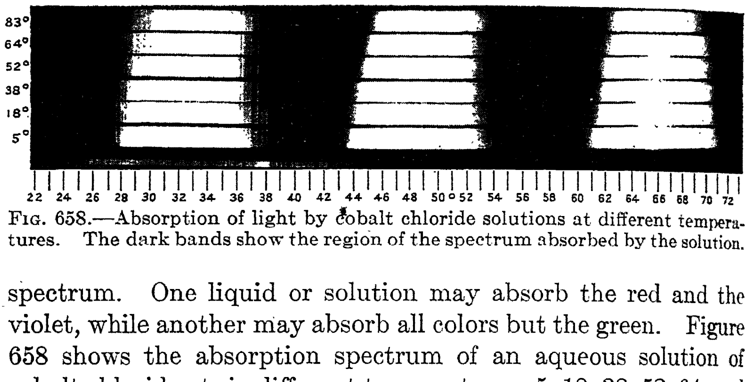


FIG. 658.—Absorption of light by cobalt chloride solutions at different temperatures. The dark bands show the region of the spectrum absorbed by the solution.

Ray filters such as are used by photographers to prevent overexposures of the sky consist of some substance which absorbs some of the blue and violet light from the sky so that it will not produce an overexposure of the photographic plate. In this way, an exposure sufficiently long to secure the landscapes and clouds may be obtained without overexposing the sky.

Where solids and liquids absorb light, the absorption usually extends through a wide range of wave lengths. Gases and vapors, on the other hand, unless they are very dense, absorb certain definite wave lengths and transmit quite freely wave

lengths which are slightly longer or slightly shorter. The absorption spectrum of iron vapor (Fig. 659) shows that the absorption lines correspond to the emission lines.

656. Absorption Spectrum of Blood.—When a solution of oxyhemoglobin of moderate strength is examined with a spectroscope, two well-marked

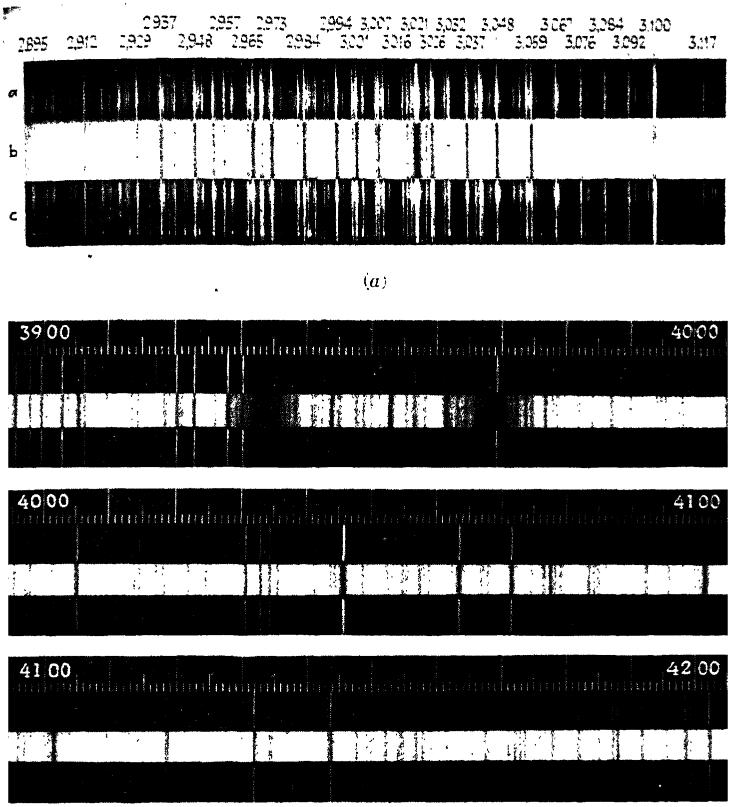


FIG. 659.—(*a*) Absorption spectrum of iron vapor, with comparison spectrum of iron above and below. (*A. S. King.*) (*b*) Spectrum of the sun and a comparison spectrum of an iron arc. The region covered extends from λλ3,900 to 4,200. (*Courtesy Mt. Wilson Observatory.*)

absorption bands are seen in the visible part of the spectrum. One of these bands has a wave length a little shorter than the Fraunhofer *D*-line and the other has a wave length a little longer than the Fraunhofer *E*-line. There is a third absorption band in the extreme violet between the Fraunhofer *H*- and *G*-lines. The position of this band can be located by means of

photography. The addition of a reducing agent, such as ammonium sulphide, causes the bands in the visible to disappear and to be replaced by a less sharply defined band with its center about equidistant between the Fraunhofer *D*- and *E*-lines. A knowledge of the absorption spectrum of hemoglobin and its derivatives is important in physiology.

657. Fraunhofer Lines.—An examination of the spectrum of the sun shows that it is crossed by a large number of fine dark lines. These lines, of unusual interest to physicists and astronomers, are known as **Fraunhofer lines** from their discoverer. These lines are produced by absorption in the atmosphere of the sun. The central part of the sun consisting of incandescent

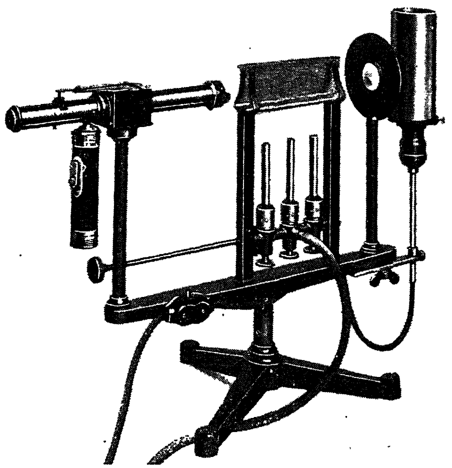


FIG. 660.—Spectroscope for showing absorption lines in vapors. (*Courtesy Eastern Science Supply Co.*)

solids or liquids emits white light. Surrounding this central part is an atmosphere made up of vapors and gases which come from the central part. These vapors and gases constituting the atmosphere of the sun are at a much lower temperature than its central part from which the light comes. When light comes from the sun and passes through this atmosphere, the wave lengths corresponding to the light which would be emitted by these luminous vapors is absorbed by them. Thus, if sodium, iron, and copper are present in the sun, vapors of these elements are present in its atmosphere. White light passing through the atmosphere containing these vapors will lose those wave lengths or colors which sodium, iron, and copper emit when

luminous. These wave lengths will, therefore, be missing when the solar spectrum is examined (Plate VI, 1). In this way, it has been possible to get a good idea of the elements which compose the sun and it has been found that there are present in the sun the same elements which are present on the earth. Indeed one element, helium, was discovered on the sun by this method before it was found on the earth (see Plate I). A convenient spectroscope for observing emission and absorption spectra as well as Fraunhofer lines is the one shown in Fig. 660.

658. Absorption Spectrum of Chlorophyll.—The absorption spectrum of a green leaf shows that light of wave length less than 5000 Å. is absorbed to the greatest extent. The amount of the absorption becomes greater as the wave length becomes shorter. There is also a well-marked absorption band in the red portion of the spectrum between 6500 and 6580 Å. The maximum of the energy in solar radiation is in the neighborhood of 6600 Å., so that the green leaf is able to utilize the light which has a wave length less than about 5000 Å. and those radiations in the red end of the spectrum with wave lengths between about 6500 and 6700 Å. Radiations with wave lengths between about 5000 and 6500 Å. and radiations with wave lengths longer than about 6700 Å. are not utilized by green leaves.

The physical and chemical changes which are brought about in the leaves by the absorption of light are not yet understood. The mechanism of converting solar radiation into the energy stored up in the leaves is not at all simple, but certain facts seem established. Elements taken from the environment of the plant are built into organic compounds under the action of solar radiation. This result is easily tested by screening a portion of a leaf from the light, but leaving a certain portion exposed to the light, and then observing any differences between the normal and the darkened portions. The part exposed to light will be rich in starch, while the screened portion is free from starch. It has also been shown that the maximum evolution of oxygen takes place where the absorption of light is a maximum and the formation of starch is a maximum. The energy necessary for the formation of these organic compounds from water vapor and carbon dioxide is supplied by the absorption of solar radiation. This is a most important process by means of which solar radiation is made available for the performance of physical work for man.

659. The Young-Helmholtz Theory of Color Vision.—According to the Young-Helmholtz theory of color vision there are three sets of nerves terminating in the rods and cones of the retina of the eye. Each of these sets responds to one of the three primary colors: red, green, blue-violet. According to this theory, any resulting color sensation from light incident on the retina of the eye depends on the relative intensities of these primary sensations. The sensation of red is thought to be excited more or

less by all wave lengths in the visible spectrum but most strongly by red or orange light. Similarly, the curve marked "green" (Fig. 661) exhibits the relative intensities of the green sensation produced by lights of different wave lengths. The other curve marked "blue" shows how this sensation varies with the wave length of the light received by the eye.

In a normal eye which possesses all three of these types of nerve centers, a given wave length of light excites three sensations to different degrees. If the wave lengths are long, the red sensation predominates, and if the wave lengths are short, the blue-violet sensation predominates. The intensity of each sensation is proportional to the ordinates of the curves in Fig. 661, at

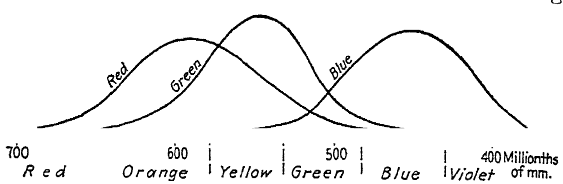


FIG. 661.—Young-Helmholtz theory of color vision.

the point corresponding to the wave length of the light which is incident on the eye.

There is no evidence from the anatomy of the eye that these three types of nerve centers exist in the retina of the eye. The theory, however, accounts for many of the essential facts in color vision.

660. Color Blindness.—The Young-Helmholtz theory of color vision accounts for the abnormal color sensations of color-blind persons. According to this theory a color-blind person is one in whom one or possibly two of these sets of color nerves are missing. A person color blind in the red is one in whom the red nerves are insensitive. The eye of such a person will respond only to the green and blue-violet sensations. Such an eye will not be sensitive to light with wave lengths which lie in the red end of the spectrum. In case the person is color blind in the green, the nerves which respond to the green light are insensitive.

661. Effects of Ultra-violet Light on Living Matter.—That radiations affect living cells is evident from sunburns, snow blindness, sterilization, and in many other ways. Ordinary visible radiation may kill some kinds of germs, but in general it is the ultra-violet radiations which are effective Such modern artificial sources as the quartz mercury-arc lamp, the carbon

arc, and the flaming arc give ultra-violet radiations which have a powerful germicidal action. The sterilization of water by means of ultra-violet radiations from a quartz mercury-arc lamp has now become an important method for purifying water in swimming pools. Various applications of ultra-violet light have been made to the sterilization of milk, butter, and certain fats.

The skin and blood of the human body are somewhat transparent to light rays. When light rays, especially those in the ultra-violet region of the spectrum, fall on the body, certain physiological effects are produced. Sometimes these effects are beneficial and sometimes harmful. Because of the absorption of the air, the spectrum of the sun stops at about 0.293μ. Sources of light such as the mercury-arc lamp and the iron arc which emit ultra-violet radiations of wave length shorter than 0.300μ cause painful inflammation of the eyes and skin. To make such sources of light safe, it is only necessary to place a piece of window glass in front of them. The injurious radiations are absorbed by the glass to such an extent that they cease to be injurious except in the case of very intense sources of radiation. In electric welding, it is desirable to wear spectacles made of a special kind of glass so that they absorb more of the light than is absorbed by ordinary glass.

662. Luminescence.—A body which emits light because it is maintained at a high temperature is said to be *incandescent*. There are, however, bodies which give off light at temperatures much below the temperature at which they become incandescent. In such cases the light is stimulated by some other means than heat. Such bodies are said to be *luminescent*. To this group of bodies belong all luminous organisms. Luminescence may be produced in a body in a variety of ways, and for this reason it is convenient to distinguish different types of luminescence according to the way in which the light is stimulated. Some substances begin to emit light of shorter wave lengths than red at temperatures well below 525°C. Diamond, marble, and fluorite are examples. Some crystals of fluorite when heated in an iron spoon will give off white light long before any trace of redness appears in the spoon. Other crystals will luminesce in hot water. In all such cases the luminescence is dependent on a previous illumination of the body. The power of a body to emit light at temperatures less than incandescence is known as *thermoluminescence*.

Many substances have the power to absorb radiation and afterward emit it as light. Such bodies are said to be *fluorescent* when they give off light only during the time the radiation is incident upon them. If the body continues to emit light after the stimulating radiation is removed, it is said to be *phosphorescent*. The distinction between fluorescent and phosphorescent bodies is purely arbitrary. Some bodies give off light for only a fraction of a second after being illuminated. Other bodies which fluoresce at ordinary temperatures are phosphorescent at low temperatures. Examples of substances which become phosphorescent at low temperatures are salicylic acid, starch, glue, and eggshells. The best known cases of phosphorescence at room temperature are the alkali earth sulphides, barium sulphide, calcium sulphide, and strontium sulphide. A rise in temperature

increases the intensity of the phosphorescent light but decreases the time of its duration. Fluorescence is most efficiently excited by cathode rays in a vacuum tube. When the tube is made of sodium glass, its walls glow with a yellow-green light. Diamonds, rubies, and many other minerals fluoresce brilliantly in the path of cathode rays. A luminous paint, frequently used, consists of zinc sulphide with a trace of radium salt. The radium rays which are emitted continuously by the radium salt cause a steady fluorescence in the zinc sulphide. Many solutions show fluorescence in strong lights. This is especially marked in quinine sulphate, mineral oils, eosin, rhodamin, chlorophyll, etc.

CHAPTER LVI

INTERFERENCE AND DIFFRACTION

663. Interference of Water Waves.—If the surface of a dish of mercury is agitated by means of the prong of a tuning fork to which is attached a piece of fine wire which dips into the surface of the mercury, a system of waves is set up. If both prongs of the tuning fork are used instead of one, two systems of waves are produced which travel across the surface of the mercury. At different points, these systems of waves reinforce or neutralize each other. They may be projected by means of an apparatus represented in Fig. 662. By means of this apparatus, the waves are produced in a surface of mercury and projected on a screen. Light from a lamp at L is reflected by means of a mirror M and illuminates the surface of the mercury where the tips of the tuning fork dip into it. The light reflected from the surface of the mercury passes through the lens S and is reflected by the mirror N to form an image on a distant screen. The resulting circular ripples go out from the points at which the prongs of the tuning fork are in contact with the surface of the mercury. These ripples move too rapidly to be seen with the unaided eye. Their position may be located by an instantaneous photograph or by projecting them through a stroboscopic disk.

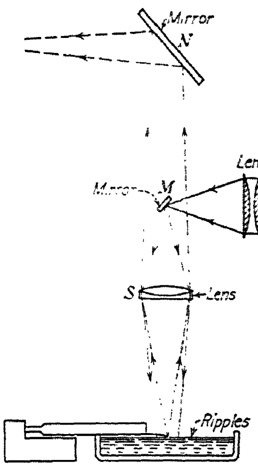


FIG. 662.—Production of ripples in the surface of mercury. The vibrations are produced by a tuning fork.

664. Interference of Light Waves.—Thomas Young showed that when two trains of waves having the same wave length and amplitude of vibration and traveling in the same direction are superposed, they do not always produce increased illumination

but may neutralize each other. The principles of this phenomenon, known as **interference,** are best understood from the original experiment of Young.

Behind a screen containing a small pinhole S (Fig. 663) is placed a source of light. A second screen having two small holes A and B is placed in front of the first screen in such a way that the openings A and B are equally illuminated by the light from S. If light is allowed to pass through either A or B, there is produced a bright spot on a third screen at D. If, however, light passes through both holes A and B at the same time, there is formed on the third screen a series of bright and dark bands. To see the meaning of this fact, consider in detail Fig. 663. Let DM be a perpendicular to AB at a point M midway between

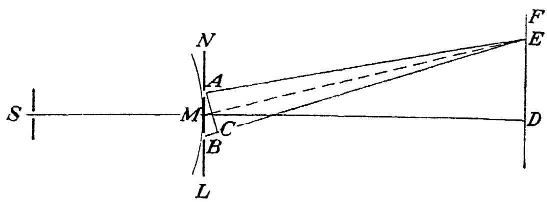
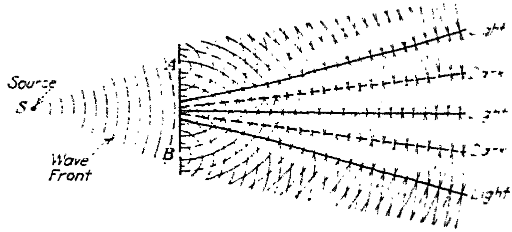


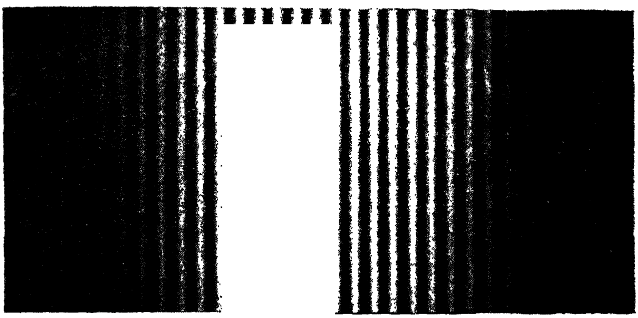
FIG. 663.—Young's interference experiment.

A and B. Since D is equidistant from A and B, light leaving A and B at the same time will reach D at the same time and be in phase. Hence, the waves from A and B will coincide and, therefore, reinforce each other. The illumination at this point will be a maximum. If some point E above or below D is chosen, the light from A in reaching E will travel the distance AE, and the light from B will travel the distance BE. Since BE is greater than AE, the light from B will arrive later than the light from A. If E is chosen far enough from D that BC, the difference between BE and AE, is equal to one-half wave length of light, then the light from A will be ahead of the light from B by one-half wave length or one-half period of vibration. When the waves coming from A produce a displacement in one direction, those coming from B will produce a displacement in the opposite direction. The two waves will thus neutralize each other and so produce no light at E. If E is chosen still farther away from D so that BC is equal to one whole wave length of light, the light from both sources A and B on reaching the screen will again be in phase and so produce an illumination of the screen at that point.

By choosing points still farther away from D, alternate dark and light regions will be found according to whether BC is equal to an odd number or an even number of half wave lengths of light.



(a)



(b)

FIG. 664.—(a) Interference of light from two identical slits A and B. (b) Interference fringes produced by two identical sources. (*From Fundamentals of Physical Optics by Jenkins and White with permission of authors.*)

Figure 664 describes interference in such a case in terms of the wave fronts from the two holes A and B.

When

$$\begin{aligned} BE - AE = BC &= \tfrac{1}{2}\lambda, \\ &= \tfrac{3}{2}\lambda, \\ &= \tfrac{5}{2}\lambda, \end{aligned}$$

the screen is dark.

When

$$\begin{aligned} BE - AE = BC &= \lambda, \\ &= 2\lambda, \\ &= 3\lambda, \text{ etc.,} \end{aligned}$$

the screen is illuminated.

If, therefore, the pinhole S be illuminated with sodium light, the screen DF, which is normal to the plane of the paper, will be

covered by a series of alternate yellow and black fringes. These fringes start with D which is yellow and extend above and below the point D. If white light is used, the central fringe at D will be white, and on either side of it there will be a set of rainbow-colored fringes. These colors arise out of the fact that the different colors in the white light have different wave lengths and reinforce or destroy each other at different points on the screen. In these rainbow fringes, the edges nearest D are violet and the edges farthest from D are red. This distribution of color indicates that the wave length of violet light is less than that of red light so that the violet rays destroy each other for a smaller path difference than is necessary for longer wave lengths.

Because of the importance of this experiment in showing that light is a wave motion, many modifications of it have been devised. These were designed to remove any criticism of the experiment and make the reality of the interference phenomenon as certain as possible. The simplest of these modifications is one in which Fresnel used a biprism having a very obtuse angle in place of the screen containing the two pinholes. By means of this prism he avoided any disturbance of the light which might arise when it passed through the small apertures A and B. He also replaced the biprism by two mirrors inclined at a small angle to each other. The results were precisely like those observed by Young, and the reality of the interference of light was established.

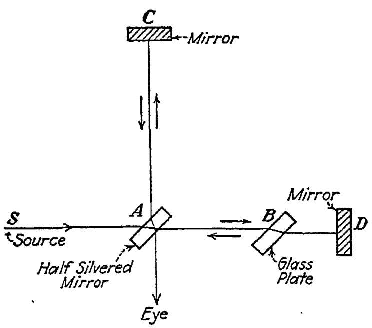


FIG. 665.—Diagram of Michelson's interferometer.

665. Michelson Interferometer.—In this interferometer (Figs. 665 and 666) light emerging from a source S falls on a glass plate A, which reflects part of it and transmits the remainder. The reflected part goes to the mirror C by which it is again reflected and returns along its original path. The part of the original ray from S, which passed through the plate A without reflection to C, passes through a second glass plate B and is then reflected at the mirror D, and later returns to the mirror A by which it is reflected in such a way that its direction coincides with the direction of the ray which was reflected at the mirror C and subsequently passed through the glass plate A. Thus, the ray of light which originally came from the source S has

been split into two rays. Both of these rays are received by the eye: one of them after reflection at C, and the other after reflection at D. The plane-parallel glass plate B is introduced to compensate for the extra thickness of glass through which the ray reflected at C has passed in reaching the eye. In this way, the thickness of glass through which the two rays have passed in reaching the eye is the same.

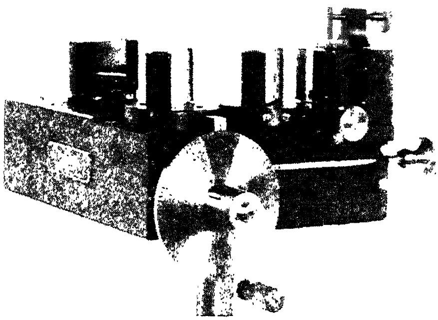


FIG. —Michelson's interferometer. (*Courtesy Gaertner Scientific Corporation.*)

If the distance from the plate A to the mirror C is the same as the distance from the plate A to the mirror D, the two rays of light have traveled the same distance, and they will, therefore, be in phase since they originated by the splitting of the ray from the source S. Under these conditions, the rays will reinforce each other. If, however, the distance AD is greater than the distance AC by a distance equal to one-quarter of a wave length of the light, the two rays will be out of phase and destroy each other. As the mirror D is moved along the line AD, there will be alternate reinforcement and destruction of the light, that is, alternate brightness and darkness. If the eye is replaced by a photographic plate, alternate bright and dark bands or interference fringes will be obtained on the plate. Such a system of interference fringes for a neon line (λ = 5852 Å.) is given in Fig. 667.

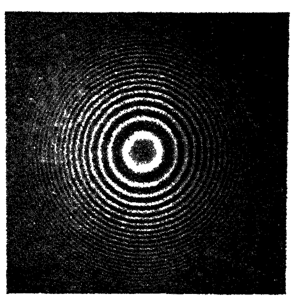


FIG. 667.—Interference fringes from one of the lines of neon (λ = 5852 Å.) (*Babcock.*)

666. Interference in Thin Films.—Consider a very thin film of some transparent body with surfaces parallel to each other (Fig. 668). If one of these surfaces is illuminated by a broad beam of light of a single wave length, light will be reflected equally from the upper and lower surface of the film. An incident ray like AB will be partially reflected from the upper surface, but most of the light will enter the film and a small part of it will be reflected at C. In a similar manner, part of the ray DE will be reflected at E and the

remainder of the light will enter the film. Some of it will be reflected at F, etc. If rays parallel to AB and DE illuminate the upper surface XY of the film, parallel rays EF, GH, etc., will be reflected from the upper surface. There will also be rays which are parallel to EF, GH, etc., which have been reflected from the lower surface MN of the film. The rays reflected from the lower surface MN are superposed on those reflected from the upper surface XY. These two sets of reflected beams have nearly the same brightness but differ in phase, because those which are reflected from the lower surface of the

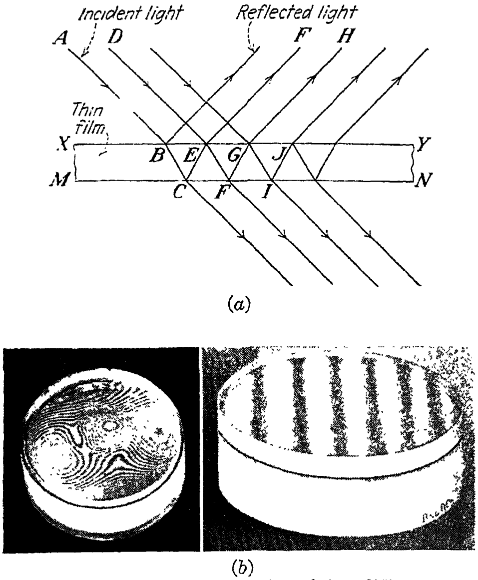


FIG. 668.—(*a*) Inteference of light produced by differences of path in thin films. (*b*) Interference produced by light reflected from nearly plane surfaces of glass separated by a very thin film of air. Parallel fringes indicate surfaces are optically plane. Irregular fringes indicate surfaces are not plane. (*Courtesy Bausch and Lomb Optical Company.*)

film have traveled twice the thickness of the film in excess of the distance traveled by the rays reflected from the upper surface. When a ray after reflection from the upper surface at E, traveling in the direction EF, is one whole period ahead of a ray which after reflection at C emerges from the upper surface of the film at E in the direction EF, these two rays will reinforce each other. If these rays differ in phase by one-half of a period, they will neutralize each other. There will thus be destructive interference when a suitable relation holds between the wave length of the light, the thickness of the film, the index of refraction, and the angle of incidence. The condition for destructive interference does not occur at the same place for different wave lengths. Hence, when the film is illuminated by white light, certain wave lengths will reinforce each other where light of other wave lengths

destroy each other. The result will be a series of colored fringes, giving the appearance of a rainbow (Plate VII, 3).

The colors of thin films of oils are an illustration of this type of interference. When a thin film of oxide is formed on a metal surface, the color of the film is due to interference of light reflected from its upper and lower surfaces. This color will vary with the thickness of the film.

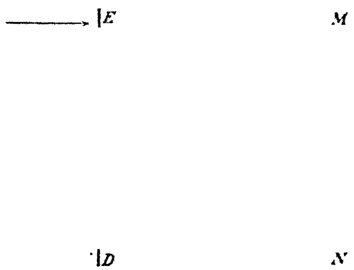


FIG. 669.—Bending of light into the geometric shadow.

667. Diffraction.—When light passes through a small opening, it is supposed not to spread out into the region XY (Fig. 669) but to proceed in straight lines and produce a sharp image on the screen MN. The light does not, however, proceed in exactly straight lines but spreads out somewhat into the region XY. In other words, light bends around the corners of an obstacle in much the same way that water waves bend around the corners of an object. This spreading of a wave motion into the geometric shadow of an object is called **diffraction.** This effect is large in the case of water waves and sound waves but small in the case of light waves. Where the wave length is large, the diffraction is large; where the wave length is small, the diffraction is small.

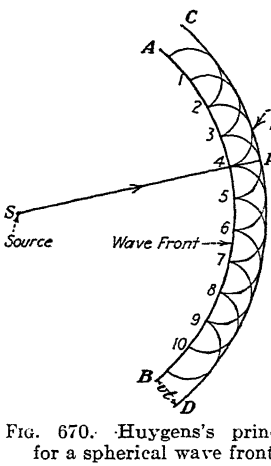


FIG. 670.—Huygens's principle for a spherical wave front.

668. Huygens's Principle.—To account for the fact that light does not travel exactly in straight lines, and for other reasons, Huygens considered that each vibrating particle in the wave front of any wave motion may be considered as a secondary source of spherical wavelets which spread out from their sources with the velocity of the primary wave. The surface which is tangent to all these secondary spherical wave fronts gives the new position of the primary wave front. Consider a source of waves originating at S (Fig. 670). Suppose that these waves at a given time have reached points 1, 2, 3, . . . 10 on the wave

front AB. It is desired to locate the wave front at some later time, t. Consider each of these points 1, 2, 3 . . . 10 as a source of secondary wavelets which spread out with a velocity v. After a time t, each of these secondary disturbances will have spread over spheres of radius vt. Now draw a sphere CD tangent to all of these spherical wavelets. This sphere will give the new position of the primary disturbance, and the line perpendicular to this surface at any point will give the direction of propagation of the wave motion. If the rays of light are parallel to each other, the wave front is a plane (Fig. 671). The way in which the second position of the wave front can be obtained from the first position is indicated in Fig. 670. The distance between these wave fronts is the distance the wave motion travels in t sec.

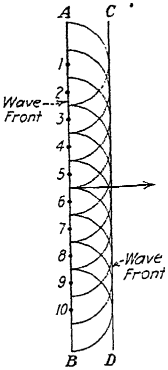
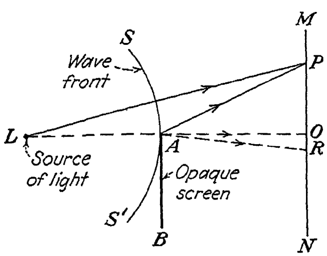
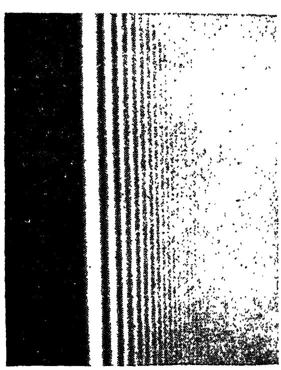


FIG. 671.—Huygens's principle for a plane wave front.

669. Diffraction by a Straight Edge.—Suppose light is diverging from a luminous point L (Fig. 672) and that it passes by the edge of an opaque screen AB. If the light were propagated accurately in straight lines, there would be uniform illumination above the line LAO and complete darkness below it on the screen. It is, however, observed that the illumination does not become zero





(a) (b)

FIG. 672.—(a) Light passing a straight edge bends into the geometric shadow. (b) Diffraction pattern due to a straight edge. (*Courtesy M. E. Hufford, University of Indiana.*)

immediately below O but fades away continuously. There is complete darkness at a small distance below O. On the other hand, immediately above O the illumination is not uniform but

shows a series of bright and dark bands. These bands correspond to maxima and minima in the illumination on the screen. The brilliant fringes thus produced run parallel to the edge of the diaphragm AB. The intensity of the fringes decreases as we proceed from the point O, and at a short distance from O the intensity of the illumination becomes uniform. It is thus seen that the geometric shadow of AB is not distinctly marked, but that the light fades away gradually on one side and passes through a series of maxima and minima on the other side. The appearance of the fringes thus produced is seen in Fig. 672*b*.

670. Diffraction of Light by a Narrow Wire.—Consider now the case of a fine wire AB (Fig. 673*a*, *b*) placed in front of a narrow slit

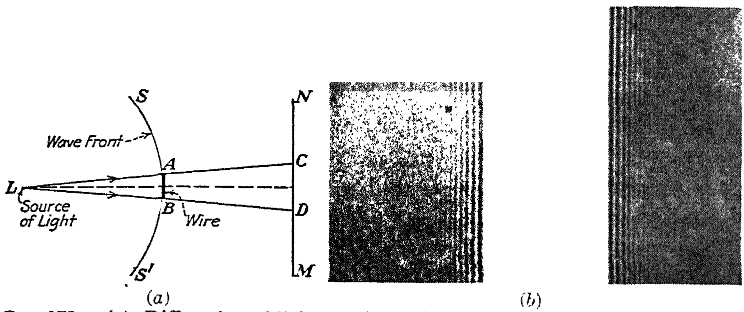


FIG. 673.—(*a*) Diffraction of light passing a fine wire. (*b*) Diffraction pattern produced by a fine wire. (*Courtesy M. E. Hufford, University of Indiana.*)

L. The shadow of this wire on the screen MN will be found to be bounded on each side by a system of parallel fringes like those shown in Fig. 673*b*. The system of fringes lying above C is due to diffraction of light from L over the edge of the obstacle at A. The fringes below D on the screen are due to the diffraction of light which has passed by the edge B. In between C and D, there may be another set of fringes, provided the wire is sufficiently narrow. In such a case, the space between C and D is illuminated by light which has passed the edge A and been bent into the geometric shadow CD, and also by light which has passed the edge B and been bent into the geometric shadow CD of the wire AB. Since the region between C and D is thus illuminated by light from two sources, these lights superpose on each other and may under suitable conditions interfere in such a way as to produce interference fringes. Figure 673*b* shows the two regions of diffraction bands which border this central region

parent, and light can pass through regularly. The plate of glass is then somewhat like a picket fence. In effect, there is a strip through which the light can pass followed by a strip through which it cannot pass. The furrows are very close together.

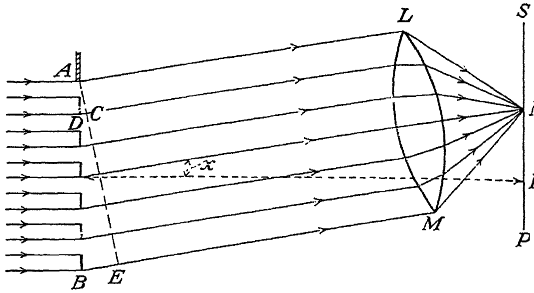


FIG. 678.—Diffraction grating. For reinforcement $n\lambda = d \sin x$.

Let AB (Fig. 678) represent such a grating on which is falling parallel light so that the direction of the rays is perpendicular to the plane of the grating. Figure 679 shows the wave fronts which will start out from these slits. These wavelets will destroy or reinforce each other, according to whether they are in phase or out of phase at a given point. Through the slits in the grating will come beams of parallel light. If these rays coming in the direction perpendicular to the plane of the grating are brought to a focus on the screen PS by the lens ML, there will be formed a bright line at K. If the light is viewed in a direction making an angle x with the normal to the grating, parallel rays of light emerging from the slits in this direction will travel unequal distances to reach the screen, and when they are brought to a focus at N by the lens LM they may either reinforce or destroy each other. If the angle x is made such that a ray of light from one slit is one-half wave length behind the corresponding ray from the neighboring slit, then the rays from one slit will be out of phase with the rays from the neighboring slit and will destroy each other. Hence, at N there will be darkness because the rays interfere in such a way that they neutralize each other. If the

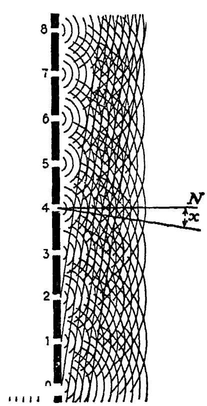


FIG. 679.—Wave fronts from a diffraction grating.

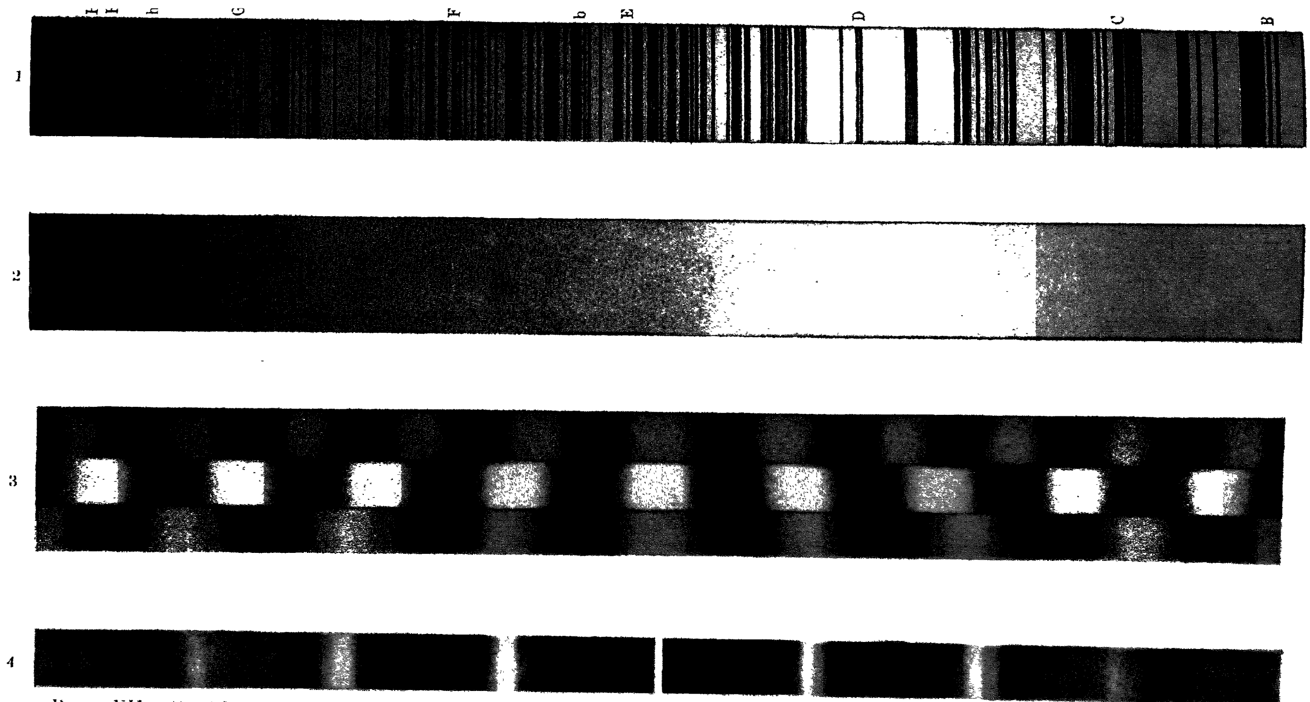


PLATE VII.—(1) Solar spectrum crossed by Fraunhofer lines. (2) Continuous spectrum from an incandescent lamp. (3) Interference fringes showing that distance between fringes decreases as the wave length decreases. (4) Spectra produced by a diffraction grating. The central image is white. The width of the spectrum increases with the distance from the central image. (Adapted from Plates I, II, and III of Michelson's *Light Waves and Their Uses*, 1902. *By permission of the University of Chicago Press.*)

of interference. The interference bands are not present in this case because the diameter of the wire was too large.

671. Diffraction through a Narrow Slit.—If light from a narrow slit at O falls on a second narrow slit which is parallel to the first slit, there will appear on the screen MN (Fig. 674) a bright central band. On each side of this central band, there will be alternate bright and dark bands which become wider as the slit O is decreased in width. By an application of Huygens's principle, these bands can be explained by the interference of light from different parts of the wave front, which has come through the slit AB. The light which reaches the points on the screen between L and S comes from points on the wave front in such a way that the light is nearly in the same phase and there is reinforcement at all points. Points on the screen above L or below S are illuminated by light which has come from points on the wave front in such a way that in some cases there is reinforcement, and in other cases there is destructive interference. Hence, alternate bright and dark bands are produced above L and below S. If the edges of the slit are not parallel the diffraction bands have the form shown in Fig. 675. Diffraction patterns produced by wire screens of different meshes are shown in Fig. 676.

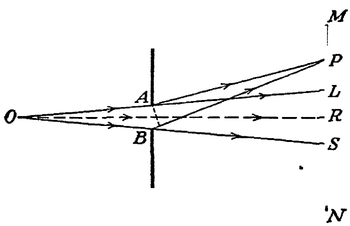


FIG. 674.—Diffraction of light passing through a narrow slit.

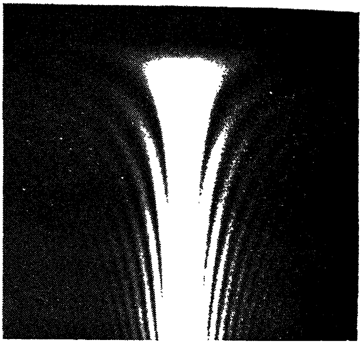
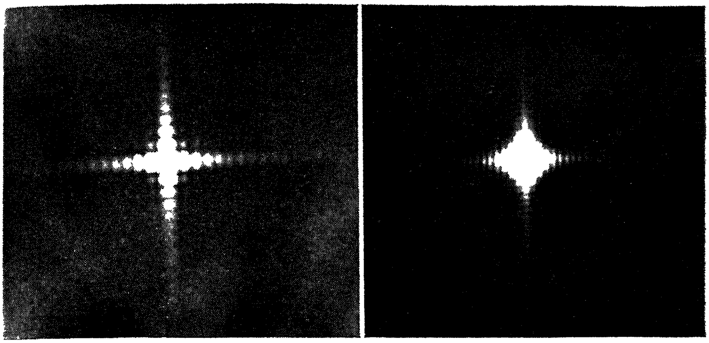


FIG. 675.—Diffraction through a wedge-shaped opening; the narrow end of the opening corresponds to the widest separation of the fringes. (*Courtesy M. E. Hufford, University of Indiana.*)

672. Diffraction by a Circular Aperture.—Another striking illustration of diffraction is observed when light from a luminous point passes through a small circular aperture like a pinhole. When this aperture is viewed by means of a magnifying glass, there appears a brilliant spot which is surrounded by a series of

bright rings. The size and appearance of these rings are altered as the eye is moved closer to the opening or farther from it. Such a system of diffraction rings about a luminous point is given in Fig. 677.



(a) (b)

FIG. 676.—Diffraction pattern through screens with rectangular openings. (a) Smaller, (b) larger rectangular opening. (*Courtesy M. E. Hufford, University of Indiana.*)

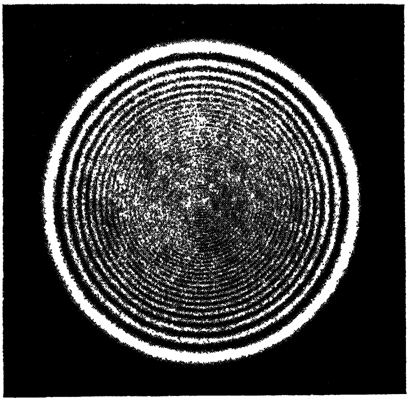


FIG. 677.—Diffraction pattern produced by a circular opening. (*Courtesy M. E. Hufford, University of Indiana.*)

673. Diffraction Grating.—If a series of very fine equidistant parallel slits be ruled on a plate of glass with a fine diamond point, we have what is known as a **diffraction grating.** Where the diamond point has made a furrow on the glass, the light cannot pass through regularly. In between the furrows where the surface of the glass has been undisturbed, the glass is still trans-

angle between the normal to the grating and the ray is increased, the rays from the lower slits will be more and more behind the rays from the upper slits. When the difference in path between corresponding rays from neighboring slits amounts to one wave length of light, the light from the slits will again be in phase and reinforce. When these rays are focused by the lens *LM*, another bright image of the slit will be formed on the screen.

From the geometry of the Fig. 678 it is seen that

$$DC = AD \sin x.$$

When

$$\begin{aligned} DC = AD \sin x &= \frac{\lambda}{2}, \\ &= \frac{3\lambda}{2}, \\ &= \frac{5\lambda}{2}, \text{ etc.,} \end{aligned}$$

the rays from different slits destroy each other, and the screen is dark.

When

$$\begin{aligned} DC = AD \sin x &= \lambda, \\ &= 2\lambda, \\ &= 3\lambda, \text{ etc.,} \end{aligned}$$

the rays from different slits reinforce each other, and the screen is illuminated (Plate VII, 4).

The angle x through which the telescope of the spectrometer must be turned before reinforcement is again obtained after leaving the central bank K can be easily measured on the divided circle of a spectrometer. The number of rulings per centimeter of the grating is given by the maker. From this the distance d between the slits on the grating can be computed. For the first reinforcement,

$$\lambda = d \sin x.$$

Since all the quantities in this equation are known except λ, the wave length of the light can be calculated. This is a very accurate method of measuring the wave length of light.

Example.—In using a grating to determine the wave length of light, it was observed that the angular separation of the second-order spectrum from

the central image was 45 deg. The number of lines per inch on the grating was 14,500. What was the wave length of the light?

$$2\lambda \quad d \sin x$$
$$d$$
$$\frac{1}{2} \times \frac{2.54}{14{,}500} \times \frac{1}{\sqrt{2}}$$
$$= 0.0000621 \text{ cm.}$$

Example.—For a certain color of yellow light, the angular separation between the central image and the first-order spectrum produced by a plane grating was 17 deg. The grating had 5,000 rulings to the centimeter. What is the wave length of the light?

$$\text{Wave length} = d \sin x$$
$$= \frac{1}{5{,}000} \times \sin 17 \text{ deg.}$$
$$= \frac{1}{5{,}000} \times 0.292$$
$$= 0.0000584 \text{ cm.}$$

Problems

1. A glass grating is ruled with 4,250 lines to the centimeter. Yellow light striking the grating normally is seen to form a second-order image diffracted at an angle of 30 deg. from the normal. What is the wave length of the yellow light?

2. A diffraction grating having 14,500 lines to the inch is used with a spectrometer in such a way that light of wave length 0.000059 cm. falls normal to its surface. What angle must the telescope of the spectrometer make with the normal to the grating so that the first order of the spectrum can be observed?

3. In observing the spectrum of a certain color with a grating which had 250 lines to the millimeter, the angular deviation of the second-order spectrum from the central image is 15 deg. when the incident light is normal to the grating. Find the wave length of the light.

4. Two spectral lines of different colors are observed by means of a grating, and it is seen that the third-order image of one line coincides with the fourth-order image of the second line. What is the ratio of the wave lengths for the two colors?

5. The fourth-order spectrum contains a certain color diffracted at an angle of 25 deg. If the grating is ruled with 200 lines to the millimeter, what is the wave length of the light?

6. Monochromatic light from a narrow slit illuminates two parallel slits 0.15 mm. apart. On a screen, 80 cm. away, interference bands are observed 3 mm. apart. Find the wave length of the light.

7. Light from a sodium flame, with a wave length of 0.0000589 cm. passes normally through a piece of glass 1 mm. thick, with a refractive index of 1.55. What difference in phase is produced, as compared with a path of 1 mm. in air?

8. Experiment shows that the index of refraction of heavy flint glass is 1.717 for *D*-light (yellow) and 1.742 for *F*-light (blue). Find the angle of dispersion of these two colors produced by a 60-deg. prism, if the light strikes the prism with an angle of incidence of 50 deg.

9. An achromatic prism is made by combining a flint-glass prism with a crown-glass prism. If the angle of the flint-glass prism is 5 deg., what must be the angle of the crown-glass prism, so that the prism will be achromatic for green light? Index of refraction for green light for crown glass is 1.514 and for flint glass, 1.626.

10. A transmission grating has 440 lines to the millimeter. It is used with light of wave length 5800 Å. If the incident light is normal, what is the angle between a ray to the undeviated image and a ray to the second-order spectrum?

CHAPTER LVII

POLARIZATION AND SACCHARIMETRY

674. Polarization.—There are various phenomena which show that light is a wave motion in the ether. It is now necessary to ask whether these waves are longitudinal, like sound waves, or transverse, like waves in a stretched string. In the former case, the displacement is in the direction in which the wave is traveling, and in the latter case it is perpendicular to this direction. There are certain phenomena which offer conclusive proof

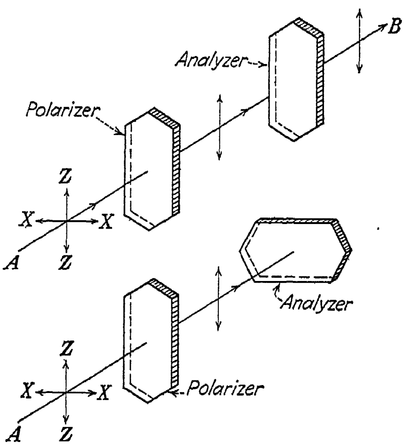


FIG. 680.—Polarization of light by tourmaline crystals.

that light waves are transverse waves, that is, that the vibrations which constitute light waves take place at right angles to the direction in which the wave is traveling. The following experiment with two crystals of tourmaline shows the nature of this evidence most simply.

When a crystal of tourmaline (Fig. 680) is cut parallel to the crystallographic axis and a ray of light passes through it, the transmitted beam in no way differs from the incident beam, so far as the unaided eye can detect. If the light which has passed through one tourmaline crystal is allowed to pass through another with its axis parallel to the first, the light will be almost

completely transmitted by the second crystal. If now the second crystal is rotated around the ray of light as an axis so that the axes of the two crystals are inclined to each other, the intensity of the transmitted light will decrease and, when the axes of the crystals are at right angles to each other, none of the light from the first crystal will pass through the second. If the rotation of the second crystal is continued until the axes of the crystals are again parallel, the light from the first crystal will be transmitted through the second. It is evident that the light in passing through the first crystal has acquired properties which ordinary light does not possess.

To see the meaning of this experiment, consider a stretched string (Fig. 681) in which the particles are vibrating in a plane perpendicular to the length of the string. If a block of wood with a slot in it is placed over the string, the vibrations will not be

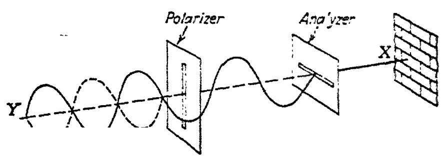
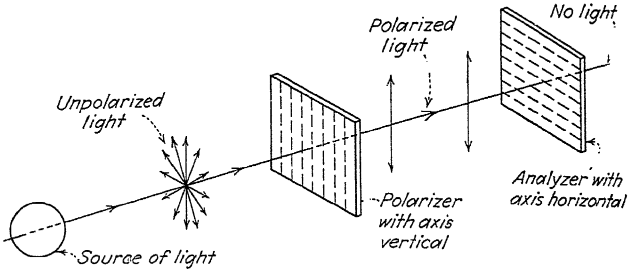


FIG. 681.—Mechanical analogue of polarization of light.

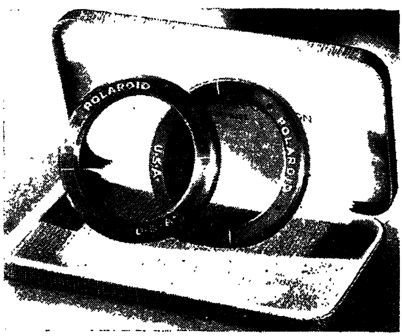
affected when the slot is parallel to the direction of vibration of the string. When, however, the slot is at right angles to the direction of vibration of the string, the vibrations will not pass beyond the slot. If the slot makes various angles with the direction of vibration of the string, then that part of the vibratory motion which is parallel to the slot will pass through. If a second slot is placed over the string, the vibrations which pass the first slot will also pass the second slot when the two slots are parallel to each other. When the slots are perpendicular to each other, the vibrations which pass the first slot will not pass through the second.

The action of the tourmaline is now understood if we consider ordinary light to consist of a transverse wave motion in which the vibrations take place in all directions in a plane perpendicular to the direction in which the light is traveling. When such a beam passes through the first tourmaline crystal, the crystal absorbs all the vibrations except those in a certain direction. These it transmits. Hence, the emerging beam differs from the

ordinary light in that all the vibrations are in one direction. Such a ray of light is said to be **polarized.** If it falls on the second crystal of tourmaline, that crystal will transmit only those vibrations which are parallel to a certain direction in the tourmaline crystal. For one position of the crystals all the vibrations are transmitted. Now if one of the crystals is rotated 90 deg.



(a)



(b)

FIG. 682.—(a) Polaroids with planes of transmission at right angles to each other. (b) Production of polarized light by means of polaroids. (*Courtesy Polaroid Corporotion.*)

from this position, none of the vibrations are transmitted. For intermediate positions, part of the vibrations are transmitted.

675. Polaroids.—A new type of polarizer known as a **polaroid** has been recently developed. It consists essentially of a flat lamination of polarizing film between plates of glass. Ordinary light falling on the film emerges as polarized light (Fig. 682*a*). The action of such a polarizer is like that of a tourmaline crystal. One component of the vibration is absorbed and the other transmitted and the vibrations of the emerging light all lie in one plane. The aperture of these polaroids may be made large and the intensity of the emerging light correspondingly increased. In this way their range of usefulness is increased. Figure 682*b* shows two of these polaroids so

oriented that in the region where they overlap light falling on one of them would not be transmitted by the other.

676. Polarization by Reflection.—It is necessary to assume that light is a transverse wave motion in order to understand the phenomena which reveal themselves when light is reflected from a clear plane glass. The light reflected in this way has very different properties from those shown by the incident light. Let M (Fig. 683) be a glass plate on which light is incident at an angle θ. Part of this light will be reflected in the direction OB. If the reflected beam then falls on another plane glass N, part of this beam is again reflected. If N is so placed that the original ray SO, the once-reflected ray OB, and the twice-reflected ray BC, lie in the same plane, the reflection at the mirror N takes place with the least loss of light since little of the light penetrates the surface of the mirror N. If this experiment is repeated for different angles of incidence, an angle of incidence will be found at which the light incident on the second mirror N will be more completely reflected than for any other angle.

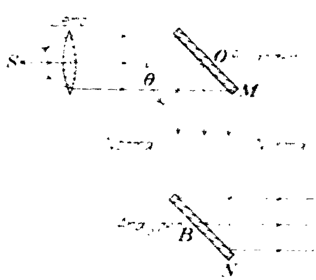


FIG. 683.—Polarization of light by reflection.

If now the mirror N is rotated through an angle of 90 deg. about the direction of the ray OB, and if M and N be set for different angles of incidence, there can be found a position for which the mirror N does not reflect any of the light which is incident upon it but transmits all of it. This can be explained if we assume that light is a transverse wave motion and that for a certain position of the mirror M only the components of the incident vibration parallel to the reflecting surface are reflected. In the reflected beam, all the vibrations will be parallel to the surface of the mirror M and at the same time perpendicular to the beam OB. In this way, the beam of light has become plane-polarized. When the beam OB is again reflected at the mirror N, the glass will transmit those vibrations which are not parallel to its surface. Since, in the first case, the mirror N is parallel to the mirror M and since the beam OB is composed only of vibrations which are parallel to the surface of the mirror

M, all of these vibrations will also be parallel to the surface of the mirror *N* and will be most largely reflected by that mirror. When, however, in the second case, the mirror *N* is rotated 90 deg. about the ray *OB* as an axis, the incident vibrations have no components parallel to the surface of the mirror *N*. All of the components are perpendicular to the surface of the mirror *N*. Hence, in this position, the glass *N* reflects none of the incident beam but transmits all of it.

Only when the light strikes both *M* and *N* at an angle of incidence of about 57 deg. can a position of *N* be found for which it reflects no light at all; *i.e.*, only for a certain angle of incidence is the reflected light found to be completely polarized. **The angle at which light is completely polarized by reflection is called the angle of polarization.**

677. Plane of Polarization.—When a beam of light is polarized, a plane can be imagined to be passed through the ray in such a way that the vibrations which constitute the wave are at right angles to the plane. This plane is known as the **plane of polarization.** If a stretched string is vibrating in a vertical plane, the horizontal plane is the plane of polarization. It would seem more natural to define the plane of polarization to be the plane in which the vibrations lie rather than the plane which is perpendicular to the plane containing the vibrations, but usage has determined that it be otherwise.

Light is after all a complex system of wave motions, consisting of an electric field and a magnetic field at right angles to each other (Fig. 593). In a plane-polarized wave, the electric vibrations all lie in the same plane, and the magnetic vibrations in a plane at right angles to the plane containing the electric vibrations. The preceding definition of the plane of polarization is still valid, if it is agreed that the direction of vibration be assumed to lie in the plane containing the electric forces and that the plane of polarization be thought of as the plane containing the magnetic forces. The plane of polarization is then perpendicular to the plane containing the electric forces.

678. Double Refraction.—In the study of refraction it was assumed that the media had the same physical properties in all directions. In crystalline substances, like Iceland spar and quartz, the physical properties of the substances are quite different in different directions. When a beam of light passes

through such a crystal (Fig. 684), it is split up into two beams. One of these beams obeys the ordinary laws of refraction and for that reason is known as the **ordinary ray**. The other beam is called the **extraordinary ray** because it does not obey the ordinary laws of refraction. An examination of these two rays shows that both are polarized and that their planes of polarization are perpendicular to each other. This fact is indicated in Fig. 684 by putting dots and crosslines on the rays to indicate the directions of the vibrations.

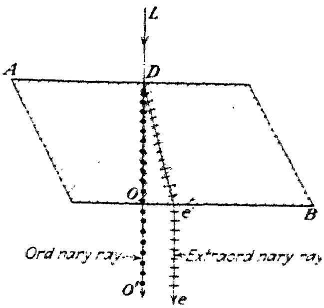


FIG. 684.—Double refraction of calcite. The incident ray is split into two rays, the ordinary ray and the extraordinary ray.

Double refraction is easily observed by placing a piece of calcite on a piece of paper containing printed matter. On looking through the calcite at the printed matter, two images are seen, one produced by the ordinary ray and the other by the extraordinary ray. The image from the ordinary ray seems nearer the observer than the image from the extraordinary ray. On rotating the calcite, the extraordinary image seems to rotate about the ordinary image.

679. Nicol Prism.—One of the best methods of separating the ordinary from the extraordinary ray is by what is known as a Nicol prism (Fig. 685). A rhomb of Iceland spar *AMBN* is

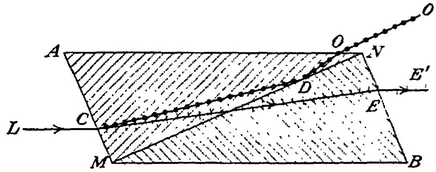


FIG. 685.—Nicol prism. The ordinary ray is eliminated by total reflection.

cut into two parts by a plane *MN*, which makes an angle of about 22 deg. with *MB*. When the cut surfaces have been polished, they are cemented together with Canada balsam. Now the index of refraction of Canada balsam is less than that of Iceland spar for the ordinary ray and greater than that of Iceland spar for the extraordinary ray. These refractive indices are:

Canada balsam = 1.55.
Iceland spar for ordinary ray = 1.658.
Iceland spar for extraordinary ray = 1.468.

The ray of light *LC* entering the face of the rhomb at *C* is broken up into the ordinary and extraordinary rays which are polarized at right angles to each other. This is indicated by the dots and crosslines in the figure. When the ordinary ray reaches the surface of separation of the Iceland spar and the Canada balsam at *D* at an angle greater than the critical angle, it is totally reflected and emerges from the rhomb in the direction *OO'*. This surface of the rhomb is ordinarily painted black and this ray is absorbed in this black coating. Because the index of refraction of the Canada balsam is greater than the index of refraction of the calcite for the extraordinary ray, total reflection in this case cannot occur, and the extraordinary ray is transmitted in the direction *CE* and emerges from the rhomb at the polished face *BN*. In this way, there is obtained a beam of light which is plane-polarized; that is, all the vibrations lie in a single plane. Its intensity is only one-half that of the incident beam, the remainder of the light having been absorbed in the Nicol.

680. Rotation of the Plane of Polarization.—When a beam of monochromatic plane-polarized light passes through certain substances, the plane of polarization is rotated. Thus, if a plate of quartz which has been cut so that the faces are perpendicular to the axis of the crystal is placed between two Nicol prisms which have been so oriented that the second one excludes the light transmitted by the first, the light will no longer be extinguished by the second Nicol prism. If, however, the second Nicol prism is rotated about the ray as an axis, a new position can be found at which the light is again extinguished.

This angle through which the plane of polarization has been rotated depends on the kind of substance interposed between the Nicol prisms, on the thickness of the substance, and on the wave length of light. The rotation may be either clockwise or counterclockwise. In this respect there are two kinds of quartz. One kind rotates the plane of polarization clockwise; the other rotates it counterclockwise. Some liquids and gases also cause a rotation of the plane of polarization. When two substances are mixed together, the observed rotation is the algebraic sum of the rotations produced by the substances separately.

681. Specific Rotation.—The rotation produced by a column of pure substance which is 10 cm. in length, is called the **specific rotation.** It depends on the wave length of the light and the temperature of the substance. It is greater for light of short wave length than for light of long wave length. **When a solution of an active substance in an inactive solvent is used, the rotation produced by a column of the solution 10 cm. in length is divided by the weight of the active substance in unit volume of the solution. This ratio gives the specific rotation of the dissolved substance.** Suppose that N g. of a substance is dissolved in an inactive solvent so that the volume of the solution is 100 c.c. If a length of 20 cm. of this solution rotates the plane of polarization θ deg., then the specific rotation of the substance is

$$\alpha = \frac{100\theta}{2.0N}.$$

The specific rotation depends not only on the wave length of light but also on the temperature.

682. Polarimeter.—The simplest way to measure the rotation of the plane of polarization is to examine the substance between two Nicol prisms. The Nicol prisms are first turned so that no light will pass through them. They are then said to be crossed. The substance to be examined is then inserted between the Nicols and the rotation of the plane of polarization observed by noting

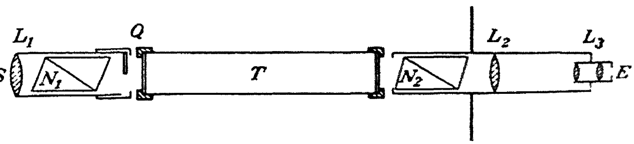


FIG. 686.—Diagram of polarimeter.

the amount which one of the Nicols must be rotated to produce extinction again. In practice this is not a sensitive arrangement for measuring the rotation of the plane of polarization, for it is impossible to tell exactly when the light is extinguished. The field of view appears dark while the Nicol is rotated through an appreciable angle. For this reason, an auxiliary piece of apparatus is added by which the sensitiveness of the instrument is increased. One of the methods of increasing the sensitiveness is represented in Fig. 686 which shows a polarimeter.

Light from a source S (Fig. 686) passes through the lens L_1 and is polarized by means of the Nicol N_1. For this reason, this Nicol is referred to as the **polarizer.** Between the polarizer and the tube T containing the solution to be studied is placed the small plate of quartz Q so that it covers half the field of view.

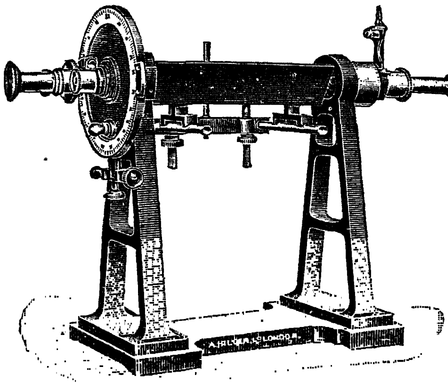


FIG. 687.—Polarimeter. (*Courtesy Hilger & Co.*)

Half the beam of light passes through the plate, and the other half does not. This plate is cut parallel to the axis and is made of such a thickness that the ordinary ray passing through it gains in phase by one-half wave length over the extraordinary ray. In this way it is possible to make the plane of polarization of light which has passed through the quartz plate inclined at a small angle to the plane of polarization of the light which has not passed through the plate. The field of view is then divided

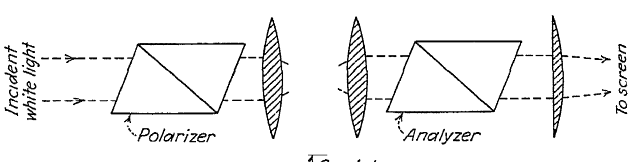


FIG. 688*a*.—Apparatus for producing interference fringes by means of converging polarized light. The crystal between the polarizer and the analyzer is doubly refracting. With white light we get colored interference patterns.

into two parts, and instead of setting the analyzer for extinction, it is set so that the two parts of the field of view are equally illuminated. This very much increases the accuracy of the setting.

After passing through the tube T, the light passes through the Nicol N_2, the **analyzer,** and is observed through the telescope

L_2L_3 with the eye at E. By observing the rotation produced by the solution in the tube T, the specific rotary power is determined. This is a convenient method of determining the strength of sugar solutions or the strength of any other solution of a substance which produces a rotation of the plane of polarization (Fig. 687). A crystal properly oriented and of proper thickness (Fig. 688*a*) gives beautiful interference patterns (Fig. 688*b*) between crossed Nicol prisms.

683. Magnetic Rotation of the Plane of Polarization.—If plane-polarized light traverses an isotropic transparent medium which is in a powerful magnetic field, the plane of polarization is rotated by the medium when the

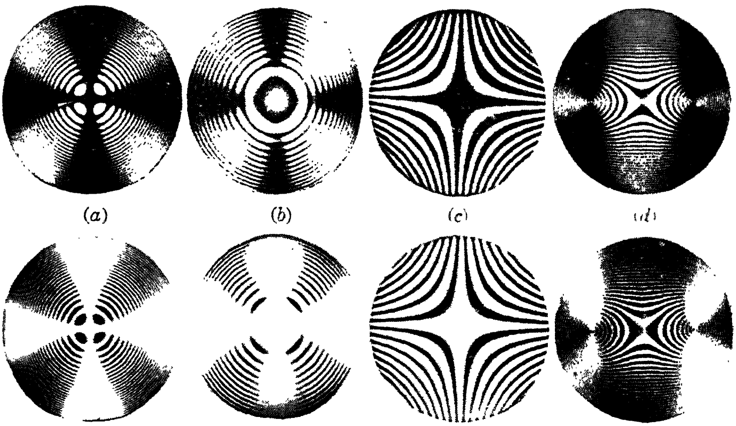


Fig. 688*b*.—Interference patterns from crystals placed in highly convergent monochromatic polarized light. Upper photographs from crossed Nicols: lower photographs from parallel Nicols. (*a*) Calcite cut perpendicular to the optic axis; (*b*) quartz cut perpendicular to the optic axis; (*c*) quartz cut parallel to the optic axis; (*d*) aragonite cut perpendicular to the bisectors of the two optic axes. (*From Fundamentals of Physical Optics by Jenkins and White with permission of the authors.*)

light passes through the medium in the direction of the lines of magnetic force. The rotation of the plane of polarization in this case differs from the natural rotation of the plane of polarization by the fact that the direction in which the plane of polarization is rotated depends on whether the light passes through the medium from the north to the south pole of the magnet or in the reverse direction. The rotation is, therefore, doubled if the light is reflected back through the medium, but in the case of natural rotation, as in quartz or sugar solutions, the direction of rotation is the same whatever the direction in which the light passes through the medium. In this case, if the light is reflected back through the medium, the rotation in passing one way just neutralizes that produced when the light passes in the opposite direction and the net rotation is zero. The magnetic rotation of the plane

of polarization is greatest in such substances as carbon disulphide or dense flint glass.

In Fig. 689 is represented a large electromagnet in which poles and cores have been bored out to allow the light rays to pass along the magnetic lines of force. Light from an arc lamp L first passes through a Nicol prism P by means of which it is plane-polarized. It then passes through the hole in the core of the magnet and then the block of glass C, by means of which the plane of polarization can be rotated. After emerging from the other end of the core of the magnet, the light passes through a second Nicol prism N. At first this second Nicol prism N is placed in such a position that it will just extinguish the emerging beam when the magnet is not excited. On allowing the current to flow through the coils on the magnet, the light again emerges from the second Nicol prism N. By turning the analyzing Nicol prism P until darkness is again produced, the amount and the direction of the rotation produced by the piece of glass can be measured.

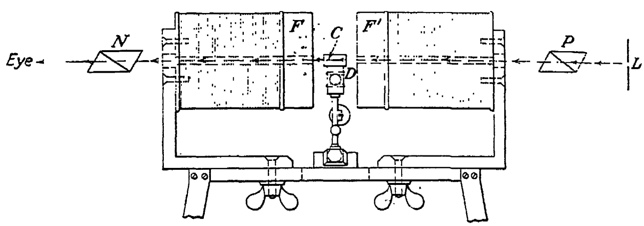


Fig. 689.—Magnetic rotation of the plane of polarization.

Problems

1. Determine the critical angle for the ordinary ray passing from Iceland spar to Canada balsam.

2. Find the specific rotation of a substance such that 20 g. dissolved in 70 g. of water, forming a solution with a density of 1.21 g. per cubic centimeter, produces a rotation of 35 deg. in the plane of polarization through a 20-cm. column of solution.

3. A solution of dextrose (specific rotation 52.5 deg.) causes a rotation of 12 deg. in a column 10 cm. long; what is the concentration of the solution?

4. Find the concentration of cane sugar in a tube which is 30 cm. long when the plane of polarization of sodium light, passing through the tube in the direction of its length, is rotated 30 deg. (Specific rotation of sugar = 66.5.)

5. Light of a certain wave length passes through a Nicol prism and is thus polarized. It then passes through a second Nicol prism whose plane section makes an angle of 60 deg. with the plane section of the first Nicol prism. What percentage of the light incident on the second Nicol prism emerges from it?

6. Two Nicol prisms have their planes parallel to each other. One of the Nicol prisms is then turned so that its principal plane makes an angle of 40 deg. with the principal plane of the other Nicol prism. What percentage of the light which is incident on the second Nicol prism is transmitted by it?

PART VII.—RADIATION AND ATOMIC STRUCTURE

CHAPTER LVIII

ORIGIN OF QUANTUM THEORY

684. Definition of a Black Body.—An ideal or perfect *black body* is defined as a body the surface of which absorbs all the radiation incident upon it. Although no such body actually exists, it may be thought of as a surface covered with a coating of lampblack. For experimental purposes a black body may be obtained by using a cavity in the side of which there is a small hole (Fig. 690). Radiant energy will be emitted from each point on the surface of the inner wall of the enclosure. This radiant energy falls on other points of the wall, where a part of it is reflected and a part is absorbed. Radiation of all wave lengths is reflected back and forth in this cavity until there is a certain uniform density of radiation throughout it. If the temperature of the wall is increased, the amount of radiation per cubic centimeter is also increased. If some of the radiation is allowed to emerge from an opening in the cavity and is examined by means of a suitable spectroscope, it is found that radiation of every wave length is present in it. The amount of energy associated with each wave length differs from wave length to wave length, and the distribution of energy in the spectrum changes with the temperature. Such an enclosed cavity is called a "black body" and the radiation enclosed in it is called **black-body radiation.**

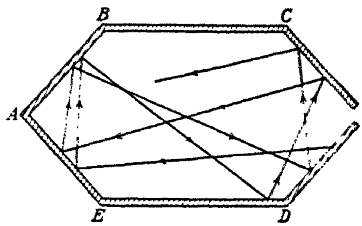


FIG. 690.—A source of black-body radiation. All wave lengths are present.

685. Stefan's Law.—The total radiation sent out by a black body increases with the temperature. The law governing this increase is known only for a body which has the power to absorb all the radiation falling on it, such a body being also an ideal or perfect radiator. The total energy emitted by such a radiator

is proportional to the fourth power of the absolute temperature. This law can be expressed as

$$E = CT^4,$$

where E is the energy emitted per unit area of the black body, C a constant, and T the absolute temperature. If one black body at a temperature T_1 sends radiation to another surrounding it at a temperature T_2, the net radiation sent by the first body to the second is

$$E = C(T_1^4 - T_2^4).$$

686. Wien's Displacement Law.—As the temperature of the radiating surface is increased, the energy emitted in every wave length increases, but not in the same proportions. Light from a hot radiating surface changes its color as the temperature of the surface is raised. When the body becomes hot enough to be visible, it first appears red. With further rise of temperature the color changes to yellow and then to white. The curves in Fig. 691 show the relation between the energy emitted and the wave length. The intensity of the energy is small for radiations of short wave lengths and also for those of long wave lengths. For intermediate wave lengths, the energy has its maximum value. As the temperature is increased, the height of the maximum is increased and at the same time shifted toward the short wave lengths. This shift of the energy curve produces an increase in the proportion of blue in the emitted light, thus producing the change of color to which reference has already been made. Because the intensity of the visible radiations in an incandescent body is greater at high than at low temperatures, for great efficiency the filaments of incandescent lamps are heated to as high temperatures as possible. Hence, the modern gas-filled tungsten lamp is much more efficient than the vacuum tungsten incandescent lamp.

Let T = the absolute temperature of the black body, and
λ_m = the wave length in microns at which the radiation is a maximum for a particular temperature (see Sec. 687).

then

$$T\lambda_m = \text{constant} = 2{,}900 \text{ micron deg.}$$

This law is known as Wien's displacement law.

Example.—What is the wave length of the maximum intensity of the radiation inside of a furnace at a temperature of 1200°C.?

$$\lambda_m \times T = \text{constant.}$$

$$\lambda_m = \frac{\text{constant}}{T} = \frac{2{,}900 \times 10^{-4}}{1{,}473}$$

$$= 0.0002 \text{ cm.}$$

687. Relation of Intensity of Radiation to Frequency.—The intensity of the radiation from a black body (Fig. 691) varies with the frequency as well as with the temperature. The theoretical interpretation of the relation between the temperature, the frequency of the radiation, and the intensity of the radiation proved to be one of the most difficult and important problems in modern physics. In 1900, Planck decided it was impossible to solve this problem on the basis of classical ideas about the nature of energy and then introduced a new and revolutionary hypothesis. He assumed that the walls of the heated cavity which is filled with black-body radiation contain a very great number of vibrators or radiators. Each of these radiators was assumed to be able to vibrate with a single well-defined frequency and thus emit radiation of a single wave length. The rate at which the energy is emitted from the walls of the black body must be related to the average amount of energy associated with each vibrator.

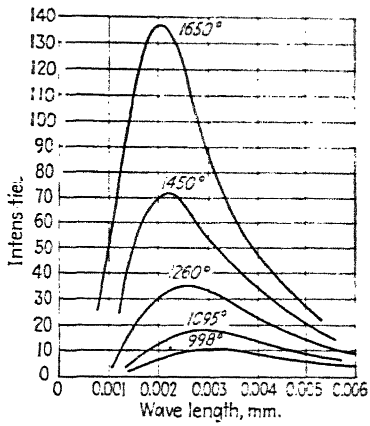


FIG. 691.—Change of intensity and distribution of radiation with temperature. The maximum shifts to shorter wave lengths at higher temperatures.

When he had found the average energy of a radiator, in terms of its frequency and the temperature, he introduced the assumption that the energy of a given vibrator at any instant is equal to an integral multiple of some unit quantity of energy. This unit of energy is called a **quantum.** It is related to the frequency by the equation,

$$\textbf{One quantum} = \epsilon = h\nu,$$

where h is a universal constant and ν is the frequency of the radiation. The energy of a radiator at any instant consists

of a whole number of quanta. When radiation is emitted or absorbed, the emission or absorption takes place as entire quanta or bundles of energy. By an application of the laws of probability to these quanta, it is possible to find the way in which they would be distributed among the different radiators. Since the walls of the cavity are in equilibrium with the radiation in the cavity, the distribution of energy between the different frequencies in the cavity can be obtained. In this way, Planck obtained the following equation for the density of radiation of frequency ν for a black body,

$$\psi_\nu = \frac{8\pi h\nu^3}{e^{\overline{kT}} - 1}$$

where $h = 6.548 \times 10^{-27}$ erg second = Planck's constant.
$k = R/N$ = Boltzmann's constant for one molecule.
N = Avogadro's number = the number of molecules in 1 g.-molecule.
c = the velocity of light in a vacuum.

It is important to note that the amount ot energy in each of these quanta is proportional to the frequency of the radiation. In the case of the yellow light emitted by sodium:

$$\begin{aligned}
\nu &= 5 \times 10^{14} \text{ vibrations per second.} \\
h &= 6.54 \times 10^{-27} \text{ erg sec.} \\
h\nu &= 6.54 \times 10^{-27} \times 5 \times 10^{14} \\
&= 32.7 \times 10^{-13} \text{ erg.}
\end{aligned}$$

688. Photoelectric Effect.—When a metal plate is illuminated by light from an arc lamp, it emits negative electricity. If the plate is completely insulated, it acquires a small positive charge and, hence, a small positive potential. When a certain potential is reached, the emission of electricity ceases. The emission is nearly stopped if a sheet of glass is interposed between the light and the metal plate. The glass absorbs nearly all the ultra-violet light from the lamp. Hence it is the ultra-violet radiations which are most effective in producing this emission. The alkali metals are very sensitive to this action of light and respond to rays from the luminous portions of the spectrum as well as to those from the ultra-violet region.

The effect is due to the emission of slowly moving negative electrons from the surface of the plate. These electrons are like those which make up the cathode rays except for the fact that they move more slowly. The ratio of their mass to their charge is the same as for cathode-ray particles. The light incident on the metal causes the emission of these electrons with small velocities, leaving the atom itself with a positive charge. These electrons then escape from the surface until the metal has acquired a sufficiently large positive charge of electricity to prevent more electrons from escaping.

If the metal plate which emits photoelectrons has a positive potential V, the work to carry an electron of charge e away from the plate is Ve. If, because of the action of the light, the electron is ejected from the atom with a velocity v, the kinetic energy with which it leaves the atom is $\frac{1}{2}mv^2$. If this initial energy of the electron is less than Ve, its motion will be reversed and it will return to the plate again. The least initial velocity an electron can have and escape from the plate is given by the equations,

$$\frac{1}{2}mv^2 = Ve.$$

$$v^2 = \frac{2Ve}{m}.$$

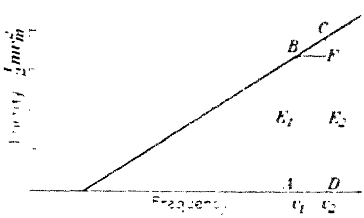


Fig. 692.—Relation between frequency of light and energy of emitted electrons

689. Relation between the Velocities of Photoelectrons and the Frequency of Light.—A linear relation has been found to exist between the maximum energy with which photoelectrons escape from the surface of the metal and the frequency of the light incident on the metal. If a curve be plotted between the frequency of the incident light and the corresponding energy of the emitted electrons, a straight line is obtained (Fig. 692), showing that the energy is proportional to the frequency of the light. In this figure, $E = \frac{1}{2}mv_m^2$ has been plotted on the vertical axis and ν the frequency of the light on the horizontal axis, where m stands for the mass of the electron and v_m the maximum velocity with which it escapes from the surface of the metal.

This straight line intersects the horizontal axis at some frequency ν_0, which is characteristic of the emitting electrode. Unless the frequency of the light is as great as this frequency ν_0, it will not cause the emission of photoelectrons. Hence, there is a long wave-length limit beyond which light does not produce a photoelectric effect.

The slope of the curve in Fig. 692 is independent of the nature of the metal. The equation of this curve is

$$E = \frac{1}{2}mv_m{}^2 = h(\nu - \nu_0) = h\nu - W_0,$$

where h is Planck's universal constant. The left-hand side of this equation is the energy with which the electrons moving with the maximum velocity v_m leave the surface; $h\nu$ is the quantum of energy received by the atom from the incident light, and W_0 is the work necessary to remove the electron from the metal.

Example.—Light of wave length 0.00007 cm. is required to cause the emission of electrons from a potassium surface. What is the energy necessary to remove one of the least firmly bound electrons?

$$\frac{1}{2}mv^2 = h\nu - w \qquad h\nu - h\nu_0,$$

where $w = h\nu_0 =$ work to remove least firmly bound electron and

$$\nu_0 = \frac{3 \times 10^{10}}{7 \times 10^{-5}}$$

$$w = h\nu_0 = 6.55 \times 10^{-27} \times \frac{3 \times 10^{10}}{7 \times 10^{-5}}$$

$$= 2.8 \times 10^{-12} \text{ erg.}$$

690. Calculation of Planck's Constant from Photoelectric Effect.—From a careful determination of the slope of this curve, Millikan has found the numerical value of the important constant h. If the ordinates are expressed in ergs and the abscissae in vibrations per second, then

$$E_1 = \frac{1}{2}mv_1{}^2 = h\nu_1 - W_0.$$

$$E_2 = \frac{1}{2}mv_2{}^2 = h\nu_2 - W_0.$$

$$h = \frac{E_2 - E_1}{\nu_2 - \nu_1} \quad \frac{\frac{1}{2}m(v_2{}^2 - v_1{}^2)}{(\nu_2 - \nu_1)} = 6.57 \times 10^{-27} \text{ erg second,}$$

where v_2 = the maximum velocity of the electrons for frequency ν_2.
v_1 = the maximum velocity of the electrons for frequency ν_1.

691. Photoelectric Cells.—A typical photoelectric cell consists of a sealed glass bulb (Fig. 693 or 694) containing an atmosphere of gas at low

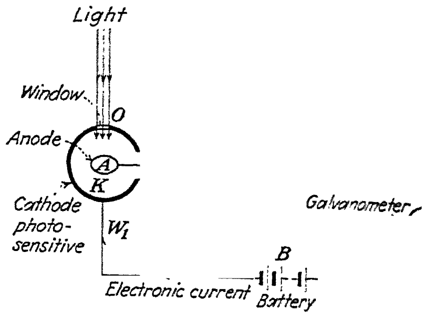


FIG. 693.—Photoelectric cell with sensitive material on walls of tube.

pressure. In this bulb there are two electrodes, one of which is sensitive to the light. In Fig. 693 the photosensitive material is an alkali metal such as potassium, which is spread on the inside wall of the glass bulb. It is connected to the exterior of the bulb by means of a sealed-in wire W_1 which in this case forms the cathode. The other electrode, which is the anode, is a simple metallic ring connected with a second wire W_2, which is carried through the stem of the bulb. The two electrodes are connected through a battery B and galvanometer G. The operation of the cell consists in allowing

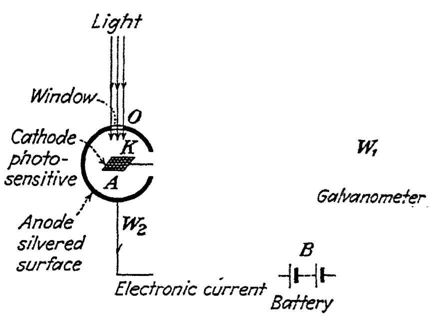


FIG. 694.—Photoelectric cell with sensitive material on central cathode.

light to fall on the cathode through the window O and measuring the resulting current by means of the galvanometer G.

In another type of photoelectric cell (Fig. 694), the walls of the cell are covered with a non-light-sensitive material such as silver. The photosensitive material is coated on a relatively small electrode placed at the center of the bulb. Just as in the other type of cell, light is allowed to pass through the window O and to fall on the photosensitive substance which

in this case is on the central electrode. The resulting current is measured in the galvanometer, but its direction is opposite to that which it had in the other type of cell.

692. Barrier-layer Photoelectric Cell.—A barrier-layer photoelectric cell (Fig. 695) consists essentially of a thin metal disk A on which there is a film of light-sensitive material B. In contact with this light-sensitive material

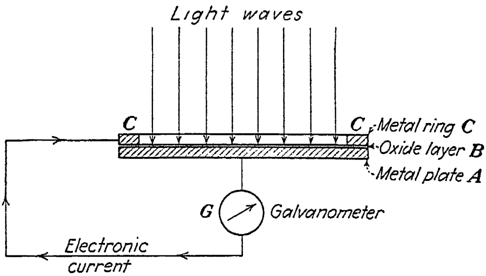
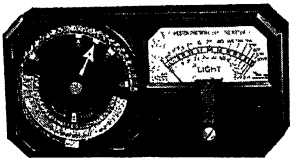
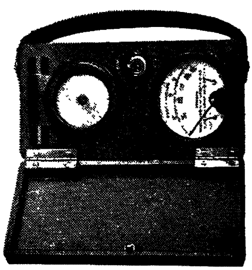


Fig. 695.—Barrier-layer photoelectric cell. The incident light develops a difference of potential.

is a collecting ring C. When the sensitive surface is illuminated with light, an electromotive force is generated between the sensitive layer B and the metal disk A. The electronic current thus produced flows from the sensitive layer to the metal plate across their surface of separation. No external electromotive force is necessary. In one form of these cells a thin coating of silver on a layer of cuprous oxide rests on a copper plate. In another form a thin coating of gold on a layer of selenium is used in contact with an iron plate. The current can be measured with the galvanometer G. The greater the intensity of light on the sensitive surface, the greater is the current in the galvanometer.



(a)



(b)

Fig. 696.—(*a* and *b*) A barrier-layer photoelectric cell used as a foot-candle meter. (*Courtesy Weston Instrument Company.*)

The principle of this cell has been applied to the construction of a simple **foot-candle meter** (Fig. 696), by which the intensity of light can be directly observed. The essential part of the instrument is one of these barrier-layer photoelectric cells which produces a current proportional to the intensity of the light falling on the sensitive layer. A milliammeter connected to the cell gives the electronic current. The instrument can be calibrated to read directly in foot-candles.

693. Transmission of Pictures over Telephone Lines.—The problem of transmitting pictures electrically from one place to another requires (1) some means for translating the lights and shades of the picture into the characteristics of an electric current, (2) an electrical transmission channel capable of faithfully transmitting the characteristics of the electric current, and (3) a means of translating the electrical signals as received into lights and shades corresponding to their values in the original picture.

The picture to be transmitted is prepared in the form of a film transparency which is bent in the form of a cylinder. This cylinder is mounted on a carriage which is moved along its axis by means of a screw at the same time the cylinder is rotated. A small spot of light thrown on the film is thus caused to traverse the entire area of the film in a long spiral. The light which passes through the cylinder varies in intensity with the tone value of the picture. Figure 698 shows the optical arrangement for projecting the spot of light upon the film transparency.

FIG. 697.—A photoelectric cell. (*Courtesy Central Scientific Company.*)

The transformation of this light of varying intensity into a variable electric current is accomplished by means of an alkali photoelectric cell. Such a cell consists of a vacuum tube in which the cathode is an alkali metal such as potassium. When this metal is illuminated, electrons are emitted. As the film on the cylinder is rotated and advanced, the illumination of the cell and, consequently, the current from it register in succession the brightness of each elementary area of the picture.

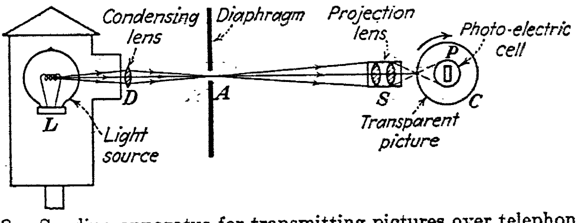


FIG. 698.—Sending apparatus for transmitting pictures over telephone lines.

The current from the photoelectric cell then traverses the communication line to some distant point. At this distant point it is necessary to have a

device for retranslating the current into light and shade. This retranslation is accomplished by means of a narrow ribbon-like conductor (Fig. 699) lying in a magnetic field in such a position as entirely to cover a small aperture. The incoming current passing through this conductor causes it to be deflected to one side by the interaction of the current and the magnetic field. The aperture beneath the ribbon is thus exposed, and the light passing through this aperture is varied in intensity. If this light then falls on a sensitive photographic film, the exposures on the film will be proportional to the lights and shades of the original picture.

In this simple scheme the photoelectric cell gives rise to direct currents of varying amplitudes. Commercial long-distance telephone lines are not designed to transmit direct currents so that in practice these photoelectric currents are superposed on a carrier current which has a frequency of about 1,300 cycles per second. The variations in this carrier current, because

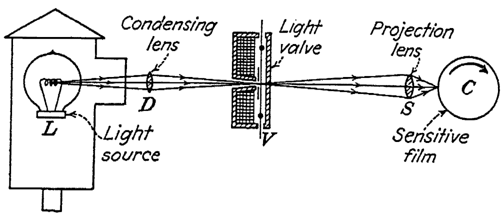


FIG. 699.—Receiving apparatus for transmitting pictures over telephone lines.

of the superposed photoelectric currents, is what is really transmitted over the line wires. Moreover, the photoelectric currents are very weak in comparison with the ordinary telephone current, and for this reason the photoelectric currents are amplified by means of vacuum-tube amplifiers. When these variations in the carrier current, caused by the changes in the photoelectric current according to the lights and shades of the picture at the sending end, traverse the ribbon at the receiving end, the aperture is opened and closed with each pulse of the alternating current. In this way, the lights and shades of the original picture are reproduced. In order that the lights and shades traced out on the receiving cylinder shall produce an accurate copy of the original picture, it is necessary that the two cylinders rotate at the same uniform rate. This condition demands the use of accurate tuning devices which are too complicated to be discussed here.

694. Series Relations in Spectra.—In the optical region of the spectrum, relations were early observed between the frequencies of the spectral lines of certain of the elements. It was found that these lines could be arranged accurately in sequences which are called *series*. The similarities among these series of spectral lines of different elements indicated that there is a fundamental way of interpreting these spectra in terms of the characteristics of the atoms. In 1913, Bohr made a

significant contribution to the understanding of such spectra by the introduction of the concept of **radiation quanta,** which had been introduced by Planck to explain the fundamental law of the distribution of energy in a black body. The concept that energy exists in the form of grains or quanta is essential to Bohr's theory and its later revisions and extensions.

Moseley found similar series relation between the lines in the characteristic X-ray spectra of the elements. An interpretation of these spectra also required the assumption that energy exists in the form of quanta. Hence, the necessity for a quantum theory of radiation becomes more and more evident.

In later sections the details of these spectral series and their significance for an understanding of the structure of the atom will be more fully discussed.

Problems

1. If a furnace is closed in such a way that it can be considered a black body and its temperature is 1500°C., what is the wave length at which the intensity of the radiation is a maximum according to Wien's displacement law?

2. To what temperature must a black body be raised in order to double the total radiation given out by it, if the present temperature is 927°C.?

3. What is the temperature of a black body which gives out radiation which has 5.5×10^{-5} cm. for the wave length at which the radiation is a maximum?

4. The surface of a radiation pyrometer receives 0.1 cal. per second from a furnace whose temperature is 727°C. How many calories per second will it receive when the temperature of the furnace is 1227°C.?

CHAPTER LIX

RADIOACTIVE SUBSTANCES

695. Radioactive Substances.—There are in nature certain substances which are in a state of spontaneous disintegration. These substances are known as *radioactive substances*. The most important of them are uranium, thorium, and radium.

In 1896, Becquerel found that compounds of uranium emitted rays which give an impression on a photographic plate which is covered with black paper. They were able to pass through thin sheets of metal and other substances which are opaque to light. They also possess the important property of discharging a body which is electrified either positively or negatively. This property of discharging an electrified body is found to be due to the fact that these rays produce ions in the gas through which they pass.

It was soon found that this property of emitting penetrating radiations is not confined to uranium and its compounds. Thorium and its compounds and minerals containing thorium have this same property.

By studying radioactive minerals which contained uranium and thorium, the Curies found that these minerals often contain a substance which is much more radioactive than either uranium or thorium. This element was finally isolated and called **radium.** It is derived chiefly from pitchblende and in the pure state possesses the property of radioactivity in an astonishing degree. In order to obtain a few decigrams of radium, it is necessary to start with several tons of pitchblende. Except for the fact that the pitchblende is examined by an electrical method, the radium in it could not be detected.

Other radioactive elements have also been isolated. These three, however, are typical of this whole group of substances.

696. Nature of the Radiations.—Three different types of radiations are emitted by radioactive substances. These radiations for brevity are called **alpha rays, beta rays,** and **gamma rays.**

1. The alpha rays are very easily absorbed by thin metal foil or by a few centimeters of air. They affect a photographic plate; cause many bodies to fluoresce brilliantly; and intensely ionize the air through which they pass. They consist of positively charged particles projected from the parent atom with a velocity about one-tenth the velocity of light. They are deflected by both an electric and a magnetic field.

2. The beta rays are more penetrating than the alpha rays. They consist of negatively charged particles which are projected from the atom of the radioactive substance with a velocity which is nearly, but not quite, as great as the velocity of light. In an electric and magnetic field they are deflected just as cathode rays are deflected. Because of their larger velocities and smaller mass they are much more penetrating than the alpha particles. They can penetrate a considerable thickness of a solid or liquid before they are completely absorbed. They produce much less ionization in the gas through which they pass than do the alpha particles and are less active photographically than alpha particles.

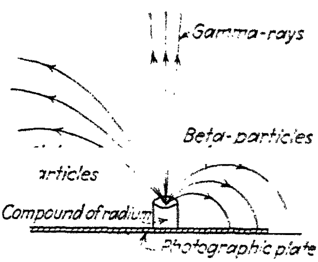


FIG. 700.—Deflection of alpha and beta particles by a magnetic field.

3. The gamma rays are extremely penetrating and are not deflected by either a magnetic or an electric field. They are not so active photographically as either the alpha or beta rays, and their fluorescent and ionizing powers are also less intense. Their nature is entirely different from that of the alpha or beta rays. They are electromagnetic pulses like very penetrating X-rays or of light of extremely short wave length.

4. Figure 700 shows the way in which the alpha particles, the beta particles, and the gamma rays are affected by a magnetic field which is perpendicular to the plane of the figure. The alpha particles which are charged positively are deflected in one direction. The beta particles which are charged negatively are deflected in the opposite direction. The gamma rays are undeflected.

697. Nature of the Alpha Particle.—The alpha particle is now known to be an atom of helium which has lost two electrons and,

therefore, carries two charges of positive electricity. The proof of this important fact is found in the following experiment. A thin-walled glass tube A (Fig. 701) was sealed into an outer tube B which was highly exhausted and connected to a small discharge tube C. To prove that there was no connection between the inner tube A and the outer tube B, the former was filled with helium under pressure and left for some hours. No trace of the spectrum which is characteristic of helium was found at that time in the discharge tube C. The helium was carefully removed from the tube A which was then filled with the emanation from radium and the tube allowed to stand for some time. The walls of the glass tube A were sufficiently thin that alpha particles from the emanation could pass through them. They were, however, stopped by the thicker walls of the tube B. After a few hours the spectrum of helium was visible in the discharge tube C. The spectrum became brighter the longer the experiment was continued. As the alpha particles were the only particles entering B during the progress of the experiment, it is evident that the alpha particles must be nothing more than the atoms of helium.

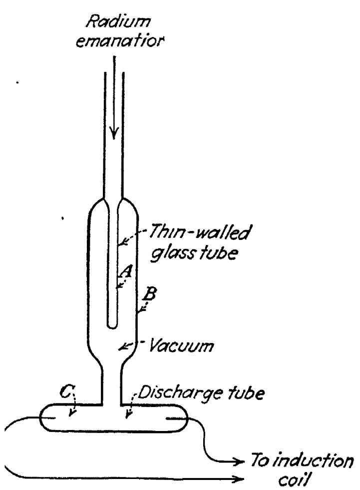


FIG. 701.—Apparatus for showing alpha particles are atoms of helium.

698. Counting Alpha Particles.—It is very important to know the number of alpha particles emitted by a radioactive material in a given time. The most obvious method is some form of scintillation method in which the number of scintillations produced each second is counted. In this method there is a possible error because it may be that some of the alpha particles, for one reason or another, do not produce scintillation when they strike the screen. The following method of counting alpha particles was devised by Rutherford and Geiger: The radioactive material supplying the alpha particles is placed on a point R (Fig. 702). The alpha particles from it pass through a small opening at S

and enter a metal cylinder. Each alpha particle produces a number of ions in passing through the gas in the cylinder. These ions increase the conductivity of the gas. Since the walls of the cylinder are connected through a battery to the earth, there will be a current through the gas in the chamber to the insulated electrode E which is inside the chamber. This electrode is connected to an electrometer by means of which the current in the gas can be measured. An increased deflection of the galvanometer will indicate that an alpha particle has entered the chamber and produced an increased supply of ions. When these ions have been removed by the electric field, the deflection of the galvanometer will return to its normal value.

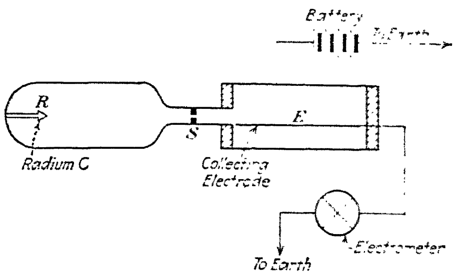


FIG. 702.—Apparatus for counting number of alpha particles.

699. Absorption of Alpha Rays by Matter.—If a layer of radioactive material is deposited on a plate and successive thin sheets of aluminum leaf are placed over this layer, the activity of the alpha rays passing through these sheets is gradually reduced. It is best to work with a pencil of rays which are nearly parallel, so that the length of path traveled by each particle is nearly the same. If such a pencil of rays is allowed to fall on a fluorescent screen, the number of particles falling on the screen can be found by counting the scintillations produced by the impact of these particles against the screen. The decrease in the number of these scintillations as different sheets of aluminum are added gives a measure of the power of the aluminum to absorb these rays. The number of scintillations per second on the screen remains constant until the thickness of the material reaches a certain critical value. When this critical value is reached, the scintillations suddenly cease. Figure 703 shows that, as the distance traveled in air is increased, the number of scintillations at first

remains the same but decreases rapidly near the end of the range. There is thus for each substance a definite thickness which all the rays can penetrate. This critical thickness is the range of the particles for that substance. This range depends only on the velocity of the rays and the nature of the absorbing substance. The range of these particles from radium C is 7.0 cm. in air at atmospheric pressure.

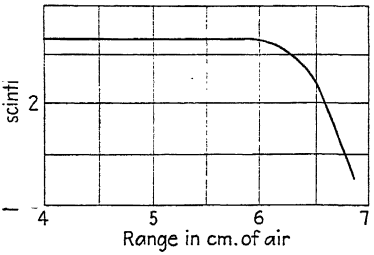


FIG. 703.—Range of alpha particles in air.

If the rays are traveling through a layer of gas, at least part of their energy is used to ionize the gas through which they pass. Figure 704 is a photographic record of the ions produced by alpha particles passing through air containing water vapor.

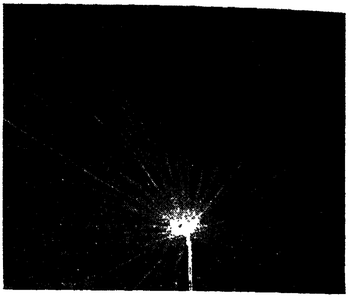


FIG. 704.—Tracks of alpha particles made visible by condensation of water vapor on ions produced by the particles.

700. Scattering of Alpha Particles.—In the discussion of the absorption of alpha particles by matter, it was assumed that these particles travel in straight lines. In the main, this assumption is true. There is, however, a possibility that a collision between an alpha particle and a molecule may produce a deflection of the former from its original path. By observing the scintillations on a fluorescent screen, Geiger found that in many cases the individual alpha particles were deflected from their original paths before reaching the end of their course. In passing through a thickness of gold equivalent in stopping power to a layer of air 3.68 cm. thick, the alpha particles suffered an average deflection of 7 deg. Once in a while a particle was deflected through a much larger angle than this. The smaller the velocity, the greater is the scattering. This deflection is shown diagrammatically in Fig. 705, where

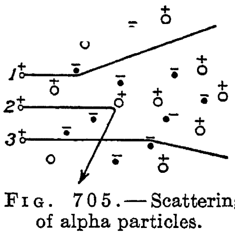


FIG. 705.—Scattering of alpha particles.

positively charged alpha particles are represented as passing through a piece of matter. Figure 706 shows a more detailed picture of a collision between an alpha particle and the nucleus of an atom and Fig. 707 a photograph of the deflection caused by the collision of an alpha particle with an atom.

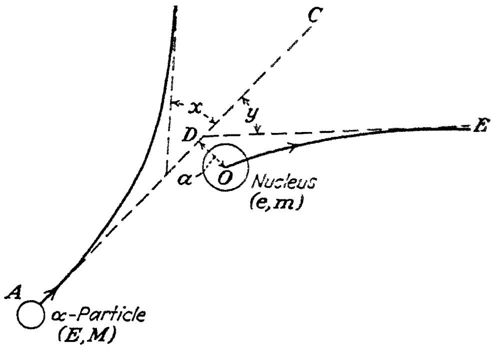


Fig. 706.—Collision of alpha particles with the nucleus of an atom

701. Nature and Properties of Beta Rays.—The beta particles are electrons moving with very high velocities. They have a penetrating power which is much larger than that of the alpha particles and will pass through absorbing sheets of matter which are 100 times as thick as that required to stop the alpha particles. The less the velocity with which these particles move, the more easily are they absorbed. The ratio of the charge to the mass of

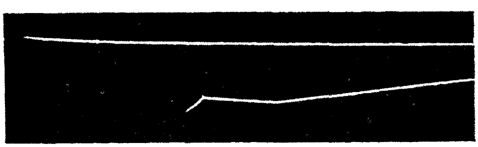


Fig. 707.—Track of an alpha particle showing collision with the nucleus of an atom.

these particles can be determined by the same methods which were used to obtain these quantities for an electron. This ratio is found to have the same value which it has for an electron.

702. Nature and Properties of Gamma Rays.—As already pointed out, the gamma rays are entirely different in nature from the alpha and the beta rays. They are electromagnetic disturbances, while the alpha and the beta particles are material parti-

cles. The gamma rays do not carry a charge of electricity and are not deflected by either an electric or a magnetic field. They are like the most penetrating type of X-rays. When their intensities are large enough, they produce a luminosity on a fluorescent screen and affect a photographic plate. Their wave lengths have been measured and are found to be less than the wave lengths of the most penetrating X-rays.

703. Disintegration of Uranium.—Uranium and its salts have been found to give off alpha, beta, and gamma rays. Crookes found that by a single chemical operation it is possible to obtain uranium which is not photographically active. At the end of this chemical operation, there is a small residue to which was

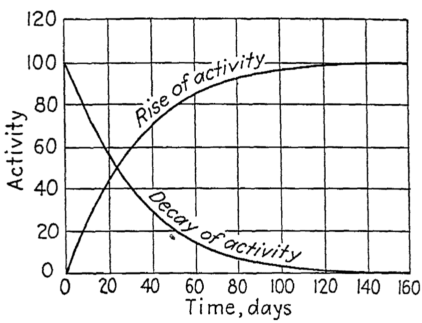


FIG. 708.—Rise and decay of the activity of a radioactive substance.

given the name of uranium *X*. This residue is many hundred times as active, weight by weight, as uranium itself. When this residue (uranium *X*) and the uranium from which it was extracted are laid aside in separate vessels for 2 or 3 months, it is found that the uranium has recovered the whole of its original activity, while the activity of the uranium *X* has completely disappeared. The loss of the activity of the uranium caused by the chemical treatment thus proved to be only temporary, and the activity of the uranium *X* was equally short lived. The total activity of the uranium and the uranium *X* at any given time is constant. The gain in the activity of the uranium is just compensated by the loss in the activity of the uranium *X*. The way in which these activities vary with the time is represented in Fig. 708.

This peculiar behavior of uranium can be easily explained by assuming that uranium disintegrates slowly and in this way produces the new substance, uranium *X*, which is very radio-

active. This new substance, uranium X, in turn disintegrates and produces other substances. Ordinary uranium is then a mixture of several substances, all of which are in a process of more or less rapid disintegration. A state of equilibrium is reached when the decrease in the amount of these products of disintegration is just equal to the rate at which these products are being formed by the parent substance. During the process of disintegration uranium gives off an alpha particle, that is, a charged atom of helium, and produces uranium X. This substance in turn disintegrates into another radioactive substance, ionium, which is also unstable. The process thus continues.

704. Nature and Properties of Radium.—The principles which explain radioactive changes will be better understood by considering in detail the radioactive changes which take place in radium. This substance is now definitely known to be an element with an atomic weight equal to 226. The conversion of uranium into radium is accompanied by the expulsion of three alpha particles, each of which has a mass of 4. Thus, on the disintegration theory, the atomic weight of radium should be less than that of uranium by $3 \times 4 = 12$ units. The atomic weight of radium on this basis should be $238.2 - 12 = 226.2$. This value is in satisfactory agreement with the observed value, 226.

1. *Radium Emanation.*—The first product of the disintegration of radium is a heavy inert gas known as radium emanation. A sample of pure radium emits this gas at a constant rate. Its formation is accompanied by the emission of an alpha particle. Radium emanation is itself an element. It has a characteristic spectrum which differs from that of radium. Its atomic weight is $226.0 - 4.0 = 222.0$. It liquefies when its temperature is reduced to $-65°$C. at atmospheric pressure. In other respects also it behaves like a gas. It belongs to the group of inert gases and is sometimes called radon.

2. *Radium A, B, and C.*—If the emanation is enclosed in a tube, it disintegrates, and the products of this disintegration collect on the walls of the tube. The immediate products of this disintegration are three in number. They are called radium *A*, radium *B*, and radium *C*. These products are short-lived. The most stable of them decays to one-half of its original value in 26.8 min. Unlike the radium emanation, these products of disintegration have the nature of solids. Radium *A* is produced

when one alpha particle is expelled from one atom of the radium emanation. The atomic weight of radium A is thus 4 units less than that of the radium emanation. This gives it an atomic weight of $222.0 - 4.0 = 218.0$. Radium B is likewise produced from radium A when one alpha particle is ejected from an atom of radium A. Consequently, the atomic weight of radium B is 4 units less than that of radium A and is therefore equal to $218.0 - 4.0 = 214.0$. When an atom of radium B gives off a beta particle, an atom of radium C is produced. Since the beta particle has a very small mass, the removal of a beta particle does not materially decrease the atomic weight so that the atomic weight of radium C and radium B is essentially the same.

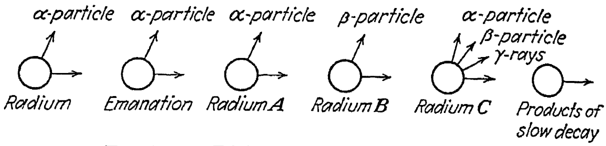


FIG. 709.—Disintegration of radium atom.

3. *Radioactive Products of Slow Change.*—Radium C breaks up with the expulsion of an alpha particle and a beta particle to form radium D, which is a radioactive product with a long life. It takes $16\frac{1}{2}$ years for its activity to decay to one-half its original value. During the transformation it gives out only slow moving beta particles. The succeeding product is radium E. In its transformation to radium F, it gives off beta particles and gamma rays. The atomic weight of radium E and radium F is thus the same. It takes 5 days for its activity to decrease to one-half of its original value. Radium F, which is the product of the decay of radium E, ejects an alpha particle and thus gives rise to a new substance of atomic weight $210.0 - 4.0 = 206.0$. From radium F there is thus finally produced radioactive lead, having an atomic weight of 206.5.

The table shown on page 671 gives in detail these transformations for radium, and Fig. 709 represents them diagrammatically.

705. Age of the Earth.—Since each alpha particle is an atom of helium which has lost two electrons, helium must be in the process of formation in all minerals containing radioactive substances. Many minerals found in the crust of the earth contain uranium, which is a radioactive substance. The number of alpha particles given out by 1 g. of uranium in equilibrium

with its radioactive products has been found to be 9.7×10^4 per second. Hence, each gram of uranium produces each year 11×10^{-5} c.c. of helium. If this helium were all occluded and retained by the mineral, the ratio of the amount of helium to the amount of uranium would give an estimate of the age of the mineral. Without doubt some of the helium escapes

URANIUM-RADIUM SERIES

Substance	Radioactive constant in $(\text{seconds})^{-1}$	Half-value period	Radiations emitted	Range of alpha particles in air in centimeters
Uranium 1........	4×10^{-18}	5×10^9 years	α	2.37
Uranium Y........	5×10^{-6}	1.5 days	β (slow)	
Uranium X_1........	3×10^{-7}	24.6 days	β (slow)	
Uranium X_2.	$.0 \times 10^{-2}$	1.15 min.	β	
Uranium 2...	$\times 10^{-14}$	2×10^6 years	α	2.75
Ionium......	$\times 10^{-13}$	2×10^5 years	α	2.85
Radium.........	$.26 \times 10^{-11}$	1730 years	α	3.13
Radium emanation.	$.085 \times 10^{-6}$	3.85 days	α	3.95
Radium A.	$.85 \times 10^{-3}$	3.0 min.		4.50
Radium B.	$.33 \times 10^{-4}$	26.7 min.	β (slow)	
Radium C^a.	$.93 \times 10^{-4}$	19.5 min.	β	
Radium C_2.	$.3 \times 10^{-3}$	1.4 min.	β	
Radium C_1.		10^{-6} sec.	α	6.57
Radium D..	$.3 \times 10^{-9}$	16.5 years	β (slow)	
Radium E.	$.6 \times 10^{-6}$	5.0 days	β (slow)	
Radium F (polonium)..........	5.90×10^{-8}	136 days		3.58
?Lead............				

[a] The production of radium C_1 from radium C is attended by the expulsion of a beta particle only. The expulsion of an alpha particle from radium C produces radium C_2. As this alpha gives rise to the branch series only, it is omitted from the table to avoid confusion.

from the mineral and this estimate of the age of the mineral would be too low. A determination of the uranium and helium in different kinds of rocks has shown that the ratio of the amount of helium to the amount of uranium is largest in those formations which, from geological considerations, are known to be the oldest. In this way Strutt has found the ages of the following rocks:

Rock	Age, in Years
Oligocene	8,000,000
Eocene	31,000,000
Carboniferous	150,000,000
Archaean	700,000,000

706. The Heating Effect of Radium.—Since the various radiations from radioactive substances are emitted with very high velocities, it is evident that radioactive substances must be giving off energy in considerable quantities. If, for example, radium is enclosed in a small vessel with walls of sufficient thickness to absorb all the alpha and beta particles which it emits, its temperature will be higher than that of the surrounding medium. Because the alpha particles have much larger masses than the beta particles, they carry most of this energy. One gram of

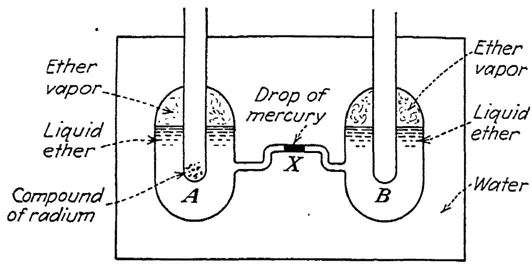


FIG. 710.—Apparatus to measure the heating effect of radium.

radium in equilibrium with its short-lived products produces from 120 to 130 cal. of heat per hour.

The heat generated by a radioactive substance can be measured by means of an apparatus designed by Duane. The radioactive substance is placed in a bulb A (Fig. 710) which is connected to a second bulb B of the same size. Both bulbs contain a volatile liquid like ether. A small drop of mercury X acts as an index to show the equality of pressure in both bulbs. Since the radioactive substance in A emits heat, the temperature of A will be greater than that of B, and the vapor pressure of the liquid in A will be greater than its vapor pressure in B. The movement of the index X gives a measure of this inequality of temperature. This heating effect due to the radioactive substance in A can be measured by passing a current of electricity through a coil of wire immersed in B. By adjusting the current in this coil until the index remains stationary, the emission of

the heat by the radioactive substance is made equal to the heat generated in the coil. An emission of heat equal to 0.0002 cal. per second can be determined in this way. The energy thus liberated by radioactive substances is derived from the internal energy of the atoms themselves.

707. Chemical Action of Radium.—Certain metals like mercury, aluminum, and lead undergo oxidation when exposed to the radiations from radium. Platinum is attacked by radium salts in solution, and radium emanation causes water partially to decompose into hydrogen and oxygen. In some cases, the beta rays from radium produce a synthetic effect. Thus, when a mixture of hydrogen and chlorine is exposed to beta rays, there is a combination of hydrogen and chlorine to form hydrochloric acid. When paraffin wax is exposed to radium and its emanations, it becomes hard and brittle and acquires a brownish tinge. If liquid paraffin is exposed to the emanation from radium, it acquires a dark color and becomes opaque. The effects produced in the more complex organic substances are extremely varied and highly complicated.

CHAPTER LX

STRUCTURE OF THE ATOM

708. Thomson's Atomic Model.—A model of the atom proposed by Sir J. J. Thomson assumed that the positive electricity is distributed uniformly throughout a sphere which has a radius equal to the radius of the atom. Inside of this sphere there is a number of electrons such that the amount of negative electricity on the electrons is just equal to the amount of positive electricity in the sphere. These electrons are supposed to be embedded in the sphere of positive electricity somewhat like plums in a pudding. This model of the atom was very successful in representing many of the more obvious chemical and physical properties of the atoms. It has, however, many limitations. For example, it cannot account for the emission of light by atoms when they are in the excited state.

709. Rutherford-Bohr Atomic Model.—The atomic model proposed by Thomson was entirely replaced by one proposed by Sir Ernest Rutherford. According to this model an atom consists of a positively charged nucleus which is very small compared to the size of the atom. In this nucleus is located nearly all the mass of the atom. Around this nucleus there is a distribution of planetary electrons in orbital motion. The total charge on these planetary electrons is just equal to the net positive charge on the nucleus. This atom is like a planetary system with the nucleus replacing the sun and the electrons replacing the planets. The size of these central suns differs from atom to atom, and the number of planetary electrons also differs from atom to atom.

The mass of the atom is almost completely determined by the mass of the nucleus. The chemical and most of the physical properties of the atoms are determined by the planetary electrons. For some problems, however, the behavior and distribution of the electrons around the nucleus are of no great importance. For example, they play no appreciable part in the large angle scattering of alpha particles. To explain the emission

of light by atoms, it is necessary to have a detailed picture of the orbits and movements of these planetary electrons. These orbits are distributed in three dimensions and become very complicated for atoms of high atomic weight.

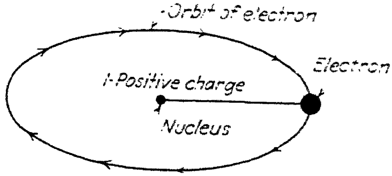


FIG. 711.—Neutral hydrogen atom.

710. The Hydrogen Atom.—In the normal condition the hydrogen atom (Fig. 711) consists of a positive nucleus and one planetary electron in orbital motion about this nucleus. Later developments in physics have shown that the electron can not be

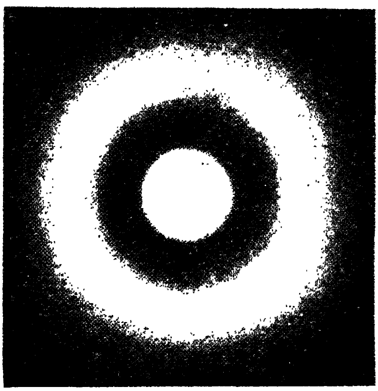


FIG. 712.—Pictorial representation of an electron as if it were a ring about the nucleus. (*H. E. White.*)

accurately considered as if it were in orbital motion about the nucleus. It must rather be considered as a ring of electricity about the nucleus (Fig. 712). For an elementary discussion, the concept of orbits is helpful.[1] When the electron is by any means

[1] *The Rutherford-Bohr atomic model has been modified and extended and in large measure replaced by more powerful methods of description and analysis. Since it is impossible to present these more accurate and sufficient ideas about the structure of the atom in an elementary text, the language and concepts of the Rutherford-Bohr atomic model have been retained as a first approach to subatomic physics.*

torn away from the hydrogen atom, the nucleus alone remains with a single positive charge of electricity on it. It is then a positive hydrogen ion which is the same as a **proton.** With it is associated nearly all the mass of the hydrogen atom. The positive charge on this single proton is just equal to the negative charge on the electron so that the atom in the normal condition is neutral.

Since there is an attractive force between the proton and the electron, the electron would fall into the proton just as the earth would fall into the sun except for the fact that the former is revolving about the latter. It is, however, assumed that the attractive force between the proton and the electron just balances the centrifugal force. Such a motion of the electron would normally be accompanied by a continuous radiation of energy. This loss in energy would cause a decrease in the speed of the electron which would gradually come closer and closer to the nucleus. Apparently this does not happen, and Bohr has assumed that the electron does not radiate energy when moving around in its orbit, but that radiation takes place only when the electron jumps from one orbit to another.

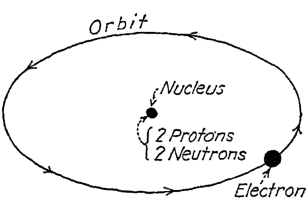


FIG. 713.—Ionized helium atom.

711. Helium.—When we pass from hydrogen, which has an atomic weight, an atomic number, and an atomic charge of unity, we next come to helium with an atomic weight of 4 and an atomic number and nuclear charge of 2. The helium nucleus, with which is associated most of the mass of the helium atom consists of two neutrons and two protons. Outside of the nucleus are two electrons. These electrons are not now considered as point charges revolving as planets about the nucleus but as diffuse rings of electricity occupying the entire orbit. These imaginary orbits do not lie in the same plane, but the older representation, which is now quite out of date, is still helpful at this stage. When the atom is in the excited state, the electrons can move in a great number of different orbits. When an electron passes from one of these orbits to another, a characteristic line of the helium spectrum is emitted. If one of the planetary electrons has been detached from the atom, a helium ion is produced (Fig. 713), with a single positive charge on it. The normal helium atom has no

affinity for other electrons. For this reason, helium does not unite with other elements. It is one of the inert gases and is monatomic.

712. Lithium.—In lithium, besides the nucleus which has three positive units of electricity on it, there are three planetary electrons. These electrons, according to the picture of the atom suggested by Bohr, revolve in different planes. The two inner electrons are bound more closely by attractive forces to the nucleus than is the outer electron. When lithium is ionized, it loses one electron, and the residue of the atom then has 1 unit of positive electricity on it and is known as a lithium ion. This outer electron determines the valency of lithium. When lithium enters into chemical union, it shares this outer electron with the element with which it combines. Under certain circumstances, a second electron may enter the outer orbit. The lithium atom is then charged negatively and becomes a negative ion. The possible orbits in which these planetary electrons can move are more complex than they are in either hydrogen or helium. The positions and characteristics of these orbits are determined from an analysis of the spectrum which the lithium atom can emit.

713. Beryllium.—If an atom of beryllium loses one of its four outer electrons, it becomes a positive ion carrying one charge of positive electricity. When it loses both of the outer electrons, it is a double charged positive ion. These outer electrons determine the valency of the beryllium atom, and it shares these two electrons with the other atoms with which it unites in forming chemical compounds. A study of the spectrum of beryllium makes it possible to locate the various orbits in which these electrons are capable of moving in agreement with the Bohr theory. The greater the number of planetary electrons, the more complex these orbits become. They are distributed in three dimensions and have such positions and shapes that they account for the observed lines in the spectrum on the basis that light of a definite frequency is emitted whenever an electron passes from one orbit to another.

714. Atoms of Higher Atomic Weight.—As the atomic weight increases and the number of planetary electrons revolving about the nucleus is also increased, the problem of determining the shape and size of the different orbits becomes more difficult. The evidence comes from spectroscopic data and the arrange-

ment of the elements in the periodic table. This evidence will be considered in detail in later sections. Figure 714 shows a diagrammatic representation of the electronic orbits in copper.

715. Periodic Table.—Table I gives a periodic classification of the elements. It is a modernized version of Mendeléeff's table and shows the similarities between the chemical and physical properties of the elements. As the atomic weight or more accurately the atomic number of the elements increases, there is a periodic variation in their physical and chemical properties. For example, beginning with lithium having an atomic weight of 6.94 and an atomic number of 3, the atomic weight and the atomic number increase until neon with an atomic weight of 20.18 and an atomic number of 10 is reached. There are eight elements in this period. A new period begins with sodium which has an atomic weight of 22.99 and an atomic number of 11 and ends with argon which has an atomic weight of 39.94. The elements in each group have similar chemical and physical properties. For example, elements in Group I have a valence of one and are chemically active. They also have similar spectra. The elements in Group VIII are inert gases which do not combine readily with other elements.

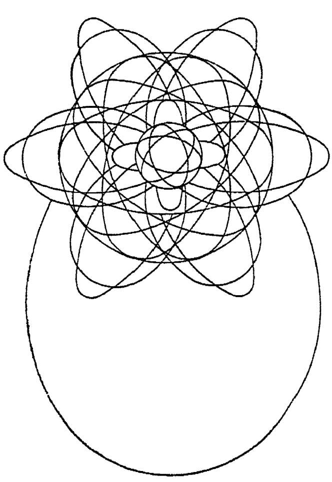


FIG. 714.—Atom of copper magnified 240 million times.

This periodic variation in the properties of the elements can be explained on the basis of the Rutherford-Bohr atomic model. The charge on each nucleus must be a multiple of the charge on the proton which can be denoted by $+e$. We must then get atoms with a nuclear charge of $+e$, $+2e$, $+3e$, $+4e$, etc. The nuclear charge increases with the atomic weight and is always equal to $+Ne$, where N is the atomic number or the ordinal number giving the position of the element in the periodic table. Thus, N for hydrogen $= 1$; N for helium $= 2$; N for lithium $= 3$; etc.

716. Arrangement of Electrons in Shells.—The chemical and spectroscopic properties of the elements are determined by the number of electrons outside of the nucleus. Elements with the same nuclear charge or the same number of electrons outside of the nucleus have the same chemical and spectroscopic properties. Now it has been seen that these properties vary periodically with the atomic number. For example, atoms which have the atomic number 2, 10, 18, 36, etc., are inert gases which do not

Period	Group I a	Group I b	Group II a	Group II b	Group III a	Group III b	Group IV a	Group IV b	Group V a	Group V b	Group VI a	Group VI b	Group VII a	Group VII b	Group VIII a	Group VIII b
I	1 H 1.0078															2 He 4.002
II	3 Li 6.940		4 Be 9.02			5 B 10.82		6 C 12.000		7 N 14.008		8 O 16.000		9 F 19.00		10 Ne 20.183
III	11 Na 22.997		12 Mg 24.32			13 Al 26.97		14 Si 28.06		15 P 31.02		16 S 32.06		17 Cl 35.457		18 A 39.94
IV	19 K 39.10		20 Ca 40.07		21 Sc 45.10		22 Ti 47.90		23 V 50.96		24 Cr 52.01		25 Mn 54.93		26 Fe 55.84 27 Co 58.94 28 Ni 58.69	
		29 Cu 63.57		30 Zn 65.38		31 Ga 69.72		32 Ge 72.60		33 As 74.96		34 Se 79.2		35 Br 79.916		36 Kr 82.9
V	37 Rb 85.44		38 Sr 87.63		39 Y 88.92		40 Zr 91.22		41 Cb 93.1		42 Mo 96.0			43	44 Ru 101.7 45 Rh 102.91 46 Pd 106.7	
		47 Ag 107.880		48 Cd 112.41		49 In 114.8		50 Sn 118.70		51 Sb 121.5		52 Te 127.5		53 I 126.932		54 Xe 130.2
VI	55 Cs 132.81		56 Ba 137.36		57 to 71 Rare earths		72 Hf 178.6		73 Ta 181.5		74 W 184.0		75 Re 188.7		76 Os 190.8 77 Ir 193.1 78 Pt 195.23	
		79 Au 197.2		80 Hg 200.61		81 Tl 204.39		82 Pb 207.22		83 Bi 209.00		84 Po 210		85		86 Rn 222
VII	87		88 Ra 225.97		89 Ac		90 Th 232.12		91 Pa		92 U 238.14					

readily combine with other elements. This fact is interpreted to mean that in each of these cases the electrons form stable systems which do not readily combine with the electrons of other atoms. Atoms, like lithium, sodium and potassium, which have one more electron outside of the nucleus than the corresponding inert gas has, have a valence of one and are strongly electropositive. This means that they easily lose one electron and combine readily with elements in which there is a strong attraction for an additional electron. On the other hand, atoms, like fluorine, chlorine and bromine have one electron less than the corresponding inert gas and are strongly electronegative. They also have a valence of one and strongly attract an additional electron to form a stable group of electrons like the groups found in the inert gases.

To explain the periodic variations of the physical and chemical properties of the atoms with an increase in the atomic number it is assumed that only a limited number of electrons can be arranged on a spherical surface with the nucleus as a center. When the number of electrons exceed the number which can be in stable equilibrium on one spherical shell, they arrange themselves on a series of concentric spherical shells. Thus, the distribution of the electrons outside the nucleus is as if they were arranged in a series of spherical layers separated from each other by finite distances. The number of these layers depends on the number of electrons in the atom. The electrons in the outside layer are held less firmly by the attractive forces due to the charges on the nucleus than are those on the interior shells. They will, therefore, be more easily displaced by the action of forces which might be exerted by neighboring atoms. It is thus the outer spherical shell of electrons which determines the chemical and spectroscopic behavior of the atom. On the other hand, the electrons near the nucleus are securely held by the electrostatic forces due to the nucleus and can be displaced only with the greatest difficulty. These electrons determine the characteristic X-ray spectra emitted by the elements. These inner electrons are little affected by the presence of neighboring atoms and change little when atoms enter into chemical combination with other atoms.

Beginning with the nucleus and moving outward, the first group is known as the *K*-shell. When it is completed it contains two electrons. This atom is the atom of helium with an atomic

number 2 and an atomic weight 4. The next group known as the *L*-shell when completed contains eight electrons. The atom which is formed when this shell is just completed is neon with an atomic number 10. In building up the *L*-shell from the *K*-shell the number of electrons in this shell increases from one to eight and this change corresponds to passing from lithium with an atomic number 3 to neon with an atomic number 10. The formation of the next group known as the *M*-shell in which there are eight electrons corresponds to a transition from sodium with an atomic number 11 to argon with an atomic number 18. A further addition of electrons after the completion of the *M*-shell begins the *N*-shell with potassium whose atomic number is 19. Beyond the next element, caesium, there are irregularities in the formation of these shells, but the essential plan of building them remains unchanged.

CHAPTER LXI

NUCLEAR PHYSICS

To get information about the structure of the nucleus, it is necessary to bombard it with different kinds of atomic projectiles which have energies sufficiently large to break it up into fragments. Alpha particles from Radium C have a speed of about 12,000 miles per second. The large energies associated with them at once suggested to Rutherford that they could be used as atomic projectiles for the disintegration of atomic nuclei.

717. Bombardment of Atoms with Alpha Particles.—When Rutherford bombarded nitrogen with alpha particles, he found

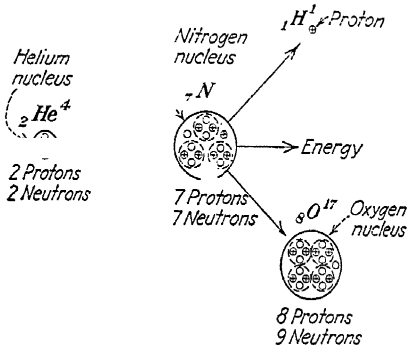


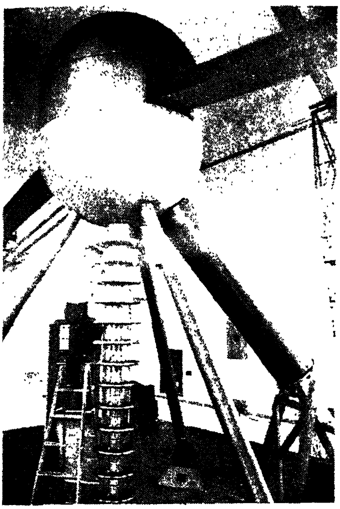
FIG. 715.—Nitrogen bombarded with alpha particles gives oxygen and hydrogen.

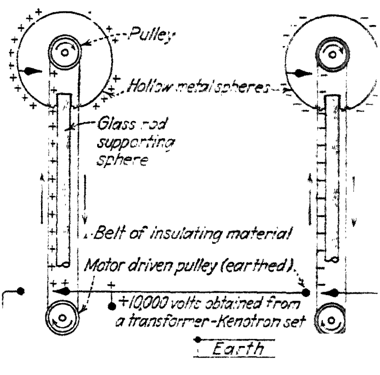
that, in the collision of an alpha particle with the nucleus of a nitrogen atom, a proton or a hydrogen nucleus is sometimes ejected. Since nitrogen has an atomic mass of 14 and a nuclear charge of 7 and the alpha particle (the nucleus of the helium atom) has an atomic mass of 4 and a nuclear charge of 2, there are altogether 18 units of atomic mass and 9 units of positive charge involved in the collision. One unit of positive charge and one unit of atomic mass are ejected as the nucleus of the hydrogen atom. There remain, therefore, 17 units of atomic mass and 8 units of positive charge. These are the units of atomic mass and positive charge which correspond to a heavy form of oxygen,

that is, to oxygen of atomic mass 17 instead of atomic mass 16. This form of oxygen was not known at the time these experiments were performed. It was later discovered as a normal constituent of chemical oxygen. In this way nitrogen with an atomic mass of 14 was transformed into oxygen with an atomic mass of 17 and hydrogen produced as a by-product. This reaction is described in the following equation and represented diagrammatically in Fig. 715.

$$_2\mathrm{He}^4 + {}_7\mathrm{N}^{14} \rightarrow {}_8\mathrm{O}^{17} + {}_1\mathrm{H}^1 + \text{energy}.$$

The subscript on each of the atomic symbols gives the atomic number of the element or the number of positive charges on the nucleus. The superscript gives the atomic mass of the element.





(a) (b)

FIG. 716.—(*a* and *b*) A Van de Graaf electrostatic machine for producing very high voltages, used in nuclear disintegrations. Differences of potential in excess of two million volts are thus available. (*Courtesy Department of Terrestrial Magnetism, Carnegie Institution.*)

In later experiments many other illustrations of similar transmutations of elements have been found by this method.

718. Acceleration of Charged Particles.—Before other atomic projectiles could be used, it was necessary to develop methods by which electrified particles could be accelerated so that they would have sufficient energy to make them effective for atomic disintegrations. To produce directly the necessary high voltages, a system of transformers with provision for rectifying the voltages

has been used by Lauretsen at the California Institute of Technology, by Cockroft and Walton at the Cavendish Laboratory, and by many other physicists. An electrostatic machine (Fig. 716) has been perfected by Van de Graaf. It has been used with great success by Tuve and his collaborators in the Department of Terrestrial Magnetism of the Carnegie Institution. Voltages in excess of a million volts can be obtained by either of these methods.

A unique apparatus for generating high voltages has been designed and developed by Lawrence at the University of Cali-

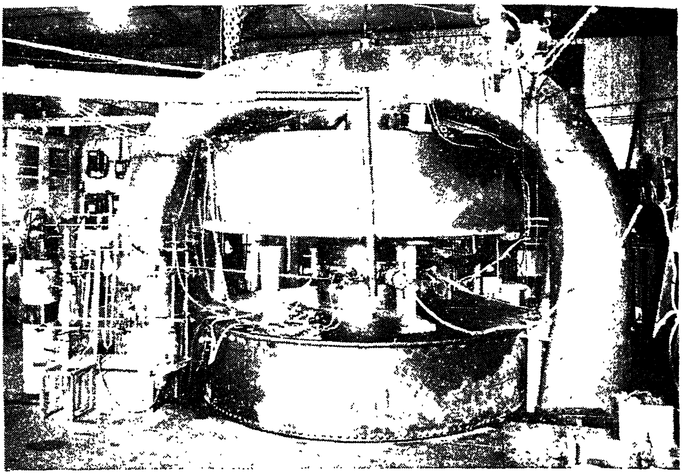
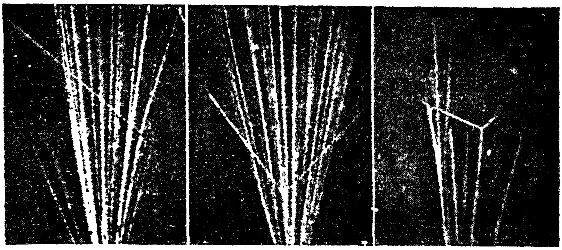


FIG. 717.—The cyclotron, used to accelerate electrified particles in atomic disintegrations. Particles with energies equal to seven or eight million electron volts have been obtained in this way. (*Courtesy E. O. Lawrence, University of California.*)

fornia. It is known as a cyclotron. It involves both a magnetic field and an oscillating electric field. The most important feature of the apparatus is a great electromagnet weighing 85 tons (Fig. 717). Ions from some suitable source are accelerated in a vacuum chamber between the poles of the magnet. The magnetic field deflects the ions so that they move most of the time in circular paths with constant angular velocity. Within the vacuum chamber are two semicircular hollow electrodes, between which is applied a high-frequency potential difference. The ions travel around from within one electrode to within the other and as they cross the region between the electrodes, they gain velocity cor-

responding to the difference of potential. The ions are thus made to spiral around in synchronism with the oscillating electric field. They go faster and faster on ever widening spirals and finally emerge from the vacuum chamber through a thin aluminum window. Deuterons with energies of about 8 million electron volts have been obtained in this way. A substance placed just outside the aluminum window can be bombarded for any desired period of time.

719. Cloud-chamber Method of Observing Collisions.—A number of ingenious methods have been devised for observing atomic collisions. One of the most important of them is the **cloud-**



(a) (b) (c)

FIG. 718.—Cloud-chamber tracks made by alpha particles showing collisions with nuclei of different atoms. (a) With hydrogen; (b) with helium; (c) with oxygen. Note in (b) that the track due to the alpha particle and that due to the helium atom are equal in length and make equal angles with the direction of the incident alpha particle. In the forked track in (c) the track to the left is due to the alpha particle and that to the right is due to the oxygen atom. (*Brackett. Courtesy Royal Society of London.*)

chamber method, by means of which the tracks of the colliding particles are photographed before and after the collision. If a rapid expansion takes place in a vessel containing saturated water vapor, a fog is formed, provided there are nuclei about which the water vapor can condense. When swiftly moving electrified particles pass through such a vessel, ions are produced along the tracks of the particles. These ions serve as centers about which the water vapor can condense when a suitable expansion takes place in the chamber. The fog which is thus produced by the expansion is confined to these tracks and serves to locate the path of the moving electrified particle. By photographing these tracks, the behavior of the particles before and after collision can be studied. Figure 718 shows the tracks made by an alpha particle before and after collision with an atom. In the case of helium the masses of the two particles are nearly equal and the

paths after collision make nearly equal angles with the path of the alpha particle before collision.

720. The Positron.—When a strong magnetic field is impressed on a cloud chamber, charged particles moving across the magnetic field are deflected and made to move in circular paths. By observing the tracks made by these particles, it is possible to determine the mass and the charge of the particles responsible for each track. In such observations it is necessary to keep in mind the direction of the magnetic field. In Fig. 719 it is away

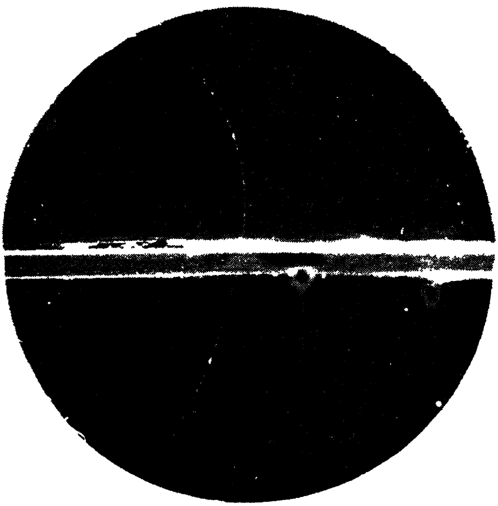


FIG. 719.—The track of a positron. Above and below the lead plate can be seen the path of a positron which has been deflected by a magnetic field perpendicular to the plane of the figure. The direction of the deflection is such that a positron and not an electron produced the track. (*Courtesy Carl Anderson, California Institute of Technology.*)

from the reader. In this case a negatively charged particle moving downward across the lead plate in the middle of the figure might give this path. On the other hand, a positively charged particle moving in the opposite direction might give this same path. If now it were possible to determine whether the particle moved upward or downward in making this track, it would be possible to determine whether the particle was positively or negatively charged.

The curvature of the track below the plate corresponds to the track of a charged particle with the rest mass of an electron and

an energy equal to 63 million electron volts. The curvature of the track above the plate, on the other hand, corresponds to the path of a particle of the same mass and charge as the electron but with an energy of only 23 million electron volts. Since the particle could not have gained energy in passing through the plate, it must have passed upward rather than downward through the plate. This direction of motion is consistent with a decrease in energy of the particle in passing through the plate. Hence, a positive particle moving from the bottom to the top of the figure must have produced this track. The characteristics of the track are those which might be expected if the track had been made by

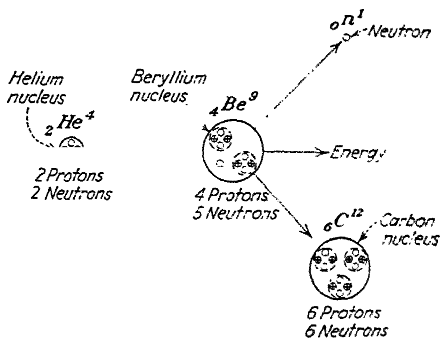


FIG. 720.—Beryllium bombarded with alpha particles gives carbon and neutrons.

an electron moving down through the plate. Since the track was made by a particle moving in the opposite direction, it can only be explained by assuming the existence of a new kind of particle, like an electron in all respects, except that it has a positive instead of a negative charge. Moving with an energy equal to 63 million electron volts, this particle passes upward through the plate in which its energy is reduced to 23 million electron volts. In this way Anderson at the California Institute of Technology discovered the **positron. A positron has the same mass as an electron but it carries a positive charge just equal in magnitude to the negative charge on the electron.**

721. The Neutron.—When an element like beryllium is bombarded by alpha particles, uncharged particles of great penetrating power are emitted. These particles have nearly the mass of a proton, that is, nearly the mass of the nucleus of the hydrogen atom. They are called **neutrons.** The name indicates that they

are uncharged particles. They may be thought of as the atoms of an element with an atomic number equal to 0 and atomic weight equal to 1. Symbolically they are characterized by ${}_0n^1$. Because these particles are without charge, they have very large penetrating power. They move with very large speeds and consequently have large energies. They transfer their momentum and energy to the lighter elements of the compounds through which they pass. If they pass through a compound containing hydrogen, protons are ejected with high velocities and great penetrating power. Chadwick assumed that, in the capture of an alpha particle by an atom of beryllium, a neutron is ejected in agreement with the following formula and diagram (Fig. 720):

$${}_2He^4 + {}_4Be^9 \rightarrow {}_6C^{12} + {}_0n^1 + \text{energy}.$$

722. Deuterium and the Deuteron.—In 1932, Urey, a physical chemist at Columbia University, found from the study of the spectrum of hydrogen that there is a second form of hydrogen which has nearly double the atomic weight of ordinary hydrogen. It combines with oxygen to form heavy water with a molecular weight of 20 as compared to 18, the molecular weight of ordinary water. This form of heavy hydrogen is present in ordinary hydrogen in the ratio of about 1 part in 5,000. It can be separated from ordinary hydrogen by an elaborate electrolytic process. The nucleus of heavy hydrogen consists of a neutron and a proton, but the nucleus of ordinary hydrogen consists only of one proton. The symbol for heavy hydrogen is ${}_1H^2$, the subscript indicating the positive charge on the nucleus and the superscript indicating the atomic weight. The following table gives a comparison of the charges and masses of these particles:

	Charge in e.s.u.	Mass in grams
Negative electron (electron)........	-4.77×10^{-10}	9.04×10^{-28}
Positive electron (positron).........	$+4.77 \times 10^{-10}$	9.04×10^{-28}
Proton (hydrogen nucleus).........	$+4.77 \times 10^{-10}$	1.7×10^{-24}
Neutron.........................	0	1.7×10^{-24}
Deuteron (heavy-hydrogen nucleus).	$+4.77 \times 10^{-10}$	3.4×10^{-24}

723. Transmutation by Capture of Protons, Deuterons, or Neutrons.—When atomic nuclei are bombarded by swiftly moving protons, deuterons, or neutrons, the nuclei may be trans-

formed into atomic nuclei having either greater or smaller atomic masses. The following are a few simple illustrations.

1. *Transmutations by the Capture of Protons.*—When lithium ($_3Li^7$) is bombarded with protons, two atoms of helium are pro-

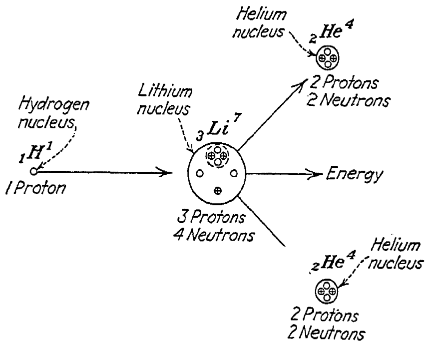


FIG. 721.—Lithium of atomic weight 7 bombarded with protons gives two atoms of helium.

duced as described in the following formula and indicated in Fig. 721:

$$_1H^1 + {}_3Li^7 \rightarrow 2\ {}_2He^4 + \text{energy}.$$

2. *Transmutation by the Capture of Deuterons.*—If the lighter form of lithium ($_3Li^6$) is bombarded with deuterons, two atoms of

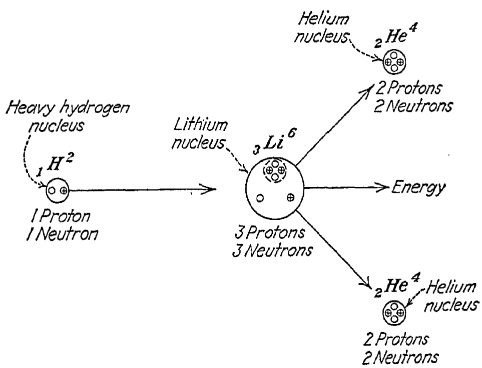


FIG. 722.—Lithium of atomic weight 6 bombarded with deuterons gives two atoms of helium.

helium are again produced. The total units of mass and the total units of charge are the same in the two cases. Hence the end result is the same as in the preceding case, as is shown by

the formula of the nuclear reaction and by Fig. 722.

$${}_1H^2 + {}_3Li^6 \rightarrow 2\ {}_2He^4 + \text{energy}.$$

3. *Transmutations by Capture of Neutrons.*—As an illustration of this type of transformation consider the bombardment of

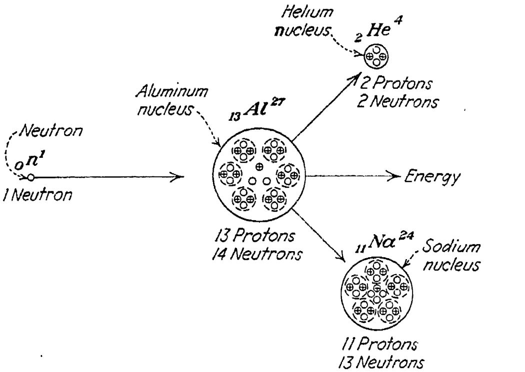


FIG. 723.—Aluminum bombarded with neutrons gives helium and sodium. The sodium has an atomic weight of 24 and is unstable.

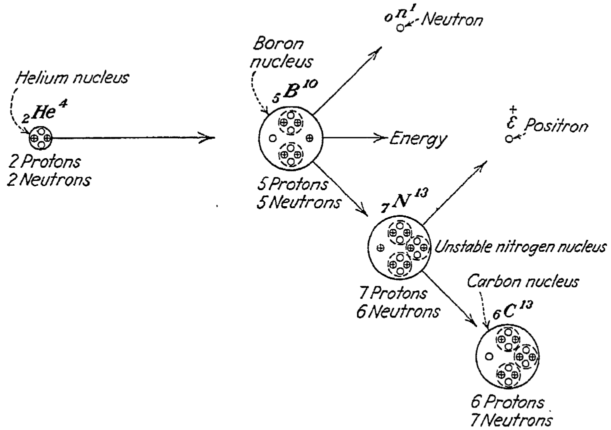


FIG. 724.—Boron bombarded with alpha particles gives nitrogen and neutrons. The nitrogen has an atomic weight of 13. It is unstable and changes into carbon with the ejection of a positron.

aluminum by neutrons. An atom of helium $_2He^4$ and an atom of sodium $_{11}Na^{24}$ are produced (Fig. 723). The sodium has an atomic weight of 24 instead of 23 and is unstable. The formula of the nuclear reaction is as follows:

$${}_0n^1 + {}_{13}Al^{27} \rightarrow {}_{11}Na^{24} + {}_2He^4 + \text{energy}.$$

724. Artificial Radioactivity.—When boron is bombarded with alpha particles, neutrons and positrons are emitted. The emission of the positrons continues for some time after the bombardment has been discontinued. The interpretation of this phenomenon is evident. The neutrons and the positrons do not come from the boron simultaneously. The neutrons are ejected first and the positrons later. The ejection of neutron leaves the nucleus with an atomic mass of 13 and an atomic charge of 7. There has thus been produced a form of nitrogen which is

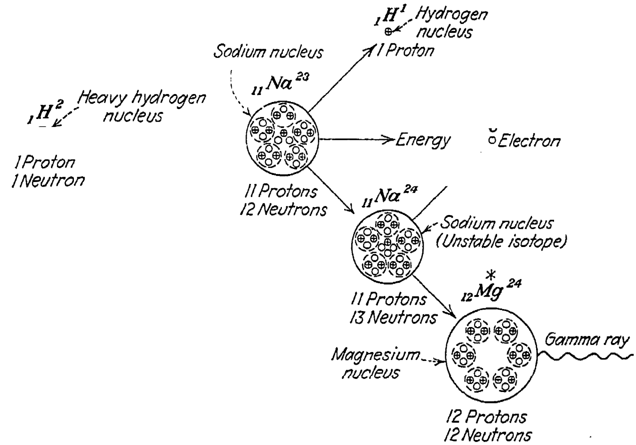


FIG. 725.—Sodium with an atomic weight of 23 bombarded with deuterons gives sodium with an atomic weight of 24. This form of sodium is unstable. It changes to magnesium with the emission of an electron.

unstable. It quickly changes into carbon which is stable. The steps which occur in these transformations are indicated in the following nuclear formulas and represented diagrammatically in Fig. 724. The radioactive nitrogen thus produced is not long-lived. It is rapidly transformed into carbon.

$$_2He^4 + {_5B^{10}} \rightarrow {_7N^{13}} + {_0n^1} + \text{energy};$$

$$_7N^{13} \rightarrow {_6C^{13}} + \overset{+}{e} + \text{energy}.$$

As a further illustration of artificial radioactivity consider the case of sodium. When ordinary sodium is bombarded with deuterons, a heavier form of sodium ($_{11}Na^{24}$) is produced and a proton ($_1H^1$) is ejected. Now sodium ($_{11}Na^{24}$) is unstable. It ejects an electron from the nucleus and becomes magnesium

($_{12}Mg^{24}$). The magnesium thus formed is left in an excited state. It emits gamma rays and returns to a stable and unexcited state. The nuclear formulas and schematic diagrams (Fig. 725) are as follows:

$$_1H^2 + {}_{11}Na^{23} \rightarrow {}_{11}Na^{24} + {}_1H^1 + \text{energy};$$

$$_{11}Na^{24} \rightarrow {}_{12}Mg^{24} + \bar{e} + \text{energy}.$$

725. Protons and Neutrons in the Nucleus.—According to the present point of view, the nucleus of an atom is made up entirely of protons and neutrons. There are no electrons as such in the nucleus. The composition of a few atomic nuclei is shown in the following table:

PROTONS AND NEUTRONS IN THE NUCLEUS

Element	Symbol	Number of protons	Number of neutrons
Hydrogen (ordinary)	$_1H^1$	1	0
Hydrogen (heavy)...	$_1H^2$	1	1
Helium.............	$_2He^4$	2	2
Lithium I..........	$_3Li^6$	3	3
Lithium II.........	$_3Li^7$	3	4
Nitrogen...........	$_7N^{14}$	7	7
Oxygen.............	$_8O^{16}$	8	8
Neon...............	$_{10}Ne^{20}$	10	10
Sodium.............	$_{11}Na^{23}$	11	12
Chlorine...........	$_{17}Cl^{35}$	17	18

The number of protons determines the atomic number. The sum of the number of protons and neutrons gives the atomic weight. An isotope of an element is formed by adding one or more neutrons to the nucleus of the element. When an alpha particle is ejected by a nucleus, two protons and two neutrons in close combination escape from the nucleus. When a beta ray is ejected from a radioactive substance such as radium, probably a neutron is transformed into an electron and a proton. The electron escapes, but the proton remains in the nucleus, leaving the nucleus with one additional positive charge on it.

726. Explanation of Isotopes.—Figures 726 and 727 show diagrammatic representations of atoms of lithium and neon. The small dark circle represents the nucleus. The number of protons and the number of neutrons in it are indicated in each

case. The small circles represent the electrons which are outside of the nucleus. There are two kinds of lithium. In one kind there are three protons and three neutrons in the nucleus and three electrons outside of the nucleus. In the other kind of lithium there are three protons and four neutrons in the nucleus and three electrons outside of it. In each case the net charge on the nucleus is 3 units of positive electricity and the number of planetary electrons is also three. The external structure of the atoms is identical. The charge on the nucleus is the same, but the mass of the nucleus is in one case six and in the other case seven times that of hydrogen. In the neon atom, there are 10 protons and 10 neutrons in the nucleus in one case, and 10 protons and 12 neutrons in the other case. Each kind of neon has 10 electrons outside of the nucleus. Since the extranuclear electrons determine the chemical and physical properties of the atoms, both kinds of neon are chemically and physically identical. The net charge on the nucleus is the same in both cases. One kind of neon has, however, an atomic weight of 20 and the other an atomic weight of 22. Ordinary neon is a mixture of these two kinds of neon and has therefore an atomic weight between 20 and 22.

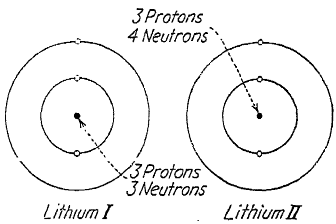


FIG. 726.—Diagrammatic representation of the isotopes of lithium.

Example.—The atomic weight of lithium as determined chemically is 6.94. What percentage of ordinary lithium has an atomic weight of 7?

Let x = per cent of lithium of atomic weight 7 in the mixture.

$$6.94 = 7\frac{x}{100} + 6\left(\frac{100 - x}{100}\right)$$

$$x = 94 \text{ per cent.}$$

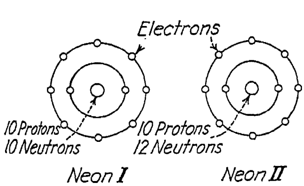


FIG. 727.—Diagramatic representation of isotopes of neon.

727. The Packing Effect.—Since the atomic weight of hydrogen which has one electron and one proton in it is 1.008, the atomic weight of helium might be expected to be $1.008 \times 4 = 4.032$. As a matter of fact, the atomic weight of helium is only 4. There has, therefore, been a decrease in mass. A similar decrease in mass occurs in the formation of the nuclei of other atoms. This decrease in mass in the formation of nuclei from protons and neutrons is called the *packing effect*. Because of this packing effect, the atomic weight of an element does not come out to be a whole number except

in the case of oxygen with an atomic weight of 16. The atomic weight of this element is taken as a standard with respect to which the packing effect of the other elements is measured. For this reason it is of necessity a whole number. From theoretical considerations, it is now believed that a decrease in mass is radiated as energy. Hence, the mass and energy of the nucleus together must be conserved; that is, when the mass decreases there must appear an equivalent amount of energy. The equation of Einstein giving the relation between the change of mass and the energy is

$$\text{Change of energy} = (\text{change of mass})\, c^2,$$
$$E \text{ (in ergs)} = m \text{ (in grams) } c^2,$$

where c = the velocity of light in centimeters per second.

If 4 g. of helium were formed from hydrogen in this way, the energy liberated would be

$$E = (0.032)(3 \times 10^{10})^2$$
$$= 0.288 \times 10^{20} \text{ ergs.}$$

728. Cosmic Rays.—In the early investigations of radioactivity a small residual ionization was found to exist in a gas even in the absence of radioactive sources. Most of this residual ionization disappeared when the ionization chamber was screened by enclosing it in a thick layer of lead. At first this residual ionization was attributed to radiations coming from the ground or the atmosphere. By sending recording electroscopes up in balloons, it was later found that this ionization increases as the height above the surface of the earth is increased. At a height of 9,000 m. it has a value about 40 times as great as at sea level. This result is inconsistent with the idea that these radiations come from the surface of the earth or the atmosphere. It thus becomes necessary to assume that they come from regions outside of the earth's atmosphere—from interstellar spaces.

In 1925 and 1926 Millikan and Cameron studied the ionization in closed vessels at different depths below the surface of water in mountain lakes and found that the ionization due to these radiations decreases with the depth of the vessels below the surface of the lake. These results together with other results obtained at high altitudes, especially those obtained in stratosphere flights and balloons of various kinds (Fig. 728), firmly established the fact that these radiations come to earth from interstellar spaces. These radiations are now known as **cosmic rays.** Most of the effects observed at sea level or at higher altitudes seem to be due to rapidly moving charged particles, probably electrons,

positrons, and protons. These particles have a wide range of energies which are dissipated as the particles pass through matter.

729. Intensity of Cosmic Rays as a Function of Latitude.—After it was established that cosmic rays come to the earth from interstellar spaces and increase in intensity with the altitude, a careful survey revealed that their intensity also changes with latitude. This dependence on latitude is seen from Fig. 729. Here the geomagnetic latitude rather than the geographic latitude has been used. From this figure it is seen that at sea level the intensity of these radiations is nearly constant from the magnetic

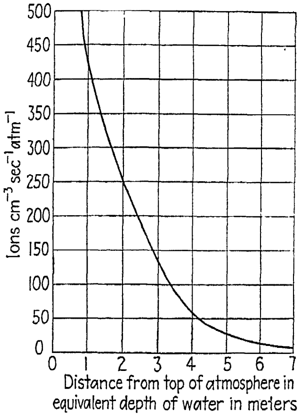


FIG. 728.

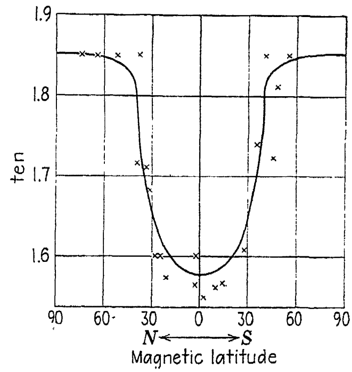


FIG. 729.

FIG. 728.—Variation of intensity of cosmic rays with distance from the top of the atmosphere. The intensity as measured by the ions produced increases rapidly with the altitude.

FIG. 729.—The intensity of cosmic rays varies with the latitude. It is a minimum at the geomagnetic equator.

pole to about 50 deg. It then decreases and reaches a minimum at the geomagnetic equator where it is about 16 per cent lower than at higher latitudes. This variation can be most easily accounted for by assuming that these rays are charged particles which are deflected as they cross the earth's magnetic field. The curvature of the paths of the particles will depend on the velocity as well as on the intensity of the earth's magnetic field. Some of the more slowly moving particles may be turned back and never reach the surface of the earth. The number of particles reaching the surface of the earth in the neighborhood of the magnetic equator is less than the number reaching it in the neighborhood of the magnetic poles.

730. Electron Pairs.—When cosmic rays, or gamma rays of very high frequency, pass through matter, pairs of electrons of opposite signs may be produced. Figure 730 shows a pair of

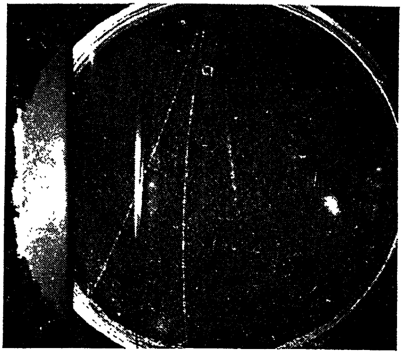
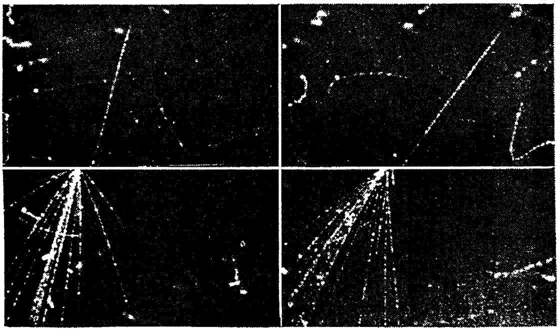


Fig. 730.—An electron pair, a negative electron and a positive electron, produced in a magnetic field. One is deflected to the right and the other to the left by the magnetic field. (*Courtesy J. C. Street and E. C. Stevenson, Harvard University.*)

electrons, one positive and the other negative, which has been produced in a magnetic field. They have been deflected in opposite directions by the magnetic field which is perpendicular



(*a*) (*b*)

Fig. 731.—(*a*) A shower of 19 rays apparently produced by an incident electron. These tracks were photographed with a cloud chamber. (*b*) A stereoscopic view of the same shower showing that the rays are emitted in all directions. (*Courtesy J. C. Street and E. C. Stevenson, Harvard University.*)

to the plane of the figure. Each pair of electrons is formed out of one photon. The energy and mass of the photon are almost completely transformed into the energy and mass of the two

electrons. A small amount of energy and momentum are transferred to the atom. A converse process may occur. A positive and negative electron may merge to produce a photon. Figures 731 (*a* and *b*) show a more complex shower which again consists of both positive and negative electrons. It was produced by cosmic rays passing through a lead plate in a cloud chamber.

CHAPTER LXII

SERIES IN OPTICAL SPECTRA

731. Units and Methods of Measurement.—In spectral measurements we are concerned either with the wave lengths of the lines or with the frequencies of the oscillations. The centimeter is the unit of length in terms of which wave lengths are ordinarily measured. Since this unit is too large various submultiples are used. The more familiar of these units are given in the following table.

Name of unit	Symbol	Value, centimeters
Micron....	μ	10^{-4}
Millimicron	$m\mu$	10^{-7}
Ångström..	Å.	10^{-8}
X-unit.....	XU	10^{-11}

The frequency is more fundamental for spectral series than is the wave length, but it is not observed directly. It must be computed from the wave length and the velocity of the radiation.

Let v = the velocity of the radiation in a vacuum expressed in centimeters.

ν = the frequency of the radiation expressed as oscillations per second.

λ = the wave length expressed in centimeters.

Then

$$c = \lambda\nu.$$

Since $c = 3 \times 10^{10}$ cm. and λ has values between 0.00007 and 0.00004 cm. for the visible region of the spectrum, the frequency involves very large numbers of the order of 10^{14}. For this reason, it is more convenient to use the number of waves per centimeter,

$$n = \frac{1}{\lambda}$$

The wave number is defined as the number of waves per centimeter. It varies with the nature of the radiation and the medium in which the radiation travels.

732. Balmer's Formula for Hydrogen.—Reasoning from the analogy of overtones in acoustics, early investigators of the relations between spectral lines tried to discover similar relation among the lines in the visible spectrum. Such attempts, however, were unsuccessful, but Balmer succeeded in making a significant beginning by showing that the wave lengths of the nine lines then known in the spectrum of hydrogen could be represented by the formula,

$$\lambda = k\frac{m^2}{m^2 - 4},$$

where k = a numerical constant, and m = an integer which can take the successive values 3, 4, 5, 6, etc.

The wave length corresponding to each integral value of m corresponds to the observed wave length of one of the lines in this spectrum. The following table shows the correspondence between the observed and the calculated values for nine lines in the spectrum of hydrogen.

m	λ (computed) Å.	λ (observed) Å.
3	6562.08	6562.10
4	4860.80	4860.74
5	4340.0	4340.10
6	4101.3	4101.2
7	3969.7	3968.1
8	3888.6	3887.5
9	3835.0	3834.0
10	3789.5	3795.0
11	3770.2	3767.5

The agreement between the observed and the calculated values suggests clearly that there is a simple relation between these spectral lines.

733. Rydberg's Formula.—Rydberg succeeded in finding a more general formula than that found by Balmer. He noted that the lines in any series tend to crowd together toward the ultra-violet according to a regular law. This crowding together toward the ultra-violet is shown in Fig. 732 for the Balmer series

of hydrogen and in Fig. 733 for a series of lines in sodium. Rydberg's formula predicted wave lengths very close to the observed wave lengths for the diffuse series of sodium, and accounted for the fact that the lines converge in a systematic way toward a head. A similar formula described the other series in sodium and

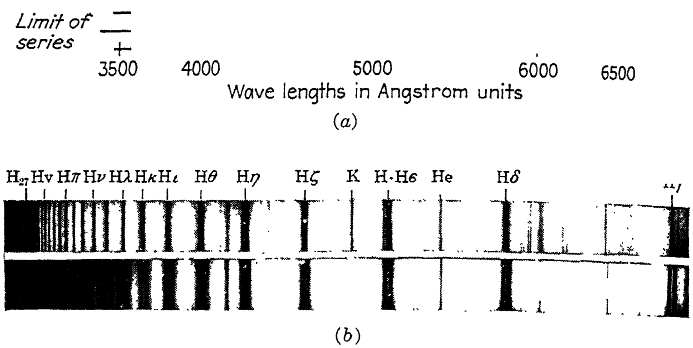


FIG. 732.—(a) Diagrammatic representation of the limit of the Balmer series. (b) Spectrum of the star ζ Tauri showing the Balmer series of hydrogen. (*Photographed by R. H. Curtiss, University of Michigan.*)

similar series in other elements, but the numerical constants were different.

Rydberg recognized three types of series (Fig. 734) which could be distinguished from each other by certain characteristics of the spectral lines. These series he denoted as the **principal** series,

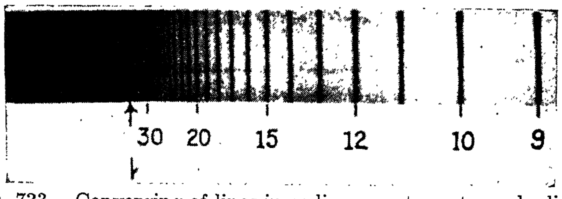


FIG. 733.—Converging of lines in sodium spectrum toward a limit.

the **sharp** series, and the **diffuse** series. Another series, called the **fundamental** series, has later been added to this classification. The lines in each of these four series are accurately described by a formula of the Rydberg type. But the numerical value of the constants is different for each series.

734. Bohr's Theory of the Hydrogen Atom.—Bohr was the first to develop a theory which would satisfactorily account for the way in which the lines in the spectrum of hydrogen are arranged in series. In order to understand the essential points of this theory, the spectrum of hydrogen must be considered in more detail. It has already been pointed out that it is possible to arrange the lines which the hydrogen atom can emit in series which are simply related to each other. Several such series have

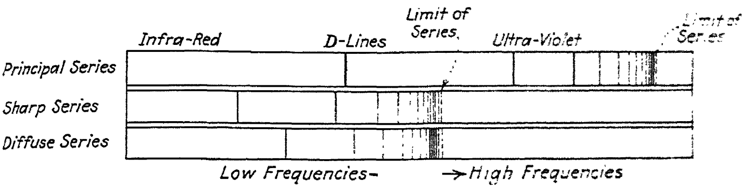


FIG. 734.—Diagrammatic representation of principal, sharp, and diffuse series in sodium.

been discovered. For our present purposes it is sufficient to consider three of them. They are known as *the Lyman series*, *the Balmer series*, and *the Paschen series*.

If ν represents the frequency of the emitted light and R a constant, then all the lines in the visible region of the spectrum can be represented by the formula,

$$= R\left(\frac{1}{2^2} - \frac{1}{n^2}\right),$$

where n takes the successive values 3, 4, 5, 6, etc. This series of lines is known as the Balmer series. As n is made larger and larger, the lines of the spectrum converge toward a limit called the limit of the series. Figure 732 shows diagrammatically this limit for the Balmer series, and Fig. 733 shows the frequency toward which lines in the spectrum of sodium converge.

There is a second series of lines which lie in the infra-red region of the spectrum. This series is known as the Paschen series. The lines in this series are accurately represented by the formula,

$$\nu = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right),$$

where n now takes the successive values 4, 5, 6, 7, etc.

Lyman discovered a third series of lines in hydrogen. This series lines in the ultra-violet region of the spectrum. The frequencies of these lines are accurately represented by the formula,

$$= R\left(\frac{1}{1^2} - \frac{1}{n^2}\right),$$

where n takes the values 2, 3, 4, 5, etc.

Bohr postulated that the hydrogen atom could exist in a number of different states due to the revolution of the electron about the nucleus in

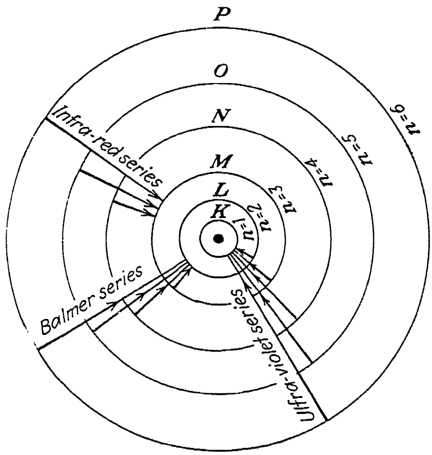


FIG. 735.—Circular orbits in hydrogen atom.

different orbits. The orbits of the electron were determined by imposing the condition

$$2\pi mva = hn, \tag{1}$$

where a = the radius of a circular orbit.
m = the mass of the electron.
v = the linear velocity of the electron in its orbit.
h = Planck's universal constant.
n = an integer like 1, 2, 3, etc.

When $n = 1$, the electron is assumed to be revolving in the circular orbit which is nearest the nucleus. When $n = 2$ (Fig. 735), it is revolving in the next largest orbit. So long as the electron remains in any one orbit its energy remains constant, but when it changes from one orbit to another its energy also changes. The energies of the electron in its different possible orbits as obtained from Bohr's theory are shown in Fig. 737 for the hydrogen atom. The lower horizontal line represents the energy which the electron has in its inner or ground orbit, and the second horizontal line represents the

energy which it has in the second orbit, etc. The difference in energy between any two levels is the energy which the electron would lose in passing from the orbit corresponding to one of these levels to the orbit corresponding to the other level. Now Bohr assumes that when the electron passes from one orbit to an inner orbit or, what is the same thing, from a higher to a

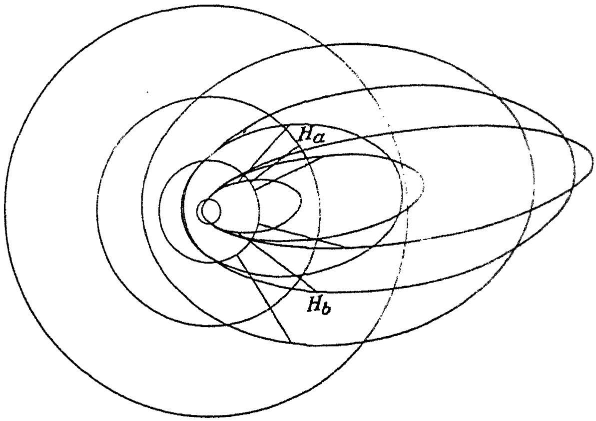


FIG. 736.—Circular and elliptic orbits in hydrogen atom.

lower energy level, the difference of energy is emitted as radiation of a definite frequency such that,

$$h\nu = E_n - E_x,$$

where h = Planck's constant.
ν = the frequency of the light.
E_n = the energy of the electron on the initial level = $2\pi^2e^4m \quad n^2h^2$.
E_x = the energy of the electron on the final level = $2\pi^2e^4m \quad x^2h^2$.

Substituting these values of E_n and E_x,

$$h\nu = \frac{2\pi^2e^4m}{n^2h} \qquad \frac{2\pi^2e^4m}{x^2h^2}$$

$$\frac{2\pi^2e^4m}{h^2}\left[\frac{1}{n^2} - \frac{1}{x^2}\right].$$

$$2\pi^2e^4m\left[\frac{1}{n^2} - \frac{1}{x^2}\right].$$

If

$$\frac{2\pi^2e^4m}{h^3} \qquad R$$

this equation becomes

$$= R\left[\frac{1}{n^2} - \frac{1}{x^2}\right].$$

If $n = 1$,

$$R\left[\frac{1}{1^2} - \frac{1}{x^2}\right],$$

where x takes the values 2, 3, 4, etc.

If $n = 2$,

$$R\left[\frac{1}{2^2} - \frac{1}{x^2}\right],$$

where x takes the values 3, 4, 5, etc.

If $n = 3$,

$$= R\left[\frac{1}{3^2} - \frac{1}{x^2}\right],$$

where x takes the values 4, 5, 6, etc.

These are precisely the equations which were found empirically to describe the Lyman series, the Balmer series, and the Paschen series for hydrogen.

735. Rydberg Constant.—The value of the constant R has been determined from spectroscopic data with great accuracy. This constant is known as the Rydberg constant and its numerical value is $R = 3.2888 \times 10^{15}$. It is also very simply derived from Bohr's theory of the hydrogen atom. According to this theory,

$$R = \frac{2\pi^2 \cdot e^4 \cdot m}{h^3} = \frac{2\pi^2 e^5}{h^3 \cdot e/m}.$$

Now, e, h, and e/m are known with great accuracy:

$$e = 4.774 \times 10^{-10} \text{ e.s.u.} = 1.59 \times 10^{-20} \text{ e.m.u.}$$
$$h = 6.555 \times 10^{-27} \text{ erg. sec.}$$
$$\frac{e}{m} = 1.76 \times 10^7$$

Hence,

$$R = 3.280 \times 10^{15}.$$

It is thus seen that the calculated and observed values of R agree within 0.25 per cent. This excellent agreement must mean that the behavior of the electron in the hydrogen atom is correctly pictured by the Bohr theory.

736. Excitation of Atoms and Energy Levels.—In the normal state, the hydrogen atom has its electron in the innermost orbit or on the lowest energy level. When the atom is bombarded by an electron or an ion or when it absorbs radiant energy, this electron may be made to occupy an orbit which lies farther away

from the nucleus than the innermost orbit, or, in other words, it may be made to occupy an energy level which is higher than the lowest energy level in the hydrogen atom. The atom is then in an excited state and possesses more energy than it does in the normal state. If the atom absorbs sufficient energy to remove the electron entirely from the atom, the atom becomes singly ionized. In the case of the hydrogen atom, only the nucleus or the proton remains. In the case of heavier atoms, like lithium, sodium, or silver, the electrons in the atom in the normal state have positions which make the atomic energy a minimum. When such atoms are excited, one or more electrons are displaced from their normal energy levels by the absorption of energy. When sufficient energy is absorbed to remove one electron, the atom is singly ionized, and there remains a singly charged positive ion. If two electrons are removed, the ion is doubly ionized. It has a deficit of two electrons and is a doubly charged positive ion.

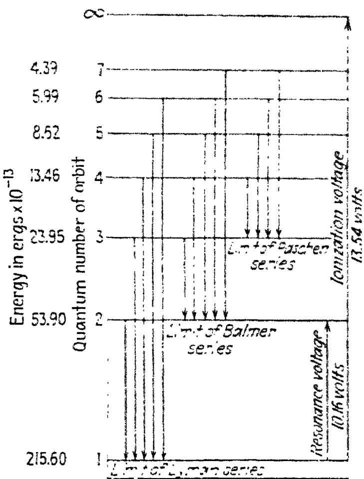


FIG. 737.—Energy levels in hydrogen atom.

From the frequencies of the spectral lines which the atom emits, it is possible to plot a system of energy levels that show the energy which the atom has in different states of excitation. The head or limit of the spectral series gives the energy necessary to remove the electron from the atom. Such a system of energy levels for hydrogen is shown in Fig. 737 and for sodium in Fig. 738. Transitions between any two of these energy levels give a line in the spectrum. The lowest level indicates the normal state of the atom.

737. Direct Measurement of Energy Levels.—According to Bohr's theory, atoms can exist only in certain stationary states, and when they change from one of these stationary states to another they emit radiation. The normal state of the atom is the state of least energy. To change from a normal state to an

excited state requires that energy be given to the atom. If E_0 = the energy of the atom in the unexcited state, and E_1 = the energy in the excited state, then the energy necessary to put the atom in the excited state is

$$E_1 - E_0 = h\nu,$$

where ν is the frequency of the light which the atom can emit in passing from the excited to the unexcited state. In the analysis of the spectra of hydrogen, a system of energy levels was constructed for the atom. Such systems of energy levels can be

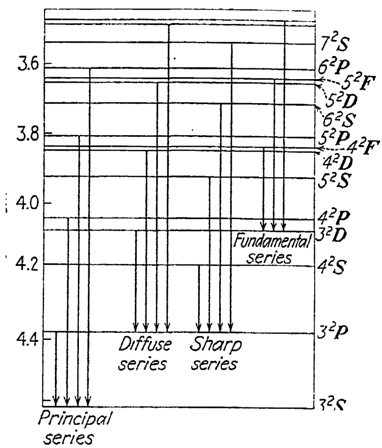


FIG. 738.—Energy levels of the sodium atom.

constructed for other atoms. They are found to describe very precisely the distribution of the lines in the spectra. It is, however, important to make sure that such energy levels really exist in the atom, and for that reason a direct measurement of these energy levels is of utmost importance.

If a stream of electrons having a velocity v is passed through a gas, the atoms of the gas may be excited by the impact of the electrons against them.

Let m = the mass of the electron.
v = the velocity of the electron.
V = the potential difference through which the electron has dropped.
e = the charge on the electron.
$\frac{1}{2}mv^2$ the kinetic energy of the electron.

If the electron has such a velocity that $Ve = \frac{1}{2}mv^2 = h\nu = (E - E_0)$, the atom will be excited and will then be able to emit a radiation of frequency ν. The potential difference required to give the electron an amount of energy sufficient to excite the atom is called the **critical potential.**

If the potential difference through which the electron has dropped is sufficiently large to completely detach another electron from the atom, one of the valence electrons may be ejected from the atom. This difference of potential is called the **ionization potential,** and the atom is said to be ionized.

Let W = the energy necessary to ionize the atom, and
V' = the ionization potential,

then

$$W = V'e = \frac{1}{2}mv'^2 = h\nu'$$

where ν' is the frequency of the radiation emitted by the atom when the electron returns to its normal position.

The critical potential and the ionization potentials have been measured by projecting electrons from hot filaments through the substance in the form of the vapor. The electrons from the filament are accelerated by means of suitably controlled differences of potential. The radiations emitted by the atoms are also observed. In this way, the potential necessary to excite a given spectral line or to ionize the atom is determined. The fact that these observed differences of potential are in excellent agreement with those calculated from spectroscopic data is a striking confirmation of the correctness of the quantum theory.

738. Work of Removing Electron from Atom.—In order completely to separate an electron from an atom, work must be expended since the electron is attracted by the positive charge on the nucleus. This work is the greater, the nearer the electron is to the nucleus. The work to carry the electron from its lowest energy level in hydrogen to infinity can be found from the expression,

Work of removing the electron

$$= \frac{\text{mass of electron} \times (\text{elementary charge})^4 \times 2\pi^2}{h^2}.$$

If the mass of the electron is taken as 9.0×10^{-28} g., the elementary charge as 4.7×10^{-10} e.s.u. and h as 6.55×10^{-27}, the work of separating the hydrogen electron from its nucleus is 20.9×10^{-12} erg.

739. Extension of Bohr's Theory to Other Atoms.—When many electrons are present in an atom, it is only the outer electrons which are responsible for the optical spectra; but the additional electrons lying near the nucleus affect the forces acting on the electrons responsible for the optical spectra. In this way, the number and distribution of the energy levels in the atom are changed. An electron which has been displaced from its normal position has then a number of different ways in which it may return to its normal position. So many different energy levels are present in these atoms, and so many different combinations of these energy levels are possible, that a large number of spectral lines can be accounted for in this way. The spectrum of iron is a good example of one of these complex spectra. In some cases, the energy levels are multiple-valued, that is, twofold, threefold, etc. When an electron shifts from one of these multiple-valued energy levels to another, two, three, or more lines lying close together are produced. These lines are known as *multiplets,* that is, *doublets, triplets,* etc. In some cases, as many as 15 lines have been observed where only one appeared in a spectroscope of low resolving power.

740. The Spectra of Ionized Atoms.—If an atom loses an electron it behaves as an entirely different atom. The positive charges are in excess of the negative charges on it. The nucleus has the same mass and charge as it had before the atom lost an electron, but the electrons outside the nucleus have been reduced in number. The spectrum which such an ionized atom emits differs from the spectrum emitted by the atom in the normal state.

If a very large difference of potential is applied to two electrodes which are near to each other in a very perfect vacuum, some of the valence electrons are completely detached from the atoms and the atoms are caused to emit spectra which correspond to the spectra of atoms that have the same number of electrons outside of the nucleus but a different nuclear charge. Thus, if the spark passes between the electrodes of magnesium, one of the valence electrons may be detached leaving the magnesium with

11 electrons outside of the nucleus and a nuclear charge of 12 on the nucleus. Now sodium has 11 electrons outside of the nucleus and a charge of 11 on the nucleus. Hence, the spectrum that is emitted by magnesium which has lost one electron will be similar to that emitted by sodium which has its entire quota of electrons. The two spectra are, however, not identical. The forces of attraction on the electrons are greater in the case of singly ionized magnesium than in the case of the sodium atom because of the greater charge on the nucleus of the magnesium atom.

741. Spinning Electron.—It has now become evident that a simple point electron moving in orbits about the nucleus of an atom is not adequate to explain all the characteristics of the spectra of the elements and to account for many other observed facts in subatomic physics. These difficulties have in large measure been overcome by assuming that the electron spins about an axis which is perpendicular to the plane of the orbit of the electron. The mechanical picture is similar to the case of the earth turning about its axis and at the same time revolving in an orbit around the sun. In the case of the electron, it is sufficient to suppose that the axis of the electron is always perpendicular to the plane of the orbit in which the electron is moving. The electron may rotate about its axis either in a clockwise or in a counterclockwise direction. Such a spinning electron would behave like a small elementary magnet with its axis perpendicular to the plane of the orbit of the electron. The direction of the spin of the electron determines the direction of the axis of the elementary magnet; that is, it determines the direction in which the north and the south pole point. For counterclockwise spins of the electron the north pole points in one direction, and for clockwise spins it points in the opposite direction. The magnetic moment of this elementary magnet is

$$\frac{eh}{4\pi mc}$$

where c = the velocity of light.
m = the mass of the electron.
e = the elementary charge on the electron.
h = Planck's constant.

742. The Zeeman Effect.—When a source of light is placed between the poles of a powerful electromagnet a single spectral line breaks up into several components (Fig. 739). This separation of a spectral line into components by the action of a magnetic field is known as the Zeeman effect because it was discovered by Professor Zeeman of the University of Leiden. The number of components depends on the particular spectral line which is

examined, and this number is not the same when the light is viewed in the direction of the magnetic field as it is when the

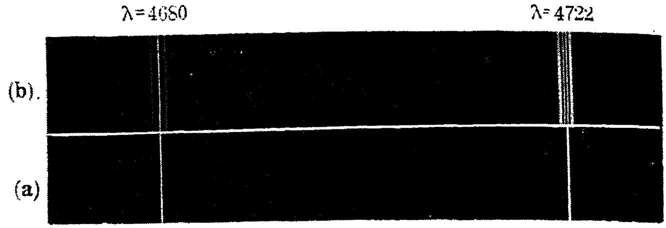


FIG. 739.—Zeeman effect in lines of the zinc spectrum, showing splitting due to magnetic field (λ = 4680 and λ = 4722 Å.) (*a*) Without magnetic field. (*b*) With magnetic field (*Courtesy J. B. Green.*)

light is viewed at right angles to the direction of the magnetic lines of force. In Fig. 739 are given two examples of the Zeeman effect for the zinc lines (λ = 4722 Å. and λ = 4680 Å.) when the light is viewed perpendicular to the magnetic field. The single spectral line has been replaced by three in one case and by six lines in another case. When the light is observed in the direction of the magnetic field a different number of components is observed. In some cases, a large number of components is observed. Owing to the interaction between the magnetic field and the magnetic fields due to the electrons in the atom, the energy levels in the atom are split up into a number of additional levels. Transitions between these new levels give rise to additional spectral lines.

FIG. 740.—Stark effect in lines of the helium spectrum, showing splitting due to varying electric field. (*Foster.*)

743. The Stark Effect.—An electric as well as a magnetic field causes the splitting up of spectral lines. In a sufficiently large electric field, a single spectral line is replaced by a number of components. The greater the electric field, the greater is the separation of these components. This separation of spectral lines into components by an electric field is known as the Stark effect from its discoverer. Figure 740 shows an example of

the Stark effect in helium from data by Foster. In the lower part of the figure, where the electric field is strong, the spectral line has been divided into a number of components. The separation decreases from zero at the top of the figure where there is no electric field, to its greatest value at the bottom where the electric field is largest. Data on the amount of this separation for different spectral lines give valuable information about the structure of the atoms which are emitting these lines.

744. Molecular Spectra.—In the preceding sections it has been seen that line spectra are emitted by atoms as a result of energy changes which take place when an electron changes its position in the system of electrons surrounding the nucleus of the atom. There is, however, another important type of spectra which is known as *molecular spectra*, due to changes in the energy of the molecules rather than to changes inside of the atom. These spectra appear both as emission spectra and as absorption spectra. Such molecular spectra may be divided into two classes: *pure-rotation spectra* and *rotation-vibration spectra*. Pure-rotation spectra lie in the neighborhood of 200μ in wave length. The rotation-vibration spectra have wave lengths which lie between 1 and 32μ.

The series of bands in the far infra-red known as the pure-rotation spectrum arise from the rotation of the molecule about an axis which is perpendicular to the line joining the two atoms in the case where the molecule is diatomic. In the case of HCl. the mass of the hydrogen atom is small in comparison with the mass of the chlorine atom, and the center of gravity of the two atoms forming the molecule would be very near the chlorine atom. The molecule as a whole would rotate about an axis through its center of gravity. Because of the rotation of the molecule about this axis, the molecule would emit a pure-rotational spectrum which would lie in the region around 200μ. The near infra-red spectrum, lying between 1 and 23μ, known as the rotation-vibration spectrum, is accounted for by supposing that it arises from vibratory motions of the atoms in the molecule combined with the rotation of the molecule about an axis. The simplest case is that of a diatomic molecule which may be thought of as a small dumb-bell which is both rotating and vibrating. The forces holding the atoms together in such a molecule are known to be large, and two nuclei in a diatomic

molecule performing oscillations along a line joining their centers may have a frequency of about 10^{14} oscillations per second.

The changes in energy which give rise to molecular spectra may be made up of three parts: (1) *Changes in the rotational energy* of the molecule about its axis of rotation; (2) *changes in the vibrational energy* of two or more molecules with respect to each other; (3) *changes in the energy of the electronic system* which is associated with the molecule. These changes in energy give rise to groups of lines called *bands.* Figure 741 shows typical

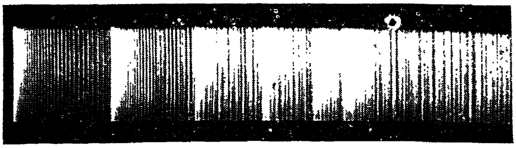


FIG. 741.—Bands in the spectrum of cyanogen.

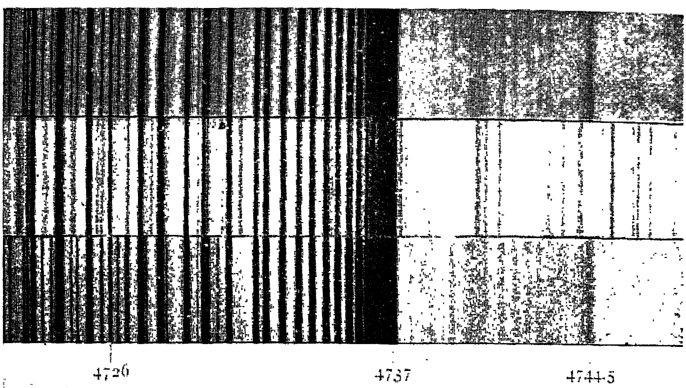


FIG. 742.—Arc spectrum (center) compared with furnace spectrum of $\lambda 4737$ band of C_2 for C = 12. Band at $\lambda 4744.5$ in furnace caused by isotopic molecule C_2 for C = 12 and C = 13.

bands in cyanogen. The frequency of the radiation emitted in a transition is given by the usual equation,

$$E_1 - E_2 = h\nu.$$

A system of energy levels may be constructed to describe the changes in energy which take place in a molecule when a band or a system of bands is emitted. The influence of isotopes on the band spectrum of CN is seen in Fig. 742. The bands do not have the same spacing in the two cases.

Problems

1. Use the Balmer formula to calculate the wave lengths of the first five lines of the Balmer series of hydrogen.

2. Find the number of waves per centimeter for the light of the first line of the Balmer series of hydrogen.

3. The orbital electron of a singly ionized helium atom is attracted by the nucleus which has a positive charge twice that of the electron. Find the velocity of an electron traveling in a circle with a radius of 5×10^{-9} cm., so that the centrifugal force is just equal to this attraction.

4. Through what difference of potential, in volts, must an electron fall in order to have an amount of energy equal to that emitted when an atom radiates the first line of the Lyman series?

5. Calculate the wave length of the first line of each of the first five series of lines predicted by the Balmer formula for the hydrogen atom.

6. Compute the longest and shortest spectral line possible in the Lyman series.

7. Calculate according to Bohr's simple theory the radius of the innermost orbit of the electron in the hydrogen atom.

Mass of an electron..............	9.01×10^{-28} g.
Planck's constant................	6.56×10^{-27} erg. second.
Charge on an electron............	4.77×10^{-10} e.s.u.

8. Calculate the wave length of the tenth and eleventh line in the Lyman series.

9. Calculate the radius of the first Bohr orbit in the case of singly ionized helium which will have two positive charges on the nucleus and one extranuclear electron.

CHAPTER LXIII

X-RAYS AND CRYSTAL STRUCTURE

745. Production of X-rays.—When electrons moving with very great velocity strike a target *T* (Fig. 743) and are stopped, they set up a series of very short electric and magnetic waves. These waves travel with the velocity of light and do not differ in their

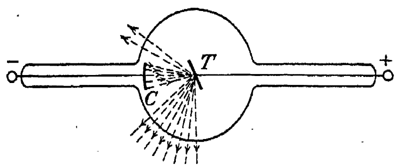


FIG. 743.—X-ray tube. The accelerated electrons are derived from the ionization of residual gases in the tube.

nature from light, except in the fact that their wave lengths are extremely short in comparison with the wave length of light. These waves, which are known as Roentgen rays or X-rays, can pass through many substances which are opaque to light and produce many important physical, chemical, and physiological effects. They have thus come to occupy an important place in physics, chemistry, medicine, and engineering.

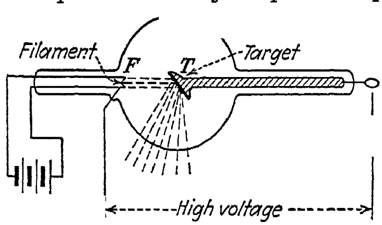


FIG. 744.—Coolidge X-ray tube. A hot filament is the source of electrons.

The most successful way to generate X-rays is by means of a Coolidge X-ray tube. It consists of a highly evacuated tube (Fig. 744) in one end of which is a small tungsten filament which is heated to incandescence by means of an electric current. This filament behaves essentially like a miniature electric lamp. A large number of electrons is emitted from the filament and by the application of a high potential, 100,000 to 200,000 volts, between the filament and the cold target at the other end of the tube, a high velocity is imparted to these electrons. When they strike the tungsten target, very

penetrating X-rays are generated. The intensity of these rays is greater than that obtained from any other tube, and the penetrating power is also very great. The tube is capable of very excellent control so that it produces rays which are well suited for most purposes. Many different types of tubes (Fig. 745) are used for different purposes.

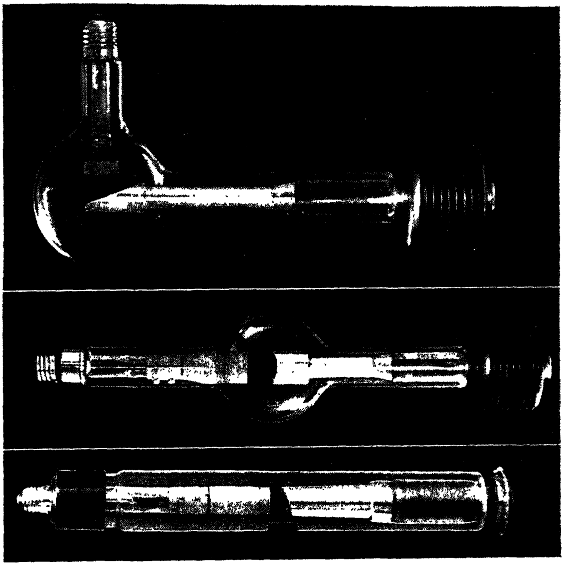


FIG. 745.—Types of X-ray tubes adapted to different uses. (a) Coolidge air-cooled tube for dental radiology, (b) Coolidge X-ray tube for general use, (c) Coolidge X-ray tube in a protective shield. *(Courtesy General Electric X-Ray Corporation.)*

Example.—Calculate the velocity with which an electron will strike the target of the X-ray tube (Fig. 744) when 20,000 volts is applied to the tube.

$$\frac{1}{2}mv^2 = Ve \times 10^7.$$

$$e = 1.592 \times 10^{-19} \text{ coulomb.}$$

$$V = 20{,}000 \text{ volts.}$$

$$m = 8.96 \times 10^{-28} \text{ g.}$$

$$\frac{1}{2} \times 8.96 \times 10^{-28} \times v^2 = 20{,}000 \times 1.592 \times 10^{-19} \times 10^7.$$

$$v^2 = \frac{3.184 \times 10^{-8}}{4.48 \times 10^{-28}}$$

$$v = 0.84 \times 10^{10} \text{ cm. per second.}$$

746. Measurement of the Intensity of X-rays.—The fact that X-rays when passing through a gas make it a conductor of electricity may be made a

basis for measuring the intensity of X-rays. A metal cylinder (Fig. 746), several centimeters in diameter, is closed at both ends except for an opening through which the X-rays may pass. A metal electrode AC is insulated from the metal cylinder and connected by means of a wire to a sensitive galvanometer G. One terminal of the galvanometer is connected to one terminal of a battery, and the other terminal of the battery is connected to the metal cylinder XY. At D there is a sulphur or amber plug which serves to insulate the electrode and its connecting wires from the metal cylinder.

When X-rays enter the metal cylinder the gas in it is ionized, and there is a current of electricity from the walls of the cylinder to the electrode. The magnitude of this current can be measured by means of the galvanometer. The current in the galvanometer gives a measure of the ionization produced in the metal cylinder by the X-rays. Since the ionization is proportional to the intensity of the X-rays, the current in the galvanometer

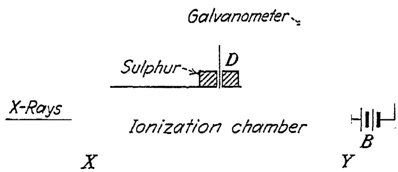


FIG. 746.—Apparatus for measuring the intensity of X-rays by observing the ionization produced by them.

is also proportional to the intensity of the X-rays. It is thus possible to measure the intensity of the X-rays by measuring the amount of ionization which they produce.

747. Medical Applications of X-rays.—In medicine and dentistry, X-rays have found many important applications. They have given the diagnostic methods of physicians, surgeons, and dentists an exactitude which formerly was entirely impossible. An atlas of nearly every part of the body is now available in most X-ray laboratories. In the surgery of the bone not only fracture but the intimate lamellar structure of the bone can be examined. Tumors in any part of the body can be located. Diseases of the alimentary canal, stomatic disorders, incipient tuberculosis of the lungs and joints, etc., can be diagnosed with certainty. In dental surgery, radiographs show clearly both the condition of the teeth and the surrounding bone. The use of X-rays for locating foreign bodies is well known. In this way it has been possible to locate pins, coins, jewelry, etc., which have been swallowed.

The radiation known as X-rays has been shown to possess valuable properties in the treatment of malignant diseases. The living cells have the power of resisting or responding to X-rays, while malignant cells disappear with suitable treatment. Inflamed glands shrink in size, and various morbid conditions of the blood clear up. In many skin diseases, X-rays have proved of great service.

748. Industrial Applications of X-rays.—By means of X-rays, the metallurgist determines the effect of heat treatment, tempering, rolling, and aging

on metals and alloys. Hidden defects in materials, concealed cracks in metals, and blow holes or other casting faults are all revealed by means of X-rays. In like manner, defective soldering, brazing, or welding can be found. The peculiarities of the structure of timbers of different kinds can be discerned. The denser heartwood is differentiated from the sapwood, the annual growth rings are easily identified and defects such as knots, resin pockets, etc., easily located. The utilization of X-rays to examine parts of aircraft is of importance. Quite small flaws can be detected in aluminum pistons, cylinder heads, and crank cases. X-rays have been turned to account by the manufacturers of rubber tires in an effort to improve the union

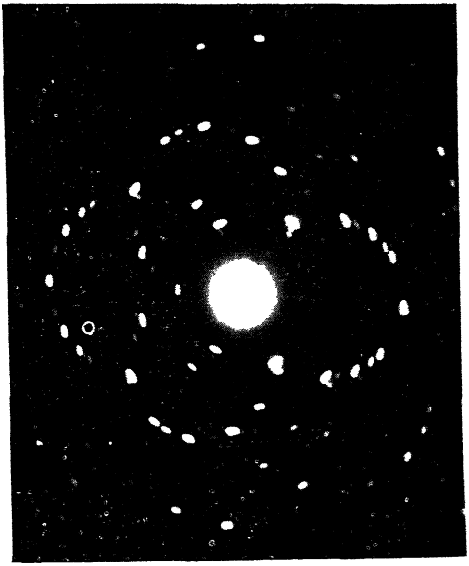


FIG. 747.—Laue photograph of calcite.

between the rubber and the cotton fabric or cord. In the field of electrical engineering, the application of X-rays has made it possible to locate defective insulation in ebonite, built-up mica, fiber, etc.

749. Diffraction of X-rays.—When X-rays are scattered by an amorphous body, the scattering takes place in a continuous manner. If the scattering is produced by a crystalline substance, the scattered rays are grouped in separate pencils which produce isolated spots on a photographic plate (Fig. 747). The arrangement of these spots is intimately connected with the crystal structure. These spots are produced by the interference of waves diffracted at a number of centers, which are determined by the atoms or molecules of which the crystal is built. These

diffracting centers make the crystal act for X-rays as a diffraction grating acts for light waves.

Suppose that a series of particles (Fig. 748) all lie in the same plane. These are the atoms or units which scatter the X-rays. When a beam of X-rays passes over these atoms, each atom emits a diffracted wave which spreads out in the form of a sphere. All diffracted wavelets are tangent to the wave front ED and thus combine to form a new wave which obeys the ordinary laws of reflection.

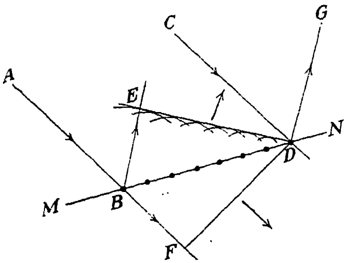


FIG. 748.—Reflection of X-rays from a series of particles.

To bring out more clearly this action, let the crystal structure be represented by a series of planes (Fig. 749) which are parallel to each other and an equal distance D apart. Now let a train of waves AB, $A'B'$ of wave length λ be incident on this surface. After reflection by the atoms of the crystal, some of these scattered waves travel along BC. The paths by which they have come are ABC, $A'B'C$, $A''B''C$, etc. Now draw BN perpendicular to $A'B'$ and produce $A'B'$ to D where D is the mirror image of B in the plane through B'. Since $B'B'' = B'D$ and $A'N = AB$, the difference between $A'B'C$ and ABC is equal to $ND = 2D \sin x$. Similarly, $A''B''C$ is greater than $A'B'C$ by the same distance, and so on. If DN is the length of a wave or a whole multiple of wave lengths, all the trains of waves reflected by the planes PP', P_1P_1', etc., are in the same phase and their amplitudes are added together. If DN differs but slightly from one wave length, many thousand reflections having taken place, there will be all sorts of phase relations and the resultant amplitude will be nearly zero.

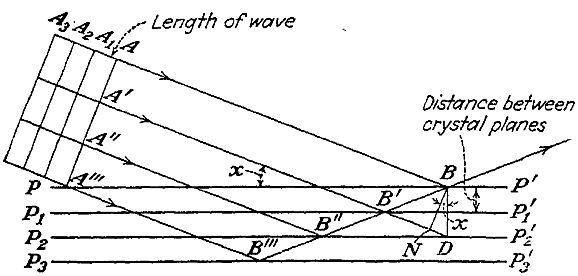


FIG. 749.—Reflection of X-rays from successive planes. The waves reinforce when $n\lambda = 2D \sin x$.

When a monochromatic train of waves strikes the face of the crystal, no reflection takes place except when the glancing angle has values given by the following equations:

$$\lambda = 2D \sin x_1.$$
$$2\lambda = 2D \sin x_2.$$
$$3\lambda = 2D \sin x_3; \text{ etc.}$$

If the crystal is slowly turned around so that the glancing angle steadily increases, in general there is no reflected beam, but as the angle assumes the values x_1, x_2, x_3, etc., there is a reflection of the rays. At another face of the crystal which has a different spacing D', these monochromatic rays will be reflected only when

$$\lambda = 2D' \sin x_1'.$$
$$2\lambda = 2D' \sin x_2'.$$
$$3\lambda = 2D' \sin x_3'; \text{ etc.}$$

If the glancing angles are measured on an X-ray spectrometer and if the wave length of the X-rays is known, it becomes possible to calculate the distance between the planes in the crystal. If, however, the distance between the crystal planes is known, the wave length of the X-rays can be calculated, just as the wave length of light is calculated from observations with a diffraction grating. For rock salt, $D = 2.81 \times 10^{-8}$ cm. For X-rays from palladium, $\lambda = 0.586 \times 10^{-8}$ cm. Hence, the distance between the planes in rock salt is about five times as large as the wave length of these X-rays.

Example.—With an X-ray spectrometer using a rock-salt crystal the glancing angle for the first reinforcement ($n = 1$) was found to be 15.8 deg. If the wave length of the incident X-rays was 1.54 Å., what is the distance between the crystal planes?

$$\lambda n = 2d \sin \theta.$$
$$n = 1$$
$$\lambda = 1.54 \times 10^{-8} \text{ cm.}$$
$$\sin \theta = 0.273.$$
$$d = \frac{\lambda}{2 \sin \theta} = \frac{1.54 \times 10^{-8}}{2 \times 0.273} = 2.8 \times 10^{-8}$$

750. X-ray Spectrometer.—An X-ray spectrometer is used for measuring the wave length of X-rays or for determining the distance between the crystal planes. Such an instrument is represented in Fig. 750. The X-ray bulb is completely enclosed in a box covered with lead. This screening is necessary to protect the rest of the apparatus and the operator from the direct action of the X-rays. A Coolidge X-ray tube with a molybdenum target is used. These rays have a wave length of 0.712 $\times 10^{-8}$ cm. for the K-line of molybdenum. When a copper anticathode is used, the K-line from copper has a wave length of 1.54×10^{-8} cm. By means of two lead slits A and B, a beam of parallel X-rays is isolated. They are taken from the target in

such a way that they are as nearly as possible perpendicular to the axis of the tube. After passing through the slits A and B, the rays fall on the crystal C which acts as a grating. After reflection from this crystal, the rays are received in the ionization chamber E. Before entering the ionization chamber, the rays fall on the slit D by means of which rays coming in the desired direction from the crystal can be selected for examination. In the ionization chamber the X-rays produce ions; and by measuring the amount of this ionization, the intensity of the X-rays can be determined. The ionization chamber can be revolved about the

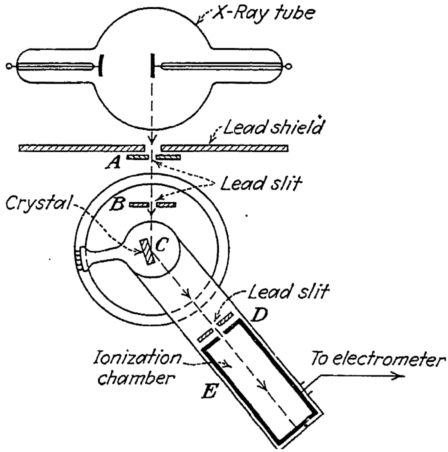


FIG. 750.—X-ray spectrometer for measuring the wave lengths of X-rays. When the X-rays enter the ionization chamber, they make the gas in it a conductor.

axis of the spectrometer and the intensity of the rays in different directions determined. The ionization chamber is filled with some gas which is easily ionized and then connected to an electrometer or an electroscope.

751. Crystal Structure.—The atoms and molecules of a crystalline substance are arranged in a definite pattern with perfect regularity and the whole crystal is built up by packing together crystal units or groups of atoms. These groups are all similar and similarly oriented in the crystal and are formed of the fewest possible number of atoms, consistent with the requirement that it is possible to build up the entire crystal out of these groups. The entire crystal can be thought of as divided into cells in the form of parallelopipeds by three sets of parallel planes. The

planes belonging to one set are all parallel and equidistant, but the distance between the planes need not be the same for all three sets of planes. The different sets of planes may make any angle with each other.

In the simplest crystal structure, the particles occupy the corners of elementary cubes. There are three kinds of lattices

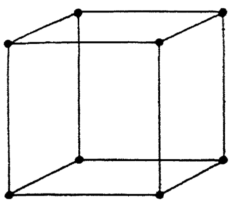


FIG. 751.—Simple cubic lattice.

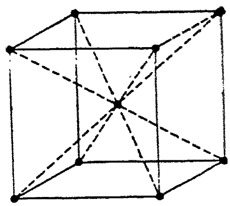


FIG. 752.—Body-centered cubic lattice.

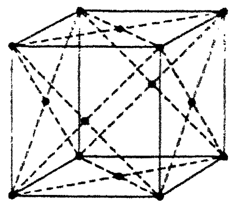


FIG. 753.—Face-centered cubic lattice.

which give cubic forms to crystals. The first structure is known as a **simple cubic lattice.** It contains eight points situated at the eight corners of a cube (Fig. 751). The second kind of cubic lattice is called a **body-centered cubic lattice.** It consists of nine points, one being at the center of the cube and the eight others at the corners (Fig. 752). The third kind of cubic struc-

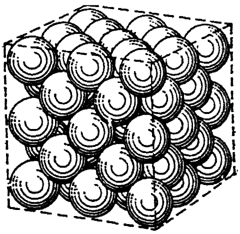


FIG. 754.—Gold atoms magnified 30 million diameters.

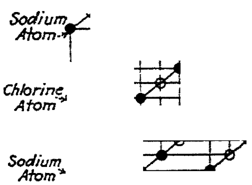


FIG. 755.—Crystal lattice of rock salt.

ture, called a **face-centered** cube, consists of fourteen points, one at the center of each face and one at each corner (Fig. 753). These lattices are similar, in so far as they have in common three rectangular axes of equal length. The external forms of crystals built around these axes may differ from one another. The crystal may become a cube, an octahedron, a tetrahedron, or a dodecahedron. The crystals of copper, silver, gold, platinum, etc., are face-centered cubic lattices. Figure 754 shows the gold

lattice magnified thirty million diameters. This is one of the closest possible forms of packing atoms. It accounts in part for the densities of these elements. Iron, sodium, tungsten, etc., are body-centered cubic lattices. The units of which these lattices are built are in all cases atoms or ions. In Fig. 755 is shown the way in which the sodium and chlorine atoms are arranged in a crystal of rock salt.

Example.—The density of rock salt is 2.16 g. per cubic centimeter and its molecular weight is 58.45. What is the distance between the atoms in a rock-salt crystal?

The number of molecules in 58.45 g. of rock salt (Avogadro's number) $= 6.06 \times 10^{23}$.

The number of molecules in 1 g. of rock salt $= 1.04 \times 10^{22}$.

The number of molecules in 1 c.c. of rock salt $= 1.04 \times 2.16 \times 10^{22} = 2.25 \times 10^{22}$.

$$\text{The number of atoms in 1 c.c. of rock salt} = 2 \times 2.25 \times 10^{22} = 4.50 \times 10^{22}.$$

$$\text{The number of atoms in a row 1 cm. long} = \sqrt[3]{4.50 \times 10^{22}} = 3.56 \times 10^{7}.$$

$$\text{Distance between atoms} = \frac{1}{3.56 \times 10^{7}} = 2.8 \times 10^{-8}\text{ cm.}$$

752. Two Types of X-ray Spectra.—When electrons projected from a cathode strike the anticathode, and the X-rays thus produced are resolved into a spectrum by means of an X-ray spectrometer, it is found that there are two spectra. One is a continuous spectrum whose upper limit of frequency is determined by the velocity of the electrons. The other is a line spectrum which is characteristic of the element that is used for the anticathode. Figure 756 shows the combination of a continuous and a line spectrum for molybdenum. In this figure, the intensities of the X-rays have been plotted on the vertical axis and the wave lengths on the horizontal axis. It is to be observed that the intensity does not change uniformly with the wave length, but that rapid increases in the intensity appear where the spectral lines characteristic of the element are

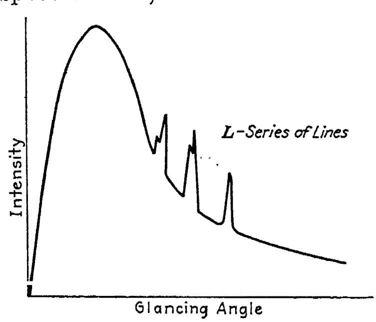


FIG. 756.—Intensity of X-rays showing the L-series of lines.

emitted. The continuous X-ray spectrum is analogous to white light in the visible region of the spectrum.

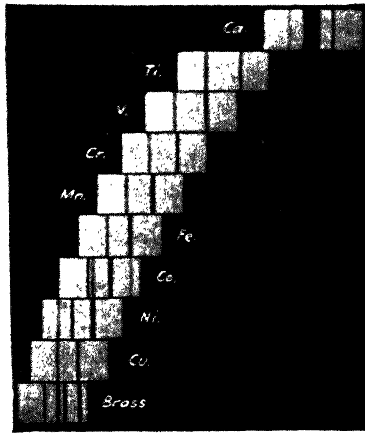


FIG. 757.—Moseley's photograph of X-ray spectra. Notice the decrease in wave length as the atomic number increases.

753. Characteristic X-ray Spectra.—The characteristic X-ray spectra of all the elements are very much alike. In all of them, the relative positions and intensities of the lines are essentially the same. The absolute positions of the lines change from element to element. As the atomic number of the element increases, the wave length of the lines shifts toward the shorter wave lengths. Figure 757 shows these spectra as they were first photographed by Moseley for a few of the elements of higher atomic numbers. The similarity of the spectra is quite evident and the shift toward the shorter wave length quite apparent. Further resolution shows that instead of single lines there is in each case a group of lines lying near to each other. One of these groups is known as the *K*-series, another as the *L*-series, and another as the *M*-series. The frequencies of the lines in the *K*-series are greater than the frequencies of the lines in the *L*-series. Similarly, the frequencies of the lines in the *L*-series are greater than those of the *M*-series. Figure 758 shows the relations between these series of lines when they have been still further resolved. Here, they have been resolved into two or three components,

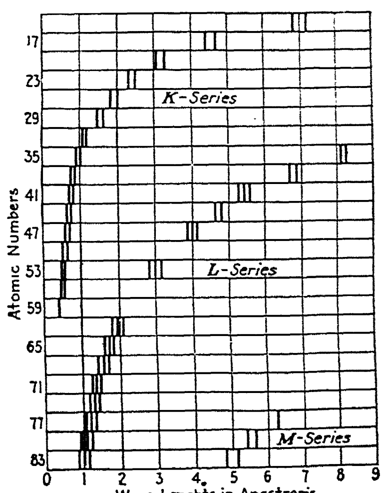


FIG. 758.—Diagrammatic representation of characteristic X-ray spectra.

but they can be resolved into a still greater number of components. A typical K-series consists of four strong lines known as $K_{\alpha}{}^{1}$, K_{α_2}, K_{β_1}, K_{β_2}, and a number of much weaker lines. The L-series consists of many more lines than the K-series and the M-series of very many lines.

To bring out the relation between the frequency of a given line and the atomic number of the element from which it originated, Moseley plotted a curve showing the relation between the square root of the frequency of the line and the atomic number of the element by which the line was emitted. Figure 759 gives such a diagram for the K_{α} and K_{β} lines. Here the atomic number has been plotted on the horizontal axis and the square root of the frequency on the vertical axis. It is at once

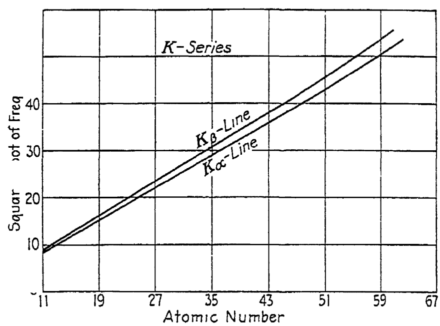


FIG. 759.—Relation between atomic number and square root of frequency.

seen from this figure that the square root of the frequency is proportional to the atomic number of the element. Similar straight lines were obtained for each of the spectral lines in the characteristic X-ray spectrum.

754. Absorption of X-rays.—If a sheet of any substance is placed in the path of a homogeneous beam of X-rays, the intensity of the beam is diminished. It falls off logarithmically with the thickness of the sheet. Let I_0 be the initial intensity of the beam, and I its intensity after passing through a thickness d of the material. Then,

$$I = I_0e^{-\mu d},$$

where μ is the coefficient of absorption of the radiation in the particular absorbing material, and e is the base of the natural logarithms. The coefficient of absorption varies with the wave

length of the X-rays used. It is found experimentally that the absorption depends only on the number of atoms present in the absorbing layer. It is independent of their state of aggregation, so that μ/ρ, where ρ is the density of the substance, is constant for a given substance for X-rays of definite wave length.

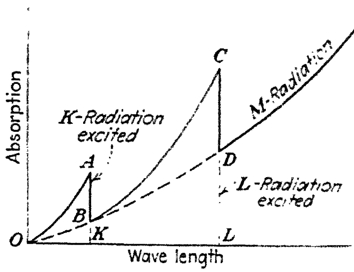


FIG. 760.—Increase in absorption critical wave lengths.

The absorption decreases as the wave length of the X-rays is decreased (Fig. 760). At a definite wave length, there is a sharp and very marked discontinuity in the curve which relates the coefficient of absorption to the wave length. At this point, the absorption rises abruptly to a high value. This increase in the absorption arises out of the fact that at this wave length the characteristic L-radiation of the absorbing substance is emitted for the first time. The energy of this characteristic radiation must come from the primary beam. After passing this discontinuity which occurs when the L-radiation begins to be emitted, the absorption again decreases with the wave length until that wave length is reached at which the characteristic K-radiation of the absorbing substance is emitted. The emission of this K-radiation again gives rise to a second discontinuity, and the absorption is again increased. But it decreases rapidly as the wave length of the radiation is still further decreased. The wave lengths at which these discontinuities occur have been measured with great accuracy. They are characteristic of the atoms which form the absorbing substance.

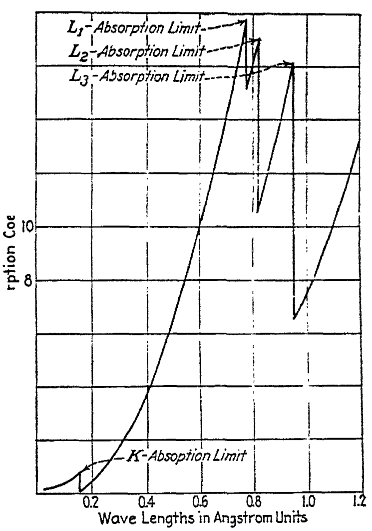


FIG. 761.—Detail of X-ray absorption limits.

755. X-ray Energy Levels.—These empirical data on the characteristic X-ray spectra can now be used to give valuable information about the arrangement of the electrons in the atom. To bring out more clearly the conclusions to be drawn from these

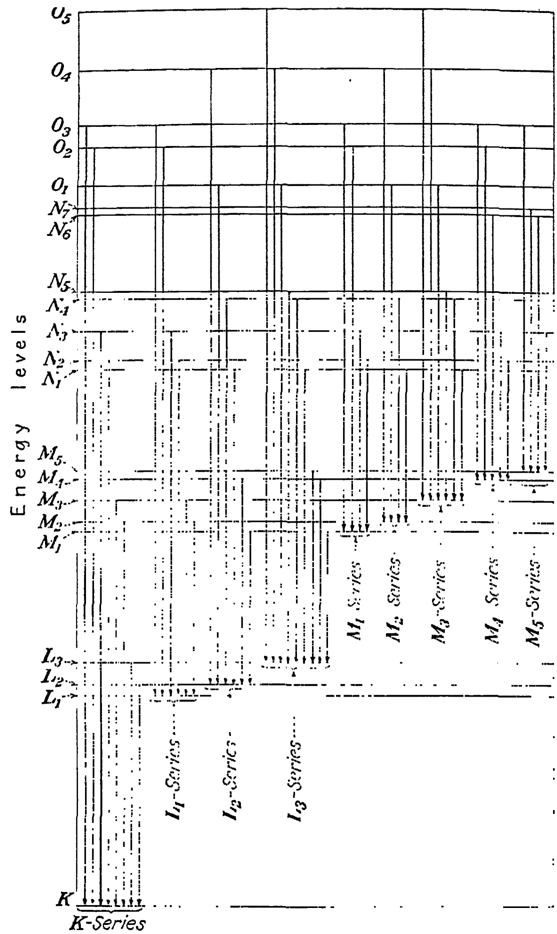


FIG. 762.—X-ray energy levels for uranium.

data it is desirable to construct an X-ray energy-level diagram for the atom like the one constructed for the hydrogen atom (Fig. 737). Such a diagram is given in 762 for uranium. This diagram shows a number of energy levels corresponding to the

energy which an electron has in one of the shells of electrons which surround the nucleus of the atom. The lowest energy level is known as the K-level. This is a single level and gives the energy associated with one of the two electrons in the K-shell, which is the shell of electrons nearest the nucleus of the atom. The next highest level is the L-level, which is a triple level and gives the energies of the electrons which lie in the L-shell. There are eight of these electrons when the shell is completed, but they are subdivided into three groups and the energy of an electron in one of these groups is not the same as the energy of an electron in another of these sub-groups. In the same way, the M-energy levels represent the energies of the electrons in the M-shell. These levels are also multiple-valued.

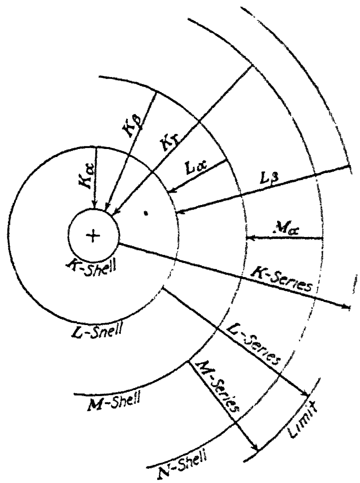


FIG. 763.—Diagrammatic representation of emission of X-rays as an electron passes from an outer to an inner shell.

Consider now the lines of the K-series. They are produced by electrons falling from an outside shell to a K-shell (Fig. 763). Before such transfers of electrons can occur, vacant spaces in the K-shell must be provided. The K-shell has normally two electrons in it. To provide a vacant space in the K-shell, one of the electrons normally in the K-shell must be removed from this shell and carried outside of the atom since there will be no vacant spaces between the K-shell and the last shell of electrons in the atom. It is here assumed that the L-shell, the M-shell, etc., are also filled with all the electrons which can enter them. Once there is a vacant space in the K-shell, this place may be filled by an electron falling into it from an outside shell. If there is a large number of atoms in this state of excitation, the entire K-series of lines will be emitted. Some of the electrons which thus fall to the K-shell will have come from energy levels in the L-shell; others from energy levels in the M-shell; etc. The intensities of the lines will be governed by the probabilities

of such transfers. Where the probability of a transfer is large, the intensity of the line will be great. Where the probability of the transition is small, the intensity of the line will be weak.

Similarly, an electron may be displaced from the L-shell to some point outside the atom. The electrons may fall back into this vacant space in the L-shell and the spectral lines in the L-series will be emitted. In this way, all the lines in the characteristic X-ray spectrum of an element may be accounted for.

756. Work to Displace an Electron from One Shell to Another Shell.—The work which must be done to extract an electron from an atom depends on the forces which hold it bound in the atom and hence on its position in the atom. Suppose, for example, that an electron in the L-ring of electrons falls into this vacant space in the K-ring of electrons. Energy will be liberated in an amount equal to the difference between the work required to remove an electron from the K-ring and the work required to remove an electron from the L-ring. Let W_K be the work to remove an electron from the K-ring, and W_L the work to remove an electron from the L-ring. The energy emitted by an electron in falling from the L-ring to the K-ring is

$$W_K - W_L = h\nu,$$

where ν is the frequency of the line in the K-radiation emitted by an electron in falling from the L-ring to the K-ring. Hence, the frequency of the emitted radiation is

$$W_K - W_L$$

In a similar way the frequencies of other lines in the X-ray spectra may be calculated.

757. The Compton Effect.—Compton showed that when a beam of monochromatic X-rays is scattered by a substance like carbon, lithium, or aluminum, the beam of scattered rays gives two spectral lines instead of one. One of these lines has the wave length of the primary beam, and the other has a longer wave length than the wave length of the primary beam. Figure 764 shows the scattered radiation from carbon when it was irradiated by the K_α-line from molybdenum. The vertical line AB shows the wave length of the primary radiation and the vertical line CD the wave length of the line produced by the scattering

of the K_α-line of molybdenum. Of the two maxima, the one at A corresponds exactly to the wave length of the primary radiation. The one at D corresponds to a shifting of the primary wave length toward the longer wave lengths. The difference between the wave lengths of the unshifted line and the shifted line is 0.0236Å.

An explanation of this shifting of a spectral line in the scattering of X-rays is obtained by assuming that the beam of X-rays consists of a stream of quanta or grains of energy, each of magnitude $h\nu$. These quanta are in motion with a velocity equal to the velocity of light. They possess both energy and momentum. When they collide with a particle of matter like an electron, the

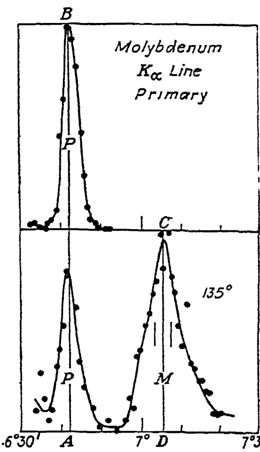


FIG. 764.—Increase in wave length of K_α-line of molybdenum due to scattering.

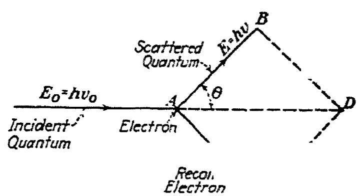


FIG. 765.—Collision of a photon with an electron.

collision may be described in terms of the law of conservation of momentum and the law of conservation of energy. By applying these two laws (Fig. 765) to the collision of a quantum with an electron, Compton calculated the change in frequency and the change in wave length which might be expected from such a collision. The calculated change in wave length was in close agreement with the observed change in wave length, indicating that a quantum of energy behaves in such a collision like a particle of matter. The results of this experiment indicate quite clearly the necessity for thinking that energy exists in quanta with many of the characteristics of particles of matter.

758. Wave Length of an Electron.—If a pencil of monochromatic X-rays passes through a powdered crystalline substance at C (Fig. 766) where the X-rays are scattered, they produce a diffraction pattern on the photographic plate MN. This diffraction pattern consists of a series of con-

centric fringes produced by X-rays which have been in phase or out of phase by an odd number of half wave lengths.

If a stream of electrons, all having the same velocity, are allowed to pass through a thin film of metal, a similar set of interference fringes is

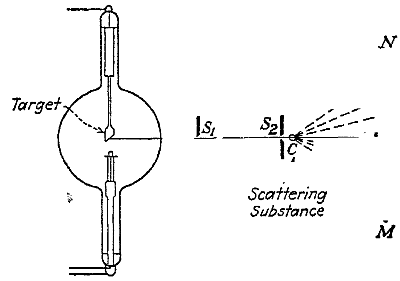
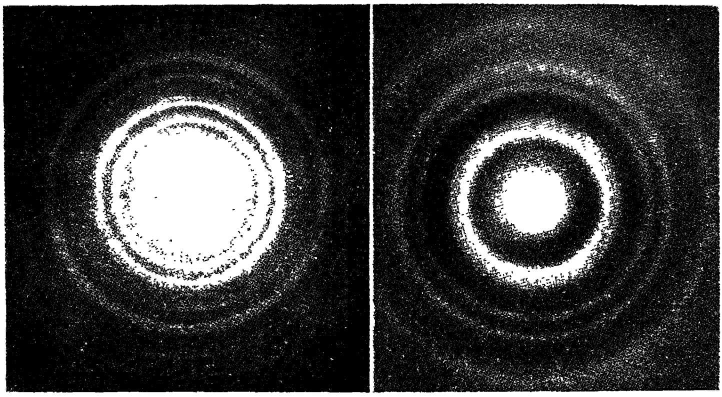


FIG. 766.—Apparatus for studying scattering of X-rays.

produced. Figure 767 gives the results when a thin film of gold was placed in the path of a beam of electrons. This set of interference fringes is nearly like those produced by the beam of monochromatic X-rays, as shown in the left side of Fig. 767 for aluminum. Now the fringes produced in the



(a) (b)

FIG. 767.—(a) Diffraction of X-rays by aluminum. (*Hull.*) (b) Diffraction of electrons by thin sheet of gold foil. (*G. P. Thomson.*)

case of the beam of X-rays are explained on the basis that X-rays are electromagnetic waves of very short wave lengths which reinforce or destroy each other. It would also seem necessary to interpret the results on the beam of electrons as showing that electrons have a wave length, and that

they also produce interference fringes. This remarkable result has led to the conclusion that an electron has some of the properties of a wave motion, and that its wave length is

$$\lambda = \frac{h}{mv},$$

where m = the mass of the electron.
h = Planck's constant.
v = the velocity of the electron.

The greater the speed of the electron, the less is its wave length.

Example.—What is the wave length of an electron traveling at a speed of 10^6 cm. per second?

$$\lambda = \frac{h}{mv} = \frac{6.55 \times 10^{-27}}{9.03 \times 10^{-28} \times 10^6}$$
$$= 7.25 \times 10^{-6} \text{ cm.}$$

Problems

1. What is the shortest wave length of X-rays produced by a tube working at a potential of 50,000 volts?

2. A potential of 10,000 volts is applied to an X-ray tube. Calculate the velocity of an electron about to strike the target.

3. Find the velocity of an electron when its mass is ten times as much as its mass would be if the electron were at rest. (See p. 520.)

4. Find the velocity imparted to an electron in falling through a difference of potential of 10 volts.

5. The electrodes of a vacuum tube are 30 cm. apart, and the difference of potential between them is 50,000 volts. Assuming that the electrons move the whole distance between these electrodes, find the acceleration and the kinetic energy of an electron when it reaches the positive electrode.

6. A current of 5 milliamp. flows from the filament of an X-ray tube to the target. If all the electrons could be collected, how long would it require to collect 1 g. of them?

7. A Coolidge X-ray tube is operating on 150,000 volts. The current in it is 25 milliamp., and the distance between the electrodes is 12 cm. With what velocity do the electrons strike the anode? Relativity correction for change of mass of electron with its speed must be made.

CHAPTER LXIV

ASTROPHYSICS

No line of separation can be drawn between physics and astronomy. In interstellar space physical phenomena manifest themselves on a tremendous scale and under conditions of pressure and temperature which cannot be found in terrestrial phenomena. Astrophysics which deals with the constitution

FIG. 768.—Large 69-in. reflecting telescope of the Perkins Observatory. (*Ohio Wesleyan University and Ohio State University.*)

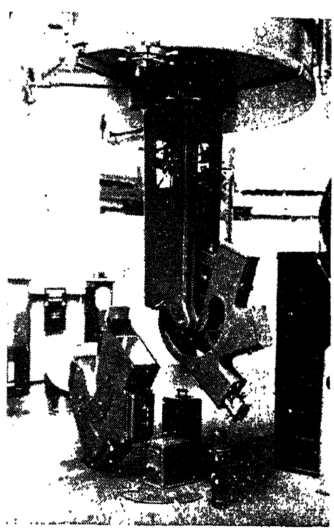


FIG. 769.—Spectrograph attached to the 72-in. reflecting telescope of the Dominion Astronomical Observatory. (*Courtesy Dominion Astrophysical Observatory.*)

and evolution of stellar bodies is essentially a branch of physics, devoted to large scale physical phenomena. Its instruments—telescopes (Fig. 768), spectroscopes (Fig. 769), photometers, etc.—are adaptations of instruments used in physical laboratories. Its principles and methods of analysis are those arrived at by the study of physical phenomena under controlled conditions. A

brief survey of astrophysics gives a good indication of the range and universality of the principles and methods of physics.

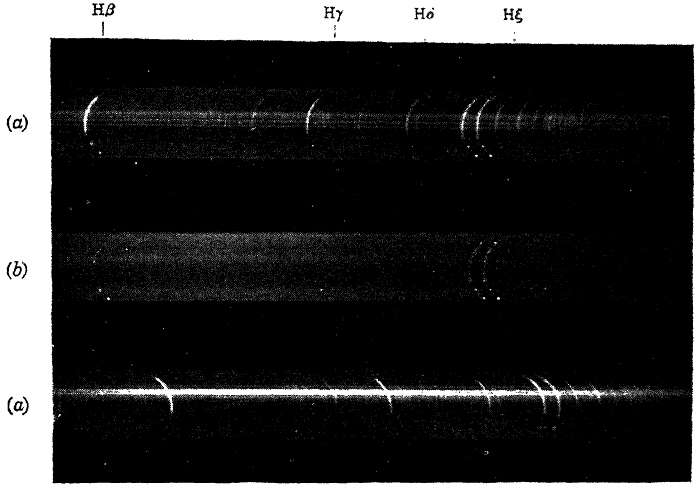


FIG. 770.—Flash spectrum and corona spectrum of the sun. (a) Flash spectrum showing prominent hydrogen lines. (b) Corona spectrum also showing the more prominent lines. The isolated bright patches are prominences rising above other parts of the moon's limb and emitting these lines. (*Courtesy Mt. Wilson Observatory.*)

759. Solar Prominences.—The "reversing layer" of the sun extends through several kilometers. Above this layer lies the chromosphere which is a layer of very bright gas several thousand kilometers in height. At the highest level it consists principally of hydrogen and calcium. It is easiest to study the chromosphere at the time of a total eclipse. The so-called flash spectrum (Fig. 770) can then be observed.

FIG. 771.—An active prominence 140,000 miles high, July 9, 1917, photographed with the *K*-line of calcium. The disk shows the size of the earth. (*Courtesy Mt. Wilson Observatory.*)

At the beginning or at the end of totality part of the chromosphere projects as an arc of varying depth. A greater depth is visible at the center of the arc than at its cusps. When this arc is photographed

with a prismatic camera, the observed spectrum consists of a number of bright arcs. Each arc corresponds to a line of the spectrum of the glowing gas of the chromosphere.

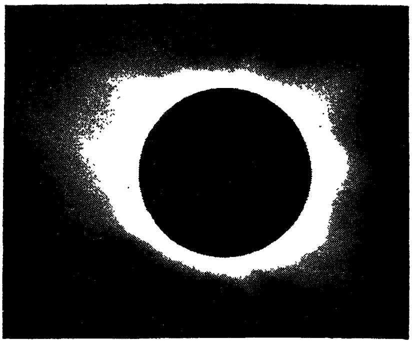


FIG. 772.—Solar corona exposed 15 sec., Jan. 24, 1925, Middletown, Conn. (*Courtesy Mt. Wilson Observatory.*)

Rising out of the chromosphere are solar prominences (Fig. 771) which are clouds of flaming gas. These clouds consist mostly of glowing hydrogen. The height of these prominences varies from several hundred to several thousands of kilometers. Some prominences are more or less quiescent. Others emerge from the chromosphere and later seem to sink back into it. Prominences have been found to extend to a height of 830,000 km. above the surface of the sun.

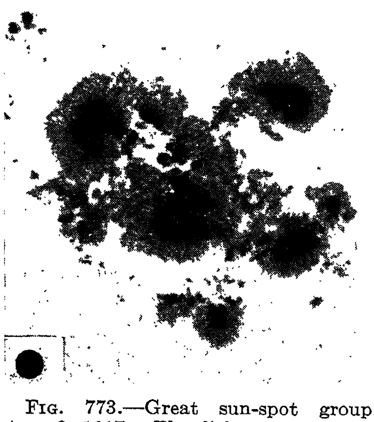


FIG. 773.—Great sun-spot group. Aug. 8, 1917. The disk represents the size of the earth in comparison with that of the sun spot. (*Courtesy Mt. Wilson Observatory.*)

760. The Solar Corona.—The corona is a pearl-colored atmosphere which may extend as much as 13 million kilometers above the limb of the sun. It is observed at the time of a total eclipse of the sun (Fig. 772). The shape of the corona depends on the number and distribution of sun spots. The spectrum of the corona is continuous except for dark absorption lines probably due to reflected sunlight. A large portion of its colored light is

scattered sunlight. In the lower layers there are a number of bright lines, owing to unidentified elements.

761. Sun Spots.—Upon the intensely brilliant visible surface of the sun are often seen relatively dark areas called **sun spots** (Fig. 773). There is great variety in the size and shape of these spots. A well-developed sun spot is roughly circular in outline. The inner part is darker than the outer border. In contrast to the photosphere the inner portion of a sun spot seems to be black. In reality it gives out about one-tenth as much light per unit area as the photosphere. A large group of sun spots may be as much as 100,000 miles in diameter. Sun spots develop rapidly. They persist a few weeks or months and then break up into a number of smaller spots. Hale showed that a sun spot is a kind of a solar storm resembling a terrestrial tornado. The hot vapors rising from the sun spots are cooled by expansion.

Hale also found that sun spots have intense magnetic fields associated with them. The presence of these magnetic fields was discovered by studying the behavior of spectral lines when the light is emitted from a sun spot. Zeeman had previously discovered in the laboratory that when spectral lines are emitted in a magnetic field, they are split up into three or more components. A similar splitting is observed in spectral lines when the light comes from a sun spot. Hale therefore concluded that sun spots must be centers of magnetic disturbances. From the separation of the components of the spectral lines he inferred the intensity of the magnetic fields in the sun spot.

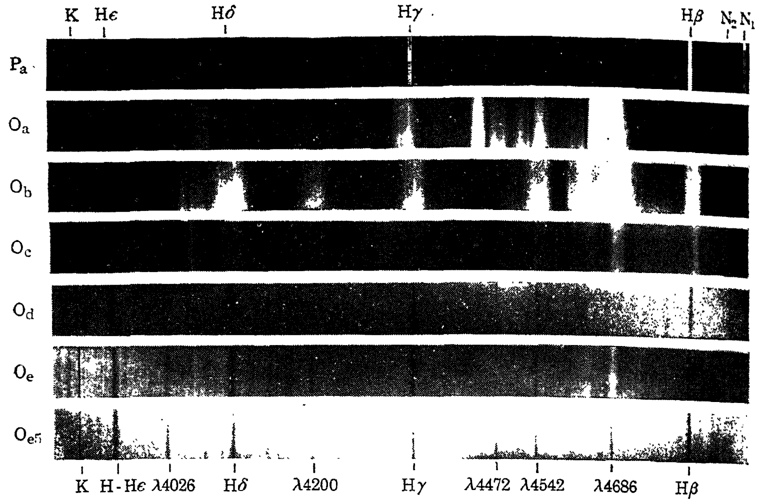
762. Types of Stellar Spectra.—Stellar spectra differ greatly among themselves. These differences are sufficient to afford a basis for the classification of stars. The following classification of stars is based on the presence or absence of certain characteristic lines in their spectra (Plates VIII and IX):

1. *Type O.*—The spectrum of stars of this type consists of a faint continuous background with bright lines superposed on it.

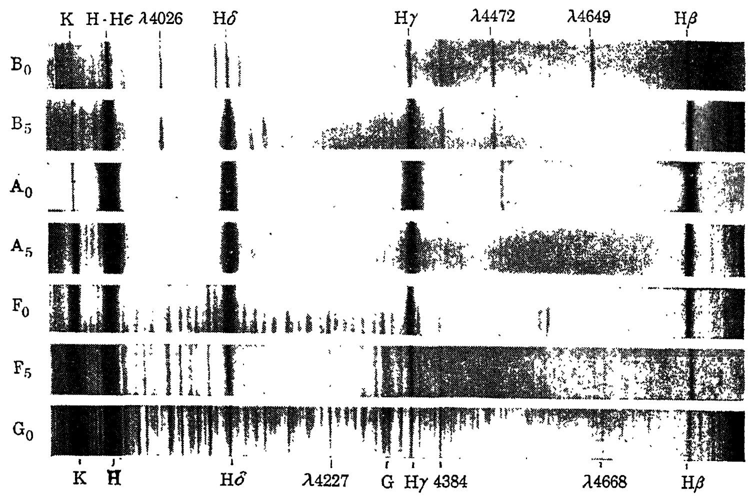
2. *Type B.*—Spectra of this type contain only dark lines. The lines due to hydrogen, neutral helium, and singly ionized oxygen and nitrogen are prominent.

3. *Type A.*—A great intensity of the hydrogen lines is the most prominent characteristic of the spectra of stars of this class.

4. *Type F.*—In the spectra of this class of stars the intensity of the hydrogen lines diminishes. The intensity of metallic lines increases and lines due to neutral atoms begin to appear.



(a)



(b)

PLATE VIII. (a) The bright-line spectrum of a nebula (Pa), the bright-band spectra of the Wolf-Rayet stars (Oa, Ob, Oc), and the dark-line spectra of classes Od, Oe, and Oe5. The spectral lines which are marked, Hβ, Hγ, Hδ, Hε are due to hydrogen; λλ4686, 4542, 4200 to ionized helium; λλ4472 and 4026 to neutral helium; and K to ionized calcium. (b) Spectra of classes of stars form Bo to Go, inclusive. The line λ4649 is due to ionized helium. The helium line λ4472 fades out in passing from Bo to Ao and a line due to ionized magnesium at λ4481 comes in. The lines λλ4384 and 4668 are due to neutral iron and the line λ4227 to neutral calcium. The lines H and K, which are very strong in class Go, are due to ionized calcium. The band at G is due to some hydrocarbon.

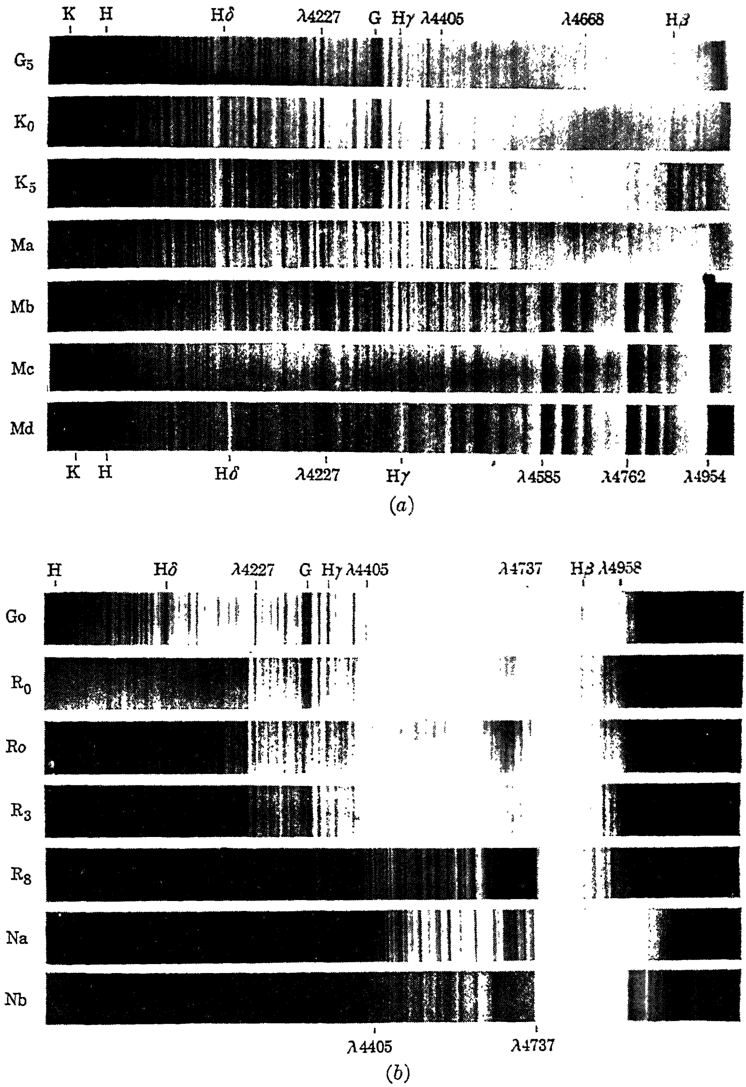


PLATE IX. (*a*) This figure shows the spectra of classes of stars from G5 to M_a. These spectra are full of arc lines of metals. The line at λ4405 is due to iron; the bands at λλ4585, 4762, 4954 are the heads of bands due to titanium oxide. The bright emission lines of hydrogen H_δ and H_γ are present in the lower spectrum. (*b*) The absence of violet light in the stars of class N means that these stars are very red. The bands at λλ4405 and 4737 are attributed to a carbon compound and the one at λ 4214 in class R stars to cyanogen.

5. *Type G.*—Numerous metallic lines are present in *G*-type stars. The hydrogen lines are conspicuous. Lines due to neutral atoms are stronger than those due to ionized atoms. The solar spectrum is of this type.

6. *Type K.*—In the spectra of type *K* stars, low-temperature metallic lines are strong and high-temperature metallic lines are weak. The lines of hydrogen are less prominent.

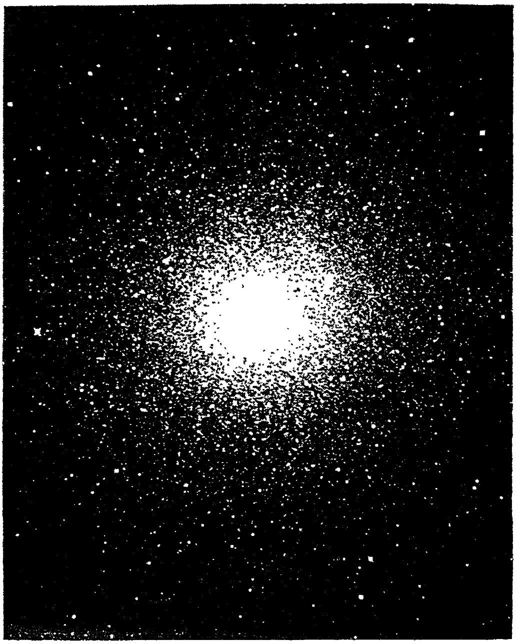


FIG. 774.—Globular cluster photographed with the 60-in. reflecting telescope. The diameter of the central part is about 15 light years and that of the outlying region 100 light years. (*Courtesy Mt. Wilson Observatory.*)

7. *Type M.*—The spectra of this class of stars are characterized by broad absorption bands which are most prominent in the violet region of the spectrum. Solar lines decrease in number and intensity.

8. *Type R.*—This is a small class of stars which is not so red those of type *M* or *N*, but most of the prominent absorption les of type *N* stars are present.

9. *Type N.*—Stars of this type are red. Their spectra have road absorption lines in the red region of the spectrum. These nes are mostly due to carbon monoxide and cyanogen.

763. Globular Star Clusters.—Globular star clusters (Fig. 774) are made up of swarms of many thousands of stars. They can be resolved with great difficulty even with a large telescope. The boundary of a cluster is approximately circular. The spectra of globular clusters show that the individual stars have large radial velocities. All globular clusters are approximately of the same size. The central part of each cluster has a diameter of about 15 light years. The diameter of the outlying region may be as much as 100 light years. The nearest globular cluster is about 20,000 light years away and the distance of the most remote one is about 220,000 light years. The stars which compose these clusters are giants much greater than our sun.

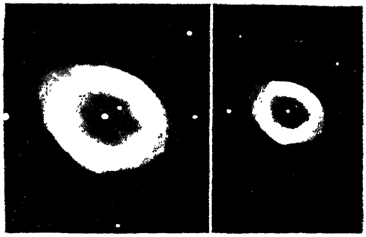


FIG. 775.—Ring nebula in Lyra photographed (a) with 100-in telescope, (b) with the 60-in. telescope. (*Courtesy Mt. Wilson Observatory.*)

FIG. 776.—Canes Venatici, spiral nebula, exposed 3 hr. with the 100-in. reflecting telescope. (*Courtesy Mt. Wilson Observatory.*)

764. Nebulae.—Nebulae appear as faint hazy clouds of light. Sometimes they are of great extent. A few of the very brightest are visible to the naked eye. The great majority are so faint that powerful telescopes are necessary to study them. Planetary nebulae (Fig. 775) are roundish and sharply defined at the edge. Diffuse nebulae have irregular outlines and often irregular shapes. Spiral nebulae (Fig. 776) are nearly flat objects. They may be seen edgewise (Fig. 777) or in plan (Fig. 778). Gaseous nebulae have sharp bright spectral lines similar to those emitted by a rarefied gas. Extragalactic nebulae give continuous spectra crossed by a large number of dark lines (Figs. 779 and 780). These

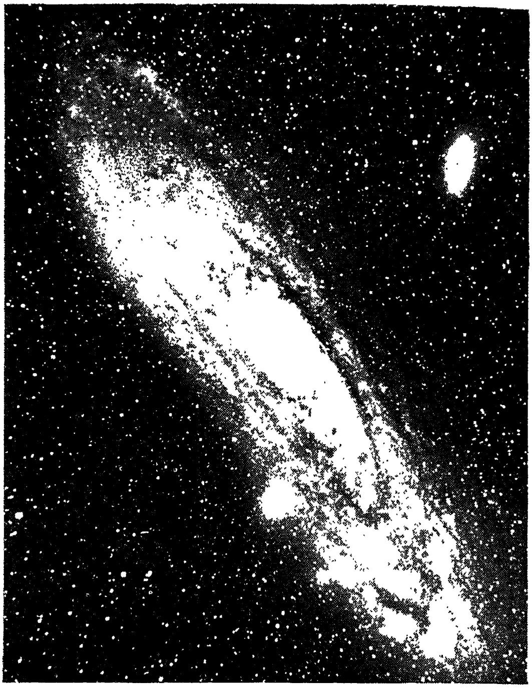


FIG. 777.—The great spiral nebula of Andromeda. It is at a distance of 930,000 light years. (*Astronomical photograph from Yerkes Observatory, reprinted by permission of Chicago University Press.*)

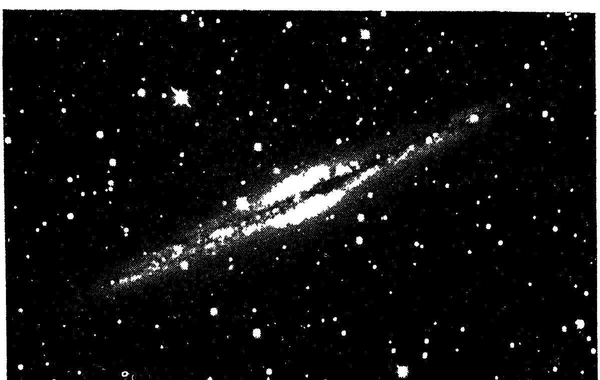


FIG. 778.—Spiral nebula of Andromeda on edgewise exposure for 7 hr. 15 min. with the 60-in. reflecting telescope. (*Courtesy Mt. Wilson Observatory.*)

spectra very closely resemble the spectra of stars such as our sun and the character of these spectra indicates that the light from these nebulae comes originally from incandescent bodies surrounded by absorbing atmospheres. Hence these nebulae

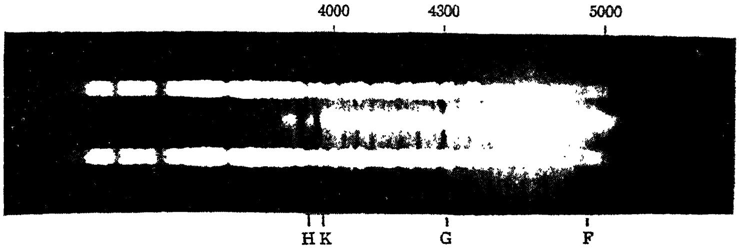


FIG. 779.—Spectrum of nebula in Andromeda with comparison spectrum. The spectrum is continuous crossed by absorption lines. (*Courtesy Mt. Wilson Observatory.*)

may be vast clouds of stars not visible as separate stars because of their great distance or they may be clouds of some form of matter illuminated by the light from great stars lying within them.

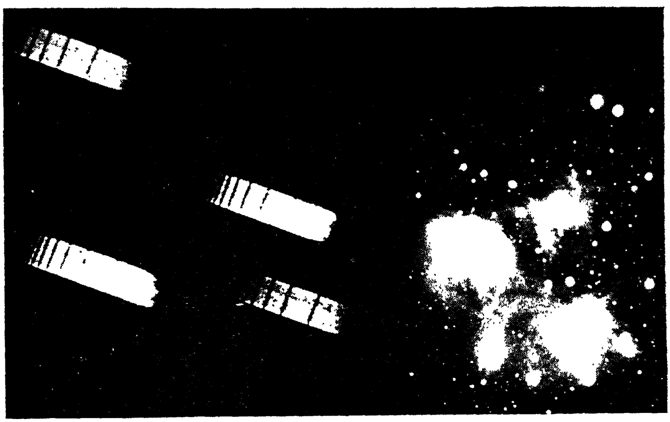


FIG. 780.—Spectra of different positions of the cluster Pleiades. (*Courtesy Mt. Wilson Observatory.*)

765. Novae.—A nova is a variable star whose brightness rises some times to a pronounced maximum and then descends more slowly with secondary maxima and minima at irregular intervals. The appearance of a nova is a rare occurrence, but several bright novae have been observed in the last quarter of a century. With

the great telescopes and spectrographs now available much information has been gained about these unusual stars. Their spectra (Fig. 781) undergo rapid changes. During the rapid rise in brilliancy the spectral lines are displaced toward the violet. The displacement of spectral lines due to the Doppler effect has

FIG. 781.—Spectrum of Novae Acquilae, showing changes in the spectrum. The hydrogen lines become increasingly prominent. (*Astronomical photograph from Yerkes Observatory, reprinted by permission of Chicago University Press.*)

already been discussed. Figure 782 gives another illustration. This displacement indicates that the shell of gas around the star is expanding with explosive violence. The sudden increase in brightness must be due to a sudden release of energy in the interior of the star. This energy may be of sub-atomic origin.

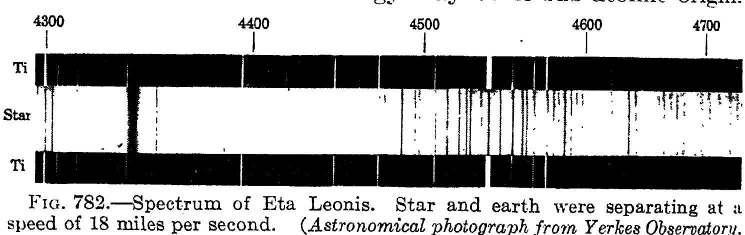


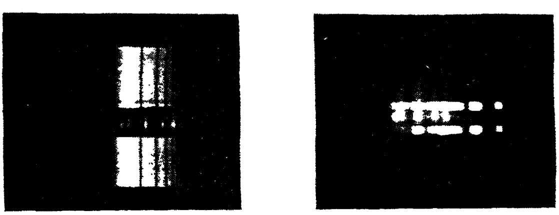
FIG. 782.—Spectrum of Eta Leonis. Star and earth were separating at a speed of 18 miles per second. (*Astronomical photograph from Yerkes Observatory, reprinted by permission of Chicago University Press.*)

766. Spectra of Comets.—A spectroscopic study of comets (Fig. 783) shows that the light of a comet's head comes partly from reflected sun light and partly from light emitted by a luminous gas. The former gives a continuous spectrum crossed by Fraunhofer lines (Fig. 784). The latter shows band spectra which arise from molecular hydrogen, from carbon monoxide, m cyanogen and various hydrocarbons. When the spectrum

of the tail is bright enough to be observed, it also shows reflected sunlight and bands due to carbon monoxide at extremely low pressures. When the comet is sufficiently near the sun, spectral lines characteristic of sodium and other metallic vapors, probably including iron, can be observed. The presence of spectral lines or bands characteristic of carbon monoxide, molecular nitrogen, and hydrocarbons shows that these compounds are present in comets. Comets are, however, probably not mainly composed of these substances. It is likely that solid or liquid particles form the greater part of them.

FIG. 783.—Comet 1908 III. Morehouse, Nov. 16, 1908. The tail which points away from the sun consists of very rarefied gas. The head is hazy cloud of faintly shining matter. (*Astronomical photograph from Yerkes Observatory, reprinted by permission of University of Chicago Press.*)

767. Interior of Stars.—The central portion of a star consists primarily of atomic nuclei from which the satellite electrons have been removed. The temperature is very high in the interior of a star. It decreases as we pass from the center to the outside of the star. At the center of the star most of the atoms have lost their satellite electrons. The number of atoms



(a) (b)

FIG. 784.—Spectra of Morehouse comet photographed (a) with a slit spectrograph, (b) with an objective prism spectrograph. (*Courtesy Mt. Wilson Observatory.*)

retaining their satellite electrons increases as the distance from the center of the star is increased. At the surface of the star

most of the atoms are complete except for one or two of the outermost electrons. At the surface of cooler stars complete molecules may be found. These molecules can be identified by studying their spectra.

The mean density of stars varies within wide limits. In some cases the density is a small fraction of the density of our sun. In other cases it is many times that of the sun. The central density of a star is much greater than its mean density. The average density of "white dwarfs" is many times that of the sun. The explanation for such a large density is found in assuming that the outer electrons have been stripped from the atoms leaving nothing but the nuclei. Such a nucleus would be a million times smaller in volume than a normal atom but its mass would be nearly equal to that of an atom. A gas composed of such nuclei could be compressed to enormous densities and there would still be an opportunity for the particles to move about as if in the gaseous state. The central density of the companion of Sirius is, according to the calculations of Eddington, two million times that of the sun.

There is a difference of opinion about the central temperature of stars. There is general agreement that these temperatures are very high. Central stellar temperatures lying between 1,000,000°C. and 40,000,000°C. are now attributed to stars depending on the spectral type to which they belong. The "White Dwarfs" have in all probability much greater central temperatures than others. The temperature of the companion of Sirius is, according to the calculations of Eddington, as much as 1,000,000,000°C.

768. Sources of Stellar Energy.—Many theories have been advanced to account for the source of stellar energy. The only type of theory which seems adequate is one which assumes the existence of some internal store of energy which decreases very slowly with time. Such a store of energy might be found in the breaking up of more complex atoms into simpler atoms or in the building up of complex atoms out of simpler atoms by a process in which matter is partially annihilated. The first of these postulates assumes that elements of higher atomic weights than those found on the earth may be found in the interior of stars. This hypothesis is scarcely more than a speculation. It is open to many objections. The theory which assumes the

building up of complex elements out of simpler elements is illustrated by what would happen if four atoms of hydrogen could be made to unite to form one atom of helium. The atomic weight of hydrogen is 1.008 and that of helium is 4. If a union of four atoms of hydrogen should take place in a way to form one atom of helium, the atomic weight of helium might be expected to be $4 \times 1.008 = 4.032$ instead of 4, the true atomic weight of helium. In such a transformation there would be a decrease of 0.032 in mass. The energy corresponding to this decrease of mass would be liberated during the process of combination. The amount of energy liberated in this way would be enough to insure that a star composed largely of hydrogen might have a very long life—thousands of millions of years—before its temperature would decrease an appreciable amount.

APPENDIX A

TRIGONOMETRIC FORMULAE

In a right triangle ABC (Fig. 785) it is convenient to define the relations between the sides in the following way. It is to be remembered that this is purely a matter of definition suggested by convenience and that by making these definitions much time is saved in finding the sides of such a triangle. Consider the angle at A. Divide the side BC, opposite A, by the hypotenuse AB and call the ratio, for brevity, **sine A.** Now divide the side AC, adjacent to A, by the hypotenuse AB and call this ratio **cosine A.** Then divide the side BC, opposite A, by the side AC, adjacent to A, and call this ratio **tangent A.**

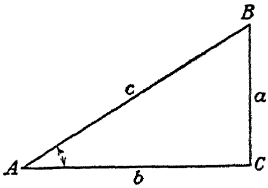


FIG. 785.—Definition of trigonometric functions.

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{BC}{AB}.$$
$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{AC}{AB}.$$
$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{BC}{AC}.$$

Let c = the hypotenuse, b = the adjacent side, and a = the opposite side.

$$\sin A = \frac{a}{c}. \qquad a = c \sin A.$$
$$\cos A = \frac{b}{c}. \qquad b = c \cos A.$$
$$\tan A = \frac{a}{b}. \qquad a = b \tan A.$$

Tables are prepared giving the values of sin A, cos A, and tan A for all values of A. With such a table at hand it is easy to find any side of a right triangle when an acute angle and one of the sides are given.

Example.—In a right triangle ABC (Fig. 785) the angle A is 30 deg., and the side BC is 2 ft. Find the hypotenuse.

$$\sin A = \frac{BC}{AB}.$$

$$\sin 30 \text{ deg.} = \frac{1}{2}.$$

$$\frac{1}{2} = \frac{2}{AB}.$$

$$AB = 4 \text{ ft.}$$

NOTE.—For numerical values of trigonometric functions see Table XXII.

APPENDIX B

COMMON UNITS

Units of Force.—There are two sets of units of force, known as (*a*) gravitational or practical units and (*b*) absolute units. The following outline shows the relation of these units to each other:

Units of Force	Gravitational	English	= pound of force.
			= weight of pound.
		Metric	= gram of force.
			= weight of gram.
	Absolute	English	= poundal.
		Metric	= dyne.

A force of 1 *lb. or a pound of force* is a force equal to the attraction of gravity on a mass of 1 lb. at sea level and 45° north latitude.

A force of 1 *g. or a gram of force* is a force equal to the attraction of gravity on a mass of 1 g. at sea level and 45° north latitude. A force of 1 kg. is equal to 1,000 g. of force.

A poundal is that force which will give a mass of 1 lb. an acceleration of 1 ft. per second per second.

A dyne is that force which will give a mass of 1 g. an acceleration of 1 cm. per second per second.

Also 1 lb. of force = 32.2 poundals, and

1 g. of force = 980 dynes.

Units of Work.—Units of work must be divided into two classes according to the units of force used in defining them.

Units of Work	Gravitational	English	= foot-pound.
		Metric	= gram-centimeter.
	Absolute	English	= foot-poundal.
		Metric	= erg.

A foot-pound of work is the work done by a force of 1 lb. acting through a distance of 1 ft.

Foot-pounds = pounds of force × feet.

A gram-centimeter of work is the work done by a force of 1 g. acting through a distance of 1 cm.

Gram-centimeters = grams of force × centimeters.

A kilogram-meter of work is the work done by a force of 1 kg. acting through a distance of 1 m.

Kilogram-meters = kilograms of force × meters.

A foot-poundal of work is the work done by a force of 1 poundal acting through 1 ft.

Foot-poundals = poundals × feet.

An erg of work is the work done by a force of 1 dyne acting through a distance of 1 cm.

Ergs = dynes × centimeters.

A joule of work is defined to be equal to 10,000,000 ergs of work.

Also 1 ft.-lb. = 32.2 foot-poundals.

1 g.-cm. = 980 ergs,

1 joule = 10^7 ergs.

Units of Power.—In the English system the unit of power is the *horse-power*. The absolute unit of power is the *watt*.

One horsepower is defined to be 33,000 ft.-lb. of work per minute or 550 ft.-lb. of work per second.

$$\text{Horsepower} = \frac{\text{work in foot-pounds}}{33{,}000 \times \text{time in minutes}}.$$

A watt is defined to be 1 joule of work per second or 10,000,000 ergs of work per second.

$$\text{Watts} = \frac{\text{work in ergs}}{10{,}000{,}000 \times \text{times in seconds}}.$$

1 hp. = 746 watts.

Units of Volume.

1 liter = 1,000 c.c.

1 gallon = 231 cu. in.

= 8.337 lb. of water.

APPENDIX C

ELECTROMAGNETIC UNITS

Unit Strength of Current.—The electromagnetic unit of current is defined to be that current which when flowing in a wire bent in the form of a circle with a radius of 1 cm. exerts a force of 2π dynes on unit magnetic pole at the center of the circle. This force is perpendicular to the plane of the circle.

Unit Difference of Potential or Electromotive Force.—An electromagnetic unit difference of potential exists between two points when it requires the expenditure of 1 erg of work to carry an electromagnetic unit of positive electricity from one point to the other against the electric force.

Unit of Resistance.—A conductor has one electromagnetic unit of resistance when one electromagnetic unit of difference of potential causes one electromagnetic unit of current to flow through it.

Unit Quantity of Electricity.—One electromagnetic unit of quantity of electricity is the quantity of electricity conveyed by one electromagnetic unit of current in 1 sec.

Unit of Capacitance.—A conductor has one electromagnetic unit of capacitance when one electromagnetic unit of charge changes its potential by one electromagnetic unit.

Unit of Induction.—A circuit has one electromagnetic unit of induction when the variation of the current in it at the rate of one electromagnetic unit per second produces one electromagnetic unit of electromotive force.

PRACTICAL UNITS

Resistance.—The *ohm* = 10^9 electromagnetic units of resistance. It is also defined as the resistance at 0°C. of a uniform column of mercury 106.3 cm. long and 14.4521 g. in mass.

Current.—The *ampere* = 10^{-1} electromagnetic unit of current. The ampere is also defined to be that unvarying current which deposits silver at the rate of 0.001118 g. per second from an aqueous solution of silver nitrate of standard concentration and at a fixed temperature.

Electromotive Force.—The *volt* = 10^8 ergs or electromagnetic units of difference of potential. It is also defined as that electromotive force which when applied to conductor having a resistance of 1 ohm produces in it a current of 1 amp.

Quantity.—The *coulomb* = 10^{-1} electromagnetic unit of quantity. The coulomb is that quantity of electricity which is conveyed by a current of 1 amp. in 1 sec.

Capacitance.—The *farad* = 10^{-9} electromagnetic unit of capacitance. A conductor or a condenser has a capacitance of 1 farad when it is charged to a

potential of 1 volt by 1 coulomb of electricity. The microfarad is the millionth part of a farad.

Inductance.—The *henry* = 10^9 electromagnetic units of inductance. It is the inductance in a circuit when an electromotive force of 1 volt is induced in the circuit by a current which is changing at the rate of 1 amp. per second.

Power.—The *watt* = 10^7 ergs per second = 1 joule per second. It is the work done in 1 sec. by a current of 1 amp. flowing under an electrical pressure of 1 volt. The *kilowatt* = 1,000 watts; 1 kw. for 1 hr. is called a *kilowatt-hour*. It is represented by a current of 10 amp. flowing for 1 hr. under a pressure of 100 volts.

MAGNETIC UNITS

Unit Magnetic Pole.—A unit magnetic pole is one which when placed at a distance of 1 cm. in air from a similar pole of equal strength repels it with a force of 1 dyne.

Unit Magnetic Field.—The intensity of a magnetic field is one oersted when the field acts on a unit magnetic pole with a force of 1 dyne. This unit was formerly called a gauss. Such a field is represented by drawing one line of force for each square centimeter.

Magnetic Flux.—The maxwell is the unit of magnetic flux. One line of magnetic force is called a *maxwell.*

Magnetomotive Force or Magnetic Difference of Potential.—The *gilbert* is the unit of magnetomotive force or of magnetic difference of potential. The magnetomotive force of a circuit is 1 gilbert when it requires 1 erg of work to carry unit pole completely around the circuit. The magnetic difference of potential between two points is 1 gilbert when it requires 1 erg of work to carry unit north pole from one point to the other.

$$1 \text{ ampere-turn} = \frac{4\pi}{10} \text{ gilberts.}$$

Magnetic Reluctance.—A magnetic circuit has a reluctance of one electromagnetic unit when a magnetomotive force of 1 gilbert sets up a magnetic flux of 1 maxwell in it.

APPENDIX D

DEFINITIONS

Many physical quantities cannot be accurately defined and many physical relations cannot be accurately stated without the use of the calculus. In order further to clarify earlier definitions and statements in the text and make their meaning more precise, some of them are restated here in the form of supplementary notes in which the methods of the calculus have been used.

1. Linear Velocity.—The linear velocity of a body is defined as

$$v = \frac{ds}{dt},$$

where v is the velocity and ds/dt the first derivative of the space with respect to the time.

2. Linear Acceleration.—The linear acceleration of a body is the time rate of change of its linear velocity.

$$\text{Acceleration} = a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

3. Newton's Second Law of Motion.—This law which states that the applied force is equal to the time rate of change of the momentum can best be written as

$$\text{Force} = F = \frac{d(mv)}{dt} = m\frac{dv}{dt} = m\frac{d^2s}{dt^2}.$$

where m = the mass of the body.
F = the force measured in absolute units.

4. Work and Power.—The work done by a force F acting on a body is,

$$W = \int F \cdot ds.$$

The integral is to be taken over the path along which the body moves.

Power, which is the time rate of doing work, is

$$\text{Power} = P = \frac{dW}{dt}.$$

5. Kinetic Energy.—The kinetic energy of translation of a body of mass m, moving with a velocity v, is obtained as follows:

$$dW = F\,ds \qquad m\frac{dv}{dt}ds$$

$$\text{Kinetic energy} = W = \int F\,ds = m\int \frac{dv}{dt}ds = m\int \frac{ds}{dt}dv$$
$$= m\int v\,dv = \frac{1}{2}mv^2 + \text{constant.}$$

6. Angular Velocity.—Angular velocity is the rate at which a radius vector drawn to the axis of the body rotates.

$$\text{Angular velocity} = \omega = \frac{d\theta}{dt},$$

where θ is the angle in radians.

7. Angular Acceleration.—Angular acceleration is the time rate of change of angular velocity.

$$\text{Angular acceleration} = A = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

8. Newton's Second Law for Rotary Motions.—For rotary motions this law states that the torque is proportional to the time rate of change of the angular momentum.

$$\text{Torque} = T = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\frac{d^2\theta}{dt^2}.$$

9. Kinetic Energy of Rotation.—The kinetic energy of rotation of a body is the work done in producing its state of rotation.

$$\text{Work} = dW = F \cdot ds = F \cdot r \cdot d\theta = T \cdot d\theta$$
$$\text{Kinetic energy} = \int T\,d\theta = \int \frac{d}{dt}(I\omega)d\theta = I\int \frac{d\theta}{dt}\omega$$
$$= I\int \omega\,d\omega = \frac{1}{2}I\omega^2 + \text{constant,}$$

where F is the force and r is the moment arm.

10. Coefficient of Linear Expansion.—The coefficient of linear expansion, k, is defined as:

$$k = \frac{1}{l}\frac{dl}{dt},$$

where l is the length and t the temperature.

11. The coefficient of volume expansion, b, is defined as:

$$b = \frac{1}{v}\frac{dv}{dt},$$

where v is the volume and t is the temperature.

12. The coefficient of thermal conductivity, k, is defined as:

$$k = \frac{Q}{A \cdot T \cdot \dfrac{dt}{dx}},$$

where Q = the quantity of heat.
A = the area of the cross section.
T = the time of flow.
dt/dx = the temperature gradient.

13. Coefficient of Mutual Inductance.—The electromotive force induced in the secondary circuit by changing the current in the primary circuit is:

$$E_s = M \cdot \frac{di}{dt},$$

where M = the coefficient of mutual induction.
E_s = the electromotive force in the secondary.
di/dt = the rate of change of the current in the primary.

14. The Coefficient of Self-inductance.—The back electromotive force induced in a circuit by a current changing in the same circuit is:

$$e = L\frac{di}{dt},$$

where L = the coefficient of self-induction.
e = the back electromotive force.
di/dt = the rate of change of current in the circuit itself.

15. Electromotive Force Due to Change of Flux.—If the flux in a circuit is changing, the electromotive force induced in the circuit is:

$$e = \frac{n}{10^8}\frac{dN}{dt}\text{volts},$$

where e = the electromotive force in volts.
n = the number of turns linking the flux.
dN/dt = the time rate of change of the flux.

16. Quantity of Electricity Induced by a Change of Flux.—If R is the resistance of a closed circuit in which the flux is changing, i, the instantaneous current in the circuit, and Q, the total quantity of electricity which flows through the circuit, then

$$\frac{e}{R} = \frac{n}{10^8R}\frac{dN}{dt}\text{ amp.}$$

$$Q = \int i\,dt = \frac{n}{10^8R}\int_{N_1}^{N_2}\frac{dN}{dt}dt = \frac{n}{10^8R}\int_{N_1}^{N_2}dN$$

$$\frac{\ }{10^8R}(N_2 - N_1)\text{ coulombs.}$$

17. Average Value of an Alternating Electromotive Force.—The instantaneous value of a sinusoidal electromotive force is:

$$e = E \sin \varphi.$$

The average value over one-half of a cycle is:

$$E_{\text{ave.}} = \frac{1}{\pi}\int_0^{\pi} e\, d\varphi = \frac{1}{\pi}\int_0^{\pi} E \sin\varphi\, d\varphi = \frac{E}{\pi}\int_0^{\pi} \sin\varphi\, d\varphi = \frac{2E}{\pi} = 0.637E.$$

18. The Effective Value of an Alternating Current.—The effective value of an alternating current is a steady current which would have the same heating effect as the alternating current. The heating effect is determined by the square of the current. Hence the effective value of an alternating current can be obtained by taking the square root of the average value of the square of the instantaneous currents.

$$i = I \sin\varphi.$$
$$i^2 = I^2 \sin^2\varphi.$$

$$\text{The square of the effective value} = \frac{1}{\pi}\int_0^{\pi} i^2\, d\varphi = \frac{I^2}{\pi}\int_0^{\pi} \sin^2\varphi\, d\varphi = \frac{I^2}{\pi}\frac{\pi}{2} = \frac{I^2}{2}.$$

$$I_{\text{eff.}} = \frac{I}{\sqrt{2}}.$$

APPENDIX E

DERIVATIONS

1. Composition of Forces Not at Right Angles.—Suppose that two forces a and b act at an angle x with each other and it is desired to find their resultant c (Fig. 14).

$$\overline{OC}^2 = \overline{CD}^2 + \overline{OD}^2.$$
$$\overline{OC}^2 = \overline{CD}^2 + (OB + BD)^2.$$
$$\overline{OC}^2 = a^2 \sin^2 x + (b + a \cos x)^2$$
$$= a^2(\sin^2 x + \cos^2 x) + 2ab \cos x + b^2$$
$$c^2 = a^2 + b^2 + 2ab \cos x.$$

2. Modulus of Volumetric Elasticity for Gases.—If dp denotes the change of pressure and dv the corresponding change of volume, the modulus of volumetric elasticity E is;

$$E = \frac{dp}{(dv/v)} = v\left(\frac{dp}{dv}\right).$$

If a gas is compressed or expands at constant temperature, Boyle's law holds and

$$pv = PV = \text{constant},$$
$$\frac{dp}{dv} \cdot v + p \quad 0,$$
$$\frac{dp}{dv}$$
$$E = v\left(\frac{dp}{dv}\right) = -\frac{p}{v} \cdot v = -p.$$

i.e., the modulus of isothermal elasticity is numerically equal to the pressure. If the gas expands or is compressed adiabatically,

$$pv^k = PV^k = \text{constant},$$

where

$$k = \frac{C_p}{C_v} = \frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}.$$

Differentiating,

$$kpv^{k-1} + v^k\left(\frac{dp}{dv}\right) = 0.$$
$$v\left(\frac{dp}{dv}\right) = -kp.$$
$$E_a = v\left(\frac{dp}{dv}\right) = -kp.$$

3. Moment of Inertia of a Disk.—The moment of inertia, which is the sum of the products of the individual masses and the square of their respective distances from the axis of rotation, can be written as

$$\int_0^M r^2 \cdot dm,$$

where the integral is to be taken over the entire mass.

For a disk of radius a, thickness l, and density ρ we have,

$$\begin{aligned} dm &= 2\pi\rho l r \, dr. \\ I &= 2\pi\rho l \int_0^a r^3 \, dr \\ &= 2\pi\rho l \frac{r^4}{4}\Big|_0^a = 2\pi\rho l \frac{a^4}{4} \\ &= \frac{\pi\rho l a^4}{2}. \\ \pi\rho a^2 l &= M = \text{mass of the disk.} \end{aligned}$$

Hence

$$Ma^2$$

4. Simple Harmonic Motion.—The method of the calculus simplifies the analysis of simple harmonic motion.

Let ω = the angular velocity of OP (Fig. 167).

$$x = R\cos\theta = R\cos\omega t. \tag{1}$$

$$\frac{dx}{dt} = V = -R\omega\sin\omega t. \tag{2}$$

$$\frac{d^2x}{dt^2} = \frac{dV}{dt} = A_x = -R\omega^2\cos\omega t. \tag{3}$$

Dividing equation (3) by equation (1)

$$\begin{aligned} \frac{A_x}{} &= -\frac{R\omega^2\cos\omega t}{R\cos\omega t} = -\omega^2 \\ &= -\left(\frac{2\pi}{T}\right)^2, \end{aligned}$$

whence

$$2\pi\sqrt{-\frac{x}{A_x}}.$$

5. Velocity of Waves in a Cord.—Suppose that a transverse wave is traveling toward the right in the cord (Fig. 786).

Let T = the tension in dynes.

m = the mass in grams per centimeter.

V = the velocity of the wave in centimeters per second.

Now assume that while the pulse or wave is moving toward the right with a velocity V, the cord is made to move toward the left with an equal velocity. As a result of these superposed velocities, the wave appears to stand still. For simplicity assume the pulse is circular in form and consider a small segment ds of the cord. The components of the tension T along the radius CO is:

$$F = 2T \sin d\theta = 2T\, d\theta \text{ approx.}$$

Since $2\, d\theta = ds/R$ and $d\theta = ds/2R$,

$$F = 2T\, d\theta = 2T\frac{ds}{2R} = T\frac{ds}{R}.$$

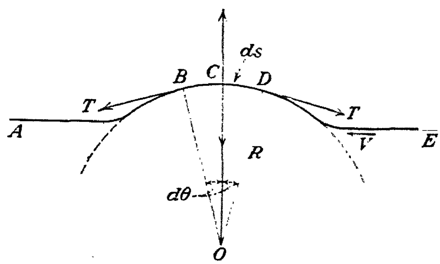


FIG. 786.—Transverse wave in a stretched string.

The centrifugal force on the element ds of the cord of mass $m\, ds$ is,

$$f = m\, ds\frac{V^2}{R}.$$

Since the cord is in equilibrium, the force F due to tension and the centrifugal force f must be equal.

Hence,

$$m\, ds\frac{V^2}{R} = T\frac{ds}{R}.$$

$$V^2 = \frac{T}{m}.$$

$$V = \sqrt{\frac{T}{m}}.$$

6. Standing Waves.—A simple harmonic motion can be represented by an equation of the form,

$$y = a \sin 2\pi\frac{t}{T},$$

where a is the amplitude, y the displacement, and T the period. Every particle of the medium through which the wave passes has a motion of this type. The vibrations, however, are not all in the same phase. The difference in phase increases with the distance from the origin. At a distance

x from the origin the difference in phase is $2\pi x/\lambda$. The complete equation for the wave then is:

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

Standing waves are produced by the superposition of an incident and a reflected train of waves. The equation of the incident waves is:

$$y_2 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

and that of the reflected waves is:

$$y_1 = -a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

The resultant displacement due to both trains of waves is,

$$\begin{aligned} y = y_1 + y_2 &= a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) - a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \\ &= -2a \sin 2\pi\frac{x}{\lambda} \cos 2\pi\frac{t}{T}. \end{aligned}$$

If we denote the resultant amplitude by $A = -2a \sin 2\pi\, x/\lambda$,

$$y = A \cos 2\pi\frac{t}{T}.$$

Hence each point in the cord has a simple harmonic motion whose amplitude is $A = -2a \sin 2\pi\, x/\lambda$. This amplitude varies from point to point along the wave train. When $x = 0$, $x = \lambda/2$, $x = \lambda$, etc., the amplitude is zero and these points are nodes.

7. Work Done in an Isothermal Expansion.—When a gas expands isothermally the work done by a piston in moving a distance ds is:

$$dW = pa\, ds,$$

where a is the area of the piston and p the pressure. Since $a \cdot ds = dv =$ the change in volume,

$$\begin{aligned} dW &= p\, dv. \\ W &= \int_{v_1}^{v_2} p\, dv. \end{aligned}$$

For any isothermal change,

$$pv = RT$$

and

$$p = \frac{RT}{v}.$$

$$\begin{aligned} W &= \int_{v_1}^{v_2} \frac{RT}{v} dv = RT \int_{v_1}^{v_2} \frac{dv}{v} \\ &= RT \log_e \frac{v_2}{v_1}. \end{aligned}$$

8. Potential at a Point.—The potential at a point is the work necessary to bring unit positive charge from infinity to that point. In the case of a point charge of Q units of electricity, the force at a distance r from the charge is $F = Q/r^2$.

The work dW to carry unit charge a distance dr against this force is:

$$dW = F\,dr = \frac{Q}{r^2}dr$$

$$\text{The potential at the point} = V = \int_a^\infty F\,dr = \int_0^\infty \frac{Q}{r^2}\,dr$$
$$= -\frac{Q}{r}\Big|_a^\infty = \frac{Q}{a}.$$

9. Energy to Charge a Body.—The energy E required to charge a body of capacity C with Q units of electricity is:

$$dE = v \cdot dq,$$

where dE = the energy required to add an infinitesimal charge dq to the body when its potential is v.

Since,

$$v = \frac{q}{C},$$

$$dE = \frac{q}{C}dq,$$

$$E = \int dE = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{C}.$$

10. Laplace's Law.—The intensity dH of a magnetic field at a point P (Fig. 787) due to a current I in a small element of a conductor ds is proportional to the current, to the length of the element, to the sine of the angle between the direction of the current and the radius from the element to the point P, and inversely proportional to the square of the distance from the element to the point P. This relation between the current and the magnetic field, known as *Laplace's law*, is:

$$dH = \frac{I\,ds}{r^2}\sin\theta,$$

where the current is measured in electromagnetic units, the magnetic field in oersteds, and the distance in centimeters. The magnetic field is perpendicular to the plane which passes through P and the element of the current. It is impossible to prove this law for a single element of the conductor, but it can be tested for a complete circuit.

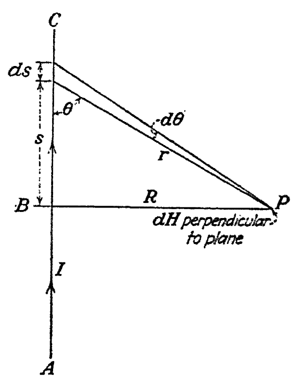


FIG. 787.—Magnetic field produced by a long wire carrying a current.

At the center of a loop of wire bent in the form of a circle of radius R,

$$dH = \frac{I\,ds}{R^2}\sin\theta.$$

Since

$$\sin\theta = 1 \text{ and } ds = R\,d\theta$$

$$H = \int_0^{2\pi}\frac{IR\,d\theta}{R^2} = \frac{I}{R}\int_0^{2\pi}d\theta$$

$$\frac{2\pi I}{R} \text{ oersteds.}$$

For a long straight wire, $H = 2I/R$ oersteds where R = distance from the wire.

11. Force on a Conductor Moving in a Magnetic Field.—When a conductor of length l is moving across a magnetic field of strength B with a velocity v, the electromotive force generated in it is;

$$e = Blv \text{ abvolts.}$$

The power to keep the conductor moving with this velocity is:

$$\text{Power} = Ie = BIlv, \text{ ergs per second,}$$

where I is the current in the conductor in abamperes. In terms of the force F acting on the conductor, the power is:

$$\text{Power} = Fv \text{ ergs per second.}$$

These two expressions for the power must be equal. Hence,

$$Fv = BIlv.$$
$$F = BlI \text{ dynes.}$$

12. Rise and Decay of Current in an Inductive Circuit.—When a constant electromotive force E is applied to a circuit which contains resistance R and an inductance L, there is an induced electromotive force which opposes the rise of the current. The relation between the current i, the impressed electromotive force E, and the back electromotive force $e = L\,di/dt$ is given by the equation,

$$E - L\frac{di}{dt} = Ri$$

Dividing by R and transposing, $\frac{E}{R} - i = \frac{L}{R}\frac{di}{dt}$.

When this equation is solved for the current, we have,

$$i = I\left(1 - e^{-\frac{R}{L}t}\right).$$

If, after the current has attained its final value, the impressed electromotive force is short-circuited leaving the circuit closed, the current in the circuit dies down exponentially. The current in the circuit at any instant is given by the equation,

$$-L\frac{di}{dt} = Ri.$$

$$\frac{di}{i} = -\frac{R}{L}dt.$$

$$\int_I^i \frac{di}{i} = -\frac{R}{L}\int_0^t dt.$$

$$\log\frac{i}{I} = -\frac{R}{L}t.$$

$$i = Ie^{-\frac{R}{L}t}.$$

13. Discharge of a Condenser Through a High Resistance.—When the terminals of a condenser which has been charged with Q units of electricity are connected through a high resistance, the condenser discharges exponentially. The relation between the instantaneous charge on the condenser and the time is given by the equation:

$$\frac{q}{C} = Ri = -R\frac{dq}{dt}.$$

$$\frac{dq}{q} = -\frac{1}{RC}dt.$$

$$\int_Q^q \frac{dq}{q} = \int_0^t -\frac{1}{RC}dt.$$

$$\log_e\frac{q}{Q} = -\frac{t}{RC}.$$

$$Qe^{-\frac{t}{RC}}.$$

14. Energy Stored up About a Circuit.—The energy stored up in a circuit for which the coefficient of self-induction is L is equal to the work required to establish the current in the circuit or the energy which will be released when the current is eliminated. The back electromotive force due to the self-inductance in the circuit is:

$$e = -L\frac{di}{dt}.$$

The work done against this back electromotive force in the time dt is,

$$e \cdot i \cdot dt = L \cdot i\frac{di}{dt}dt = L \cdot i \cdot di.$$

The whole work is,

$$W = \int_0^I e \cdot i \cdot dt = \int_0^I L \cdot i \cdot di$$

$$\frac{1}{2}LI^2.$$

15. The Lens Formula.—In Fig. 788 let V be a point source of light on the principal axis. A ray VA on entering the glass will be bent toward BA, the normal to the surface at A. By the law of refraction,

$$\frac{\sin VAC}{\sin DAB} \quad \frac{\sin i}{\sin r} = n = \text{index of refraction of glass.}$$

If A lies near the principal axis, the following approximate relations may be written down:

$$\frac{AP}{PV} - \frac{AP}{\text{object distance}} \quad \frac{AP}{U}.$$

$$\frac{AP}{CP} = \frac{AP}{\text{radius of curvature}} \quad \frac{AP}{R_1}.$$

$$\theta - i + r \quad \frac{AP}{PU} = \frac{AP}{\text{image distance}} \quad \frac{AP}{V}.$$

$$\therefore i = \frac{AP}{U} - \frac{AP}{R_1}.$$

$$= \frac{AP}{V} - \frac{AP}{R_1}.$$

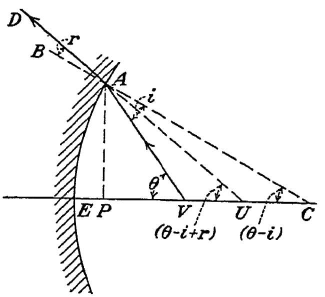


FIG. 788.—Refraction of light at a spherical surface.

Since i and r are small,

$$\frac{\sin i}{\sin r} = \frac{i}{r} = n, \text{ approximately}$$

and

$$i = nr.$$

$$\left(\frac{AP}{U} - \frac{AP}{R_1}\right) = n\left(\frac{AP}{V} - \frac{AP}{R_1}\right).$$

Hence

$$\frac{1}{U} - \frac{1}{R_1} = n\left(\frac{1}{V} - \frac{1}{R_1}\right)$$

and

$$\frac{n}{V} - \frac{1}{U} = \frac{n-1}{R_1}$$

This gives the equation for a single spherical surface.

To get the equation for a lens consider Fig. 789.

Suppose that the ray now emerges from the glass at a convex spherical surface and that the curvature of the convex surface is greater than the curvature of the concave surface. The refraction at the concave surface produces an image of the object V and this image is located at W, a distance V' from the surface. In this case the relation between the image and the object is given by

$$\frac{n}{V'} \quad \frac{1}{U} \quad \frac{n-1}{R_1}.$$

For the convex surface, W may be considered the object and V' the object distance. Refraction at the second surface of radius of curvature R_2 produces an image at U. The light now goes from glass to air and the index of refraction in such a case is the reciprocal of the index of refraction when the light goes from air to glass. For this case.

$$\frac{\frac{1}{n}}{V} - \frac{1}{V'} = \frac{\frac{1}{n} - 1}{R_2}$$
$$\frac{1}{V} - \frac{n}{V'} = \frac{n - 1}{R_2}$$

Combining these equations give

$$\frac{1}{V} - \frac{1}{U} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

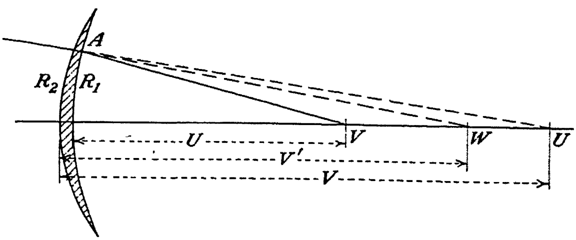


FIG. 789.—Refraction of light by two spherical surfaces.

If the rays are parallel, $U \quad \infty$ and $1/U = 0$, $V = f$. Hence,

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$
$$\therefore \frac{1}{V} \quad \frac{1}{U} \quad \frac{1}{f}.$$

In the case of a double-convex lens the surfaces are curved in opposite directions and the radius of curvature R_1 must be considered as reversed in direction. A real image produced by such a lens is on the side opposite to the source of light. The formula for a double-convex lens, producing a real image, becomes,

$$\frac{1}{U} + \frac{1}{V} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{f}$$

Since regard must be had for the signs of R_1, R_2, U, V, and f, the following convention is generally accepted: Consider R_1, R_2, U, V, and f positive, when conditions are those found in the typical case of a double-convex lens producing a real image and consider either of them negative when it is measured in a direction opposite to that found in the typical case of a double-convex lens producing a real image.

APPENDIX F

TABLES

TABLE I.—DENSITIES OF SOLIDS AND LIQUIDS

Substance	Density Per c.c.	Density Per cu. ft.	Substance	Density Per c.c.	Density Per cu. ft.
	grams	*pounds*		*grams*	*pounds*
Alcohol at 20°C.	0.789	49.3	Iron (wrought)	7.80	480.
Aluminum......	2.65	164.	Kerosene......	0.82	51.2
Balsawood.......	0.16	10.	Leather.......	0.90	56.
	8.60	535.	Lead..........	11.37	710.
Brick...........	2.10	131.	Mercury......	13.60	840.
Copper..........	8.93	555.	Nickel........	8.90	556.
Cork............	0.24	15.	Oak..........	0.80	50.
Diamond........	3.52	220.	Pine..........	0.50	31.2
Ether at 0°C....	0.735	45.9	Sandstone.....	2.25	140.
Glass (crown)....	2.50	156.	Silver.........	10.50	655.
Glass (flint).....	3.70	230.	Turpentine....	0.87	54.3
Gasoline.........	0.79	49.4	Tin...........	7.29	455.
Gold............	19.32	1200.	Water (fresh)..	1.00	62.5
Glycerine........	1.26	78.7	Water (sea)...	1.03	64.4
Ice.............	0.917	57.2	Zinc..........	7.15	446.2
Iron (cast)......	7.20	450.			

TABLE II.—DENSITIES OF GASES

Gas	Density at 0°C. Per liter	Density at 0°C. Per cu. ft.	Gas	Density at 0°C. Per liter	Density at 0°C. Per cu. ft.
	grams	*pounds*		*grams*	*pounds*
Air..............	1.2930	0.0807	Chlorine.........	3.214	0.2011
Acetylene.........	1.1791	0.0736	Helium..........	0.1785	0.01116
Ammonia.........	0.7708	0.0481	Hydrogen........	0.0899	0.00561
Carbon dioxide...	1.9768	0.1234	Nitrogen.........	1.2507	0.0781
Carbon monoxide.	1.25	0.0781	Oxygen..........	1.4291	0.0892

Table III.—Elastic Constants

Substance	Young's modulus, Per sq. cm.	Young's modulus, Per sq. in.	Breaking strength per sq. in.
	grams	*pounds*	*pounds*
Aluminum...	$7. \times 10^8$	9.8×10^6	1.3 to 2.1×10^4
Brass........	$10. \times 10^8$	$14. \times 10^6$	4.4 to 5.6×10^4
Copper.......	$12. \times 10^8$	$17. \times 10^6$	4.0 to 4.4×10^4
Gold.........	$8. \times 10^8$	11.2×10^6	1.56×10^4
Cast iron.....	$13. \times 10^8$	$18. \times 10^6$	1.1 to 3.3×10^4
Wrought iron	$20. \times 10^8$	$28. \times 10^6$	4.1 to 6.4×10^4
Steel.........	$22. \times 10^8$	$31. \times 10^6$	10 to 11×10^4
Nickel.......	$20. \times 10^8$	$28. \times 10^6$	7.6×10^4
Silver........	7.9×10^8	11.1×10^6	4.1×10^4

Table IV.—Compressibility of Liquids

Substance	Temperature	Compressibility per atmosphere per sq. cm.	Substance	Temperature	Compressibility per atmosphere per sq. cm.
	°C.			°C.	
Ethyl alcohol.	14	0.0000987	Mercury....	20	0.0000039
Ethyl ether..	0	0.000143	Turpentine..	20	0.000075
Kerosene....	20	0.0000543	Water......	20	0.000048

Table V.—Coefficients of Friction

Substances	Limits of coefficient	Substances	Limits of coefficient
	$\mu = \frac{F}{P}$		$\mu = \frac{F}{P}$
Wood on dry wood.	0.25 to 0.50	Leather on iron....	0.30 to 0.50
Metals on dry wood	0.20 to 0.60	Leather on wood..	0.30 to 0.50
Metals on dry metals..........	0.15 to 0.20	Iron on stone......	0.30 to 0.70
		Wood on stone....	0.40 to 0.60

TABLE VI.—SURFACE TENSION

Substance	Temperature	Surface tension per cm.	Substance	Temperature	Surface tension per cm.
	°C.	*grams*		°C.	*grams*
Acetic acid.....	20	23.9×10^{-3}	Mercury.....	18	556×10^{-3}
Alcohol (ethyl)..	20	22.1×10^{-3}	Turpentine...	20	28.9×10^{-3}
Ether (ethyl)...	20	17.2×10^{-3}	Petroleum....	20	26.4×10^{-3}
Glycerine.......	18	64.5×10^{-3}	Water.......	20	74.3×10^{-3}

TABLE VII.—PROPERTIES OF AQUEOUS VAPOR

Boiling points of water at pressures near standard atmospheric pressure. The pressures are given in millimeters of mercury at 0°C. at the sea level in latitude 45°.

Pressure	Temperature	Pressure	Temperature	Pressure	Temperature	Pressure	Temperature
mm.	°C.	*mm.*	°C.	*mm.*	°C.	*mm.*	°C.
733	98.99	745	99.44	757	99.89	769	100.33
735	99.07	747	99.52	759	99.96	771	100.40
737	99.14	749	99.59	761	100.04	773	100.47
739	99.22	751	99.67	763	100.11	775	100.55
741	99.29	753	99.74	765	100.18	777	100.62
743	99.37	755	99.82	767	100.26	779	100.69

TABLE VIII.—PROPERTIES OF SATURATED STEAM (CENTIGRADE UNITS)

Temperature	Pressure: Weight per sq. cm.	Pressure: Weight per sq. in.	Volume: Per kilogram	Volume: Per pound	Heat units per unit mass: Of water	Heat units per unit mass: Latent heat	Heat units per unit mass: Total heat of vapor
°C.	kg.	lb.	cu. m.	cu. ft.			
0	0.0063	0.089	204.970	3283.00	0.	594.7	594.7
10	0.0125	0.178	106.620	1707.90	10.	589.4	599.4
20	0.0236	0.336	58.150	931.48	20.	584.1	604.1
30	0.0429	0.61	33.132	530.72	30.	578.8	608.8
40	0.0747	1.06	19.650	314.77	40.1	573.4	613.5
50	0.125	1.78	12.091	193.68	50.1	567.9	618.
60	0.202	2.88	7.695	123.26	60.1	562.4	622.6
70	0.317	4.51	5.050	80.89	70.2	556.8	627.
80	0.482	6.86	3.4085	54.60	80.3	551.	631.5
90	0.714	10.16	2.3592	37.79	90.4	545.2	635.6
100	1.033	14.70	1.6702	26.754	100.5	539.1	639.7
110	1.462	20.79	1.2073	19.339	110.7	532.9	643.6
120	2.027	28.83	0.8894	14.247	120.9	526.6	647.4
130	2.760	39.26	0.6664	10.675	131.1	520.	651.
140	3.695	52.56	0.5071	8.123	141.3	513.2	654.5
150	4.868	69.24	0.3917	6.274	151.6	506.2	657.8
160	6.323	89.93	0.3065	4.91	161.9	498.9	660.8
170	8.104	115.27	0.2429	3.891	172.2	491.4	663.7
180	10.258	145.90	0.1945	3.116	182.6	483.7	666.3
190	12.835	182.56	0.1575	2.523	193.1	475.7	668.8
200	15.890	226.00	0.1288	2.063	203.6	467.5	671.1
210	19.490	277.20	0.1063	1.703	214.1	459.1	673.2

TABLE IX.—MELTING AND FREEZING POINTS OF FATS AND WAXES

Substance	Melting point	Freezing point	Substance	Melting point	Freezing point
	°C.	°C.		°C.	°C.
Butter.........	28–33	20–23	Beeswax......	61–64	60–63
Lard...........	36–40	27–30	Paraffin (soft).	38–52	....
Tallow (mutton).	44–45	36–41	Paraffin (hard).	52–56	....

TABLE X.—HEAT CONSTANTS OF LIQUIDS

Substance	Boiling point	Cubical expansion per °C.	Specific heat, cal. per gram	Heat of vaporization	
				Per gram	Per pound
	°C.			*calories*	*B.t.u.*
Ammonia	−34.	....	....	294.	529
Aniline	184.	....	0.514	110.	198
Alcohol (ethyl)	78.1	0.0011	0.55	205.	369
Benzine	80.3	0.00124	0.34	94.4	170
Chloroform	61.	0.00126	0.232	58.	106
Ether (ethyl)	34.5	0.00163	0.56	88.4	159
Gasoline	70–90	0.0012	....	71–81	128–146
Glycerine	290.	0.00053	0.58	....	....
Mercury	358.	0.000182	0.0332	68.	122
Turpentine	159.	0.00094	0.42	70.	126
Water	100.	0.000301[1]	1.00	538.	970

[1] Between 20° and 40°C.

TABLE XI.—HEAT CONSTANTS OF SOLIDS

Substance	Melting point	Coefficient of linear expansion per °C.	Specific heat, cal. per gram	Heat of fusion	
				Per gram	Per pound
	°C.			*calories*	*B.t.u.*
Aluminum	657.	0.0000255	0.22	76.8	140.
Bismuth	268.	0.0000157	0.0304	12.6	22.7
Brass	....	0.0000193	0.09	....	....
Copper	1084.	0.0000167	0.0909	43.	77.
Glass	....	0.0000083	0.2	....	....
Gold	1063.	0.0000139	0.0303	....	....
Ice	0.	0.000051	0.502	79.8	144.
Iron	1503.	0.0000119	0.104	30.	54.
Lead	327.	0.0000276	0.0302	5.4	9.7
Mercury	−38.8	....	0.033	2.8	5.4
Nickel	1452.	0.0000128	0.109	4.6	8.3
Platinum	1756.	0.0000089	0.0324	27.	48.6
Silver	960.	0.0000188	0.0556	22.	39.
Steel	....	0.0000132	0.107	....	....
Tungsten	3360.	0.0000044	0.034	....	....
Zinc	418.	0.0000263	0.0918	28.1	50.6

Table XII.—Heat of Combustion

Substance	Calories per gram	B.t.u. per pound
Lignite (low grade)	3,520	6,350
Lignite (high grade)	3,990	7,190
Bituminous coal (low grade)	6,090	10,960
Bituminous coal (high grade)	7,850	14,134
Anthracite coal (low grade)	6,990	12,580
Anthracite coal (high grade)	7,420	13,360
Wood (oak)	4,620	8,320
Gasoline (liquid)	11,100	20,000
Alcohol (liquid)	6,440	11,590
Acetylene (gas)	11,900	21,400
Hydrogen (gas)	34,460	62,030
Natural gas (density 0.86 g. per liter)	11,800	21,240
Carburetted water gas (density 0.85 g. per liter)	6,240	11,230

Table XIII.—Specific Heats of Gases at Constant Pressure

Substance	Specific Heat *Calories per gram*
Air	0.2374
Argon	0.1233
Carbon dioxide	0.2025
Carbon monoxide	0.2425
Hydrogen	3.4090
Nitrogen	0.2438
Oxygen	0.2175

Table XIV.—Thermal Conductivities

Substance	Thermal conductivity	Substance	Thermal conductivity
	c.g.s. units		*c.g.s. units*
Aluminum	0.504	Lead	0.083
Air	0.000054	Sand, white dry	0.00093
Brass	0.204	Sawdust	0.00012
Copper	0.918	Silver	0.974
Cork	0.00011	Soil, dry	0.00033
Glass	0.0015	Water	0.00143
Iron	0.161	Zinc	0.265
Ice	0.00396		

TABLE XV.—CRITICAL DATA

Substance	Critical temperature	Critical pressure atmospheres	Critical volume
	°C.		c.c.
Ammonia.....	130.	115.	0.00481
Carbon dioxide	31.1	73.	0.0066
Ether........	197.	35.8	0.158
Helium.......	−268.	2.3	0.00299
Nitrogen......	−146.	33.	0.00517
Oxygen.......	−118.	50.	0.00426
Water.........	365.	194.6	0.00386

TABLE XVI.—DIELECTRIC CONSTANTS

Substance	Temperature	Dielectric constant	Substance	Temperature	Dielectric constant
	°C.			°C.	
Air (at 76 cm. pressure)............	0	1.000586	Dry paper.	....	2.0–2.5
Hydrogen (at 76 cm. pressure)....	0	1.000264	Paraffin...	....	2.0–2.3
Benzene...........	18	2.29	Quartz....	....	4.5
Ordinary glass.....	....	7.0–8.0	Shellac....	....	3.0–3.7
Mica..............	....	5.7–7.0	Sulphur...	....	3.6–4.3
			Olive oil...	....	3.1–3.2

TABLE XVII.—ELECTROCHEMICAL EQUIVALENTS

Substance	Valency	Per coulomb	Substance	Valency	Per coulomb
		grams			grams
Aluminum.....	3	0.0009357	Mercury......	1	0.002075
Copper........	1	0.0006588	Mercury......	2	0.001037
Copper........	2	0.0003294	Nickel........	2	0.0003042
Gold..........	3	0.000681	Oxygen.......	2	0.0000829
Hydrogen......	1	0.0000104	Potassium.....	1	0.0004054
Iron...........	2	0.0002895	Silver.........	1	0.001118
Iron...........	3	0.000193	Sodium.......	1	0.0002387
Lead..........	2	0.001072	Zinc..........	2	0.0003387
Magnesium....	2	0.0001261			

TABLE XVIII.—SPECIFIC RESISTANCES AND TEMPERATURE COEFFICIENTS

Substance	Resistance at 0°C. per centimeter cube	Mean temperature coefficient, 0–100°C.
	ohms	
Aluminum	2.906×10^{-6}	0.00435
Bismuth	108.000×10^{-6}	0.00450
Copper	1.584×10^{-6}	0.00420
Iron	9.696×10^{-6}	0.00625
Gold	2.088×10^{-6}	0.00377
Mercury	94.340×10^{-6}	0.00090
Nickel	12.350×10^{-6}	0.00622
Platinum	9.035×10^{-6}	0.00367
Silver	1.561×10^{-6}	0.00400
Tin	10.500×10^{-6}	0.00440
Zinc	5.750×10^{-6}	0.00406
German silver	20.890×10^{-6}	0.00027

TABLE XIX.—RESISTANCE OF SOFT OR ANNEALED COPPER WIRE

B & S gauge	Diameter in inches	Ohms per 1,000 ft. at 20°C. or 68°F.	B & S gauge	Diameter in inches	Ohms per 1,000 ft. at 20°C. or 68°F.
0000	0.46	0.04893	19	0.03589	8.038
000	0.40964	0.0617	20	0.031961	10.14
00	0.3648	0.0778	21	0.028462	12.78
0	0.32486	0.09811	22	0.025347	16.12
1	0.2893	0.1237	23	0.022571	20.32
2	0.25763	0.156	24	0.0201	25.63
3	0.22942	0.1967	25	0.0179	32.31
4	0.20431	0.248	26	0.01594	40.759
5	0.18194	0.3128	27	0.014195	51.38
6	0.16202	0.3944	28	0.012641	64.79
7	0.14428	0.4973	29	0.011257	81.7
8	0.12949	0.6217	30	0.010025	103.
9	0.11443	0.7908	31	0.008928	129.9
10	0.10189	0.9972	32	0.00795	163.8
11	0.090742	1.257	33	0.00708	206.6
12	0.080808	1.586	34	0.006305	260.5
13	0.071961	1.999	35	0.005615	328.4
14	0.064084	2.521	36	0.005	414.2
15	0.057068	3.179	37	0.004453	522.2
16	0.05082	4.009	38	0.003965	658.5
17	0.045257	5.055	39	0.003531	830.4
18	0.040303	6.374	40	0.003145	1047.

TABLE XX.—VELOCITY OF SOUND

Substance	Temperature °C.	Velocity per second	
		Meters	Feet
Air	0	331.8	1,088
Brass		3,500.	11,480
Steel	20	4,990.	16,360
Water	0	1,435.	4,710

TABLE XXI.—INDEX OF REFRACTION

Substance	Index of refraction for sodium *D*-line	Substance	Index c refraction sodium *D*-
Air	1.0002918	Glycerine	1.47
Carbon dioxide	1.000334	Water	1.333
Canada balsam	1.530	Crown glass	1.52
Ethyl alcohol	1.362	Flint glass	1.64
Carbon bisulphide	1.632	Heavy flint glass	1.9

TABLE XXII.—TRIGONOMETRIC FUNCTIONS

A	Sin	Cos	Tan	A	Sin	Cos	Ta
0	0.000	1.000	0.000	21	0.358	0.934	0.3
1	0.017	0.999	0.017	22	0.375	0.927	0.4
2	0.035	0.999	0.035	23	0.391	0.921	0.4
3	0.052	0.999	0.052	24	0.407	0.914	0.4
4	0.070	0.998	0.070	25	0.423	0.906	0.4
5	0.087	0.996	0.087	26	0.438	0.898	0.4
6	0.105	0.995	0.105	27	0.454	0.891	0.5
7	0.122	0.993	0.123	28	0.469	0.883	0.5
8	0.139	0.990	0.141	29	0.485	0.875	0.5
9	0.156	0.988	0.158	30	0.500	0.866	0.5
10	0.174	0.985	0.176				
				31	0.515	0.857	0.6
11	0.191	0.982	0.194	32	0.530	0.848	0.6
12	0.208	0.978	0.213	33	0.545	0.839	0.6
13	0.225	0.974	0.231	34	0.559	0.829	0.6
14	0.242	0.970	0.249	35	0.574	0.819	0.7
15	0.259	0.966	0.268	36	0.588	0.809	0.7
16	0.276	0.961	0.287	37	0.602	0.799	0.7
17	0.292	0.956	0.306	38	0.616	0.788	0.7
18	0.309	0.951	0.325	39	0.629	0.777	0.810
19	0.326	0.946	0.344	40	0.643	0.766	0.839
20	0.342	0.940	0.364				

TABLE XXII.—TRIGONOMETRIC FUNCTIONS—*Continued*

A	Sin	Cos	Tan	A	Sin	Cos	Tan
41	0.656	0.755	0.869	66	0.914	0.407	2.25
42	0.669	0.743	0.900	67	0.921	0.391	2.36
43	0.682	0.731	0.933	68	0.927	0.375	2.48
44	0.695	0.719	0.966	69	0.934	0.358	2.61
45	0.707	0.707	1.000	70	0.940	0.342	2.75
				71	0.946	0.326	2.90
46	0.719	0.695	1.04	72	0.951	0.309	3.08
47	0.731	0.682	1.07	73	0.956	0.292	3.27
48	0.743	0.669	1.11	74	0.961	0.276	3.49
49	0.755	0.656	1.15	75	0.966	0.259	3.73
50	0.766	0.643	1.19				
51	0.777	0.629	1.23	76	0.970	0.242	4.01
52	0.788	0.616	1.28	77	0.974	0.225	4.33
53	0.799	0.602	1.33	78	0.978	0.208	4.70
54	0.809	0.588	1.38	79	0.982	0.191	5.14
55	0.819	0.574	1.43	80	0.985	0.174	5.67
				81	0.988	0.156	6.31
56	0.829	0.559	1.48	82	0.990	0.139	7.12
57	0.839	0.545	1.54	83	0.993	0.122	8.14
58	0.848	0.530	1.60	84	0.995	0.105	9.51
59	0.857	0.515	1.66	85	0.996	0.087	11.43
60	0.866	0.500	1.73				
61	0.875	0.485	1.80	86	0.998	0.070	14.30
62	0.883	0.469	1.88	87	0.999	0.052	19.08
63	0.891	0.454	1.96	88	0.999	0.035	28.64
64	0.898	0.438	2.05	89	0.999	0.017	57.28
65	0.906	0.423	2.14	90	1.000	0.000	Infinity

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